Induction Motor Fault Detection and Diagnosis using KDE and Kullback-Leibler Divergence

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Abstract—The present paper proposes a novel data-driven Fault Detection and Diagnosis algorithm for induction motors based on Motor Current Signature Analysis. Principal Component Analysis is used to reduce the three-phase currents space in two dimensions. Then, Kernel Density Estimation is adopted to estimate the Probability Density Function of healthy and of each faulty motors, which will give typical patterns that can be used to identify each fault. Kullback-Leibler divergence is used as an index to identify the dissimilarity between two determined probability distributions, that allows the automatic identification of distinct fault types. Several simulations and experimental results are carried out using two benchmarks in order to verify the effectiveness of the proposed methodology: the first is used to prove appropriateness of the method for air gap eccentricity fault diagnosis and the second is used to prove suitability of the method for rotor broken bars and connectors fault diagnosis. Simulations and classification results prove that the proposed Fault Detection and Diagnosis procedure is able to detect and diagnose different induction motor fault types.

I. INTRODUCTION

Induction motors are the most important electric machineries in many industrial applications. In modern industrial world, the increasing demand for efficiency and quality has influenced production technology and management strategies [1]. Especially, drive systems conditions and reliability assessment are crucial to avoid unexpected failures. The issue of preventive maintenance and noninvasive diagnosis of induction motor conditions is of great concern. A maintenance system should be able to monitor the operating conditions of a machine, to issue warnings of possible faults, and predict the life span of a defective machine component before a fatal breakdown [2]. A sophisticated maintenance system should not only be able to detect the existence of faults and to identify the cause, but should also be able to forecast the future conditions of a machine with a deteriorating fault. [3] and [4]. In this context, the maintenance approach has changed substantially, moving its focus from fault Repair Maintenance (RM) to Preventive Maintenance (PM) [5]. Failure surveys report that failures, in induction motors, are: stator related (38%), rotor related (10%), bearing related (40%) and others (12%) [6]. A topic of great interest in industrial application concerns the identification of incipient (or early) faults. Fast and accurate diagnosis of these types of faults allows actions to protect the power system and the machine. Many investigations exploit methods for fault detection in induction motors, as highlighted below.

A. Bearing Faults

Some electrical machines use ball or rolling element bearings. Even under normal operating conditions, i.e. balanced load and good alignment, fatigue failures may take place [7]. Bearing problems are often connected with a wrong installation [8], this produces physical damage in the form of brinelling or false brinelling of raceways, which leads to premature failure. Bearing faults might exhibit themselves as rotor asymmetry faults, which are usually covered under the category of eccentricity related faults. Otherwise the ball bearing-related defects can be categorized as outer or inner bearing race defects, ball and train defects. The relationship between bearing vibration and stator current spectra is determined by the air gap eccentricity, that produces anomalies in the air gap flux density.

B. Stator or Armature Faults

Stator and armature faults are usually related to insulation failure. Many stator windings faults result in a gradual electrical insulation deterioration. They are known as phase-to-ground or phase-to-phase faults. It is believed that these faults start as undetected turn-to-turn faults that finally grow [9]. About 30% – 40% of all reported induction motor failures are included in this category. Armature or stator insulation can fail due to following reasons:

- high stator core or winding temperatures;
- slack core lamination, slot wedges and joints;
- partly conductive pollution.

Penman et al. [10] are able to detect turn-to-turn faults by analyzing the machine axial flux components using a large coil concentrically around the shaft of the machine. Toliyat and Lipo [11] have shown through both modeling and experimentation that these faults result in asymmetry in the machine impedance causing the machine to draw unbalanced phase currents.

C. Broken Rotor Bar and End-Ring Faults

These faults occurred in squirrel cage induction motors. During the course of running, starting or load changing or voltage fluctuation or torque oscillation, large current may flow through the bars or end ring and large heat will be generated in the end ring joints or bars. This thermal stress cracks or breaks end ring or bars. If the motor runs further, the large current will flow and excessive heat breaks the next bar as the previously broken bar being open. This type of fault may not show any
early symptoms, propagating to the next bars and leading to the sudden collapse unless continuous fault monitoring has been carried out. In [12]–[15], spectrum analysis of machine line current is used to detect broken bar faults. They investigated the sideband components $f_b$ around the fundamental frequency for detecting broken bar faults:

$$f_b = (1 \pm 2s)f$$  \hspace{1cm} (1)

while the lower sideband is specially due to a broken bar, the upper sideband is due to consequent speed oscillation. Broken bar detection using state and parameter estimation techniques have also been reported [16]. Interbar currents of uninsulated rotor cages reduce the magnetic imbalance caused by broken bars. This makes detection of broken bars more difficult, particularly at early stages.

D. Eccentricity Related Faults

Machine eccentricity is the condition of unequal air gap that exists between the stator and rotor. When eccentricity becomes large, the resulting unbalanced radial forces (also known as Unbalanced Magnetic Pull or UMP) can cause stator and rotor rub, and this can result in damage of the stator and rotor. There are two types of air gap eccentricity: the static and the dynamic, which exist simultaneously due to manufacturing tolerances and installation procedures. In the case of the static air gap eccentricity, the position of the minimal radial air gap length is fixed in space. In the case of dynamic eccentricity, the rotor centre is not in the shaft rotation centre and the minimum air gap revolves with the rotor [17]. Both static and dynamic eccentricities tend to co-exist, which is defined as mixed eccentricity (ME). Some of the main failures on these include, but are not limited to, broken bars, broken end-ring connectors and air gap static and dynamic eccentricities.

E. Proposed Contribution

Motor Current Signature Analysis (MCSA) monitoring strategy involves detection and identification of current signature patterns that are indicative of normal and abnormal motor conditions. Current measurements are used because in electric drives these are available by inverters. However, the motor current is influenced by many factors including: electric supply, static and dynamic load conditions, noise, motor geometry, and fault conditions. Authors focus attention on detect and diagnose levels of air gap eccentricity and broken bars and connectors in induction motors. In this approach the three-phase motor currents are measured in order to extract all statistic informations contained in these current signals. Principal Component Analysis (PCA) is used for data processing, it reduces the currents space in two dimensions. This is clear from the fact that each stator current is perfectly correlated to the sum between others. As a consequence the PCA reduce data to a two-dimensional space. Then Kernel Density Estimation (KDE) is applied to estimate the Probability Density Function (PDF) of PCA-transformed data. Finally Kullback-Leibler (K-L) divergence has been used as a measure of distance between classified statistic signatures obtained by KDE. K-L is an index that allows to identify the dissimilarity between two determined probability distributions (that can be multidimensional also), in this work one is related to the modeled signatures and the other is related to the acquired data samples. By K-L divergence, the classification of each motor conditions is performed. The paper is presented as follows. In Section II PCA, KDE and K-L divergence are briefly introduced respectively. In Section III the procedures for Fault Detection and Diagnosis training and monitoring are discussed. The considered benchmarks are described in Section IV. Results are then reported and discussed in Section V. Final remarks and possible future research prospectives are given at the end of the paper.

II. RECALLED RESULTS

A. Principal Component Analysis

PCA is a dimensionality reduction technique, it produces a lower-dimensional representation in a way that preserves the correlation structure between the process variables, and it can capture the variability in the data. In PCA, the correlation among sensors is used to transform the multivariate space into a subspace which preserves maximum variance of the original space in minimum number of dimensions. In other words, PCA rotates the original coordinate system along the direction of maximum variance. Considering a data matrix $X \in \mathbb{R}^{N \times m}$ of $N$ sample rows and $m$ variable columns that are normalized to zero mean, with mean values vector $\mu$, the matrix $X$ can be decomposed as follows:

$$X = \hat{X} + \tilde{X},$$  \hspace{1cm} (2)

where $\hat{X}$ is the projection on the Principal Component Subspace (PCS) $S_p$, and $\tilde{X}$, the residual matrix, is the projection on the Residual Subspace (RS) $S_r$ [18]. Defining the loading matrix $P$, whose columns are the right singular vectors of $X$, and selecting the columns of the loading matrix $P \in \mathbb{R}^{m\times p}$, which correspond to the loading vector associated with the first $p$ singular values, it follows that:

$$\tilde{X} = XP\tilde{P}^T \in S_r,$$  \hspace{1cm} (3)

The residuals matrix $\tilde{X}$ is the difference between the data matrix $X$ and its projection into the first $p$ principal components retained in the PCA model:

$$\tilde{X} = X(I - PP^T) \in S_r,$$  \hspace{1cm} (4)

therefore the residual matrix captures the variations in the observations space spanned by the loading vectors associated with the $r = m - p$ smallest singular values. The projections of the observations in $X$ into the lower-dimensional space are contained in the score matrix:

$$T = XP \in \mathbb{R}^{N \times p}.$$  \hspace{1cm} (5)

A new observation vector $x \in \mathbb{R}^m$ can be projected into the lower-dimensional score space with:

$$t = xP \in \mathbb{R}^p.$$  \hspace{1cm} (6)

In the case of healthy motor, with three-phase without neutral connection, ideal conditions for the motor and a balanced voltage supply, the stator currents are given by Eq. 7, where $i_a, i_b$ and $i_c$ denote the three stator currents, $I_{max}$ their maximum value, $f$ their frequency, $\phi$ their phase angle and $t$ denotes the...
time.
\[
\begin{align*}
ia(t) &= I_{\text{max}} \sin(2\pi ft - \phi) \\
ib(t) &= I_{\text{max}} \sin(2\pi ft - 2\pi/3 - \phi) \\
ic(t) &= I_{\text{max}} \sin(2\pi ft - 4\pi/3 - \phi).
\end{align*}
\]  

The PCA transform (5), applied to the signals in Eq. (7), makes the smallest singular values equal to zero. This imply that the information of the principal component, captured by the smallest singular values, is null then the last principal component could be deleted and the original space reduced from three to two without miss information. This is justified by the fact that in Eq. 7, each stator current is perfectly correlated to the sum of the others. Adding Gaussian white noise, with standard deviation $\sigma$, to the stator current signals (7), the smallest singular values will not be equal to zero, but it will depend by the ratio between $I_{\text{max}}$ and $\sigma$. In the present work, in which $I_{\text{max}}$ is 6.22 $A$ and $\sigma$ is 0.4, the average percentage information, among all motor conditions, is 0.64% with standard deviation 0.25 and maximum value 1.64%.

B. Kernel Density Estimation

Given $X = [x_1, x_2, \ldots, x_N]$, be $N$ independent and identically distributed (i.i.d) random vectors, whose distribution function $F(y) = P[X \leq y]$ is absolutely continuous,
\[
F(y) = \int_{-\infty}^{y} f(y')dy',
\]
with unknown PDF $f(y)$ and considering the Parzen window as kernel function, the estimated density at $y$ is given by [19]:
\[
f(y) = \frac{1}{N} \sum_{j=1}^{N} \frac{1}{h^D} K \left( \frac{y-x_j}{h} \right),
\]
where:
\[
K \left( \frac{y-x_j}{h} \right) = \begin{cases} 
1 & \left| \frac{y-x_j}{h} \right| \leq 1/2 \\
0 & \text{otherwise}
\end{cases}.
\]

In the present study a Gaussian kernel function is used so that, the estimated density $f(y)$ becomes:
\[
f(y) = \frac{1}{N} \sum_{j=1}^{N} \frac{1}{(2\pi h^2)^{1/2}} e^{-\frac{||y-x_j||^2}{2h^2}}.
\]

where $y \in \mathbb{R}^{n_{\text{grid}}}$ and $n_{\text{grid}}$ is the number points in which the PDF is estimated. It is well known that the value of the bandwidth $h$ is of critical importance, while the shape of the kernel function has little practical impact [20]. In many computational-intelligence methods that employ KDE, the problem to find the appropriate bandwidth $h$ is the issue [20–25]. In the present work the Asymptotic Mean Integrated Squared Error (AMISE) with plug-in bandwidth selection procedure is used to choose automatically the bandwidth $h$ [21]. In the proposed algorithm, KDE is used to model a specific pattern for each motor condition. As shown in Fig. 1 the frequency features of the current signals are mapped in the two-dimensional principal components space, representing a specific signature of the motor condition. The Fig. 1(a), 1(b) and 1(c) represent the patterns of three motor conditions: healthy, one broken bar and mixed eccentricity; these figures show as the PDFs, estimated by KDE in the principal compo-


C. Kullback-Leibler distance

Given two continuous PDFs $f_1(x)$ and $f_2(x)$ a measure of “distance” or “divergence” between $f_1(x)$ versus $f_2(x)$ is given by [26]:
\[
I_{1:2}(X) = \int_{-\infty}^{\infty} f_1(x) \log \frac{f_1(x)}{f_2(x)} dx,
\]
and between $f_2(x)$ versus $f_1(x)$ is given by:
\[
I_{2:1}(X) = \int_{-\infty}^{\infty} f_2(x) \log \frac{f_2(x)}{f_1(x)} dx.
\]

Therefore the Kullback-Leibler (K-L) divergence between $f_1(x)$ and $f_2(x)$ is:
\[
J(f_1; f_2) = I_{1:2}(X) + I_{2:1}(X) = \int_{-\infty}^{\infty} (f_1(x) - f_2(x)) \log \frac{f_1(x)}{f_2(x)} dx.
\]

The above equation is known as the symmetric K-L divergence, which represents a non negative measure between two PDFs. In the present work, K-L divergence is used as a fault index therefore, defining $f_0$ as the PDF in the PCs space estimated by KDE of the oncoming current measurements, the present motor condition is that which minimizes the K-L distance between $f_0$ and the $i$-th PDF $f_i$ related to different motor conditions:
\[
c = \arg \min_i J(f_0; f_i),
\]
where $c$ is the classification output.

III. DEVELOPED ALGORITHM

The developed Fault Detection and Diagnosis (FDD) procedure based on KDE consists of two stages: training and FDD monitoring. In the first, one KDE model is computed by feature signals for each motor condition, in order to have one KDE model in the case of healthy motor and one KDE model for each faulty case. The training steps are summarized below:

T1. Stator current signals for each motor condition are acquired;
T2. PCA transform (5) is applied to stator current signals, which are projected into the two-dimensional principal components space;
T3. The matrices $P$ and $\mu$ are stored;
T4. KDE is performed on the lower-dimensional principal components space (5) using a grid of $n_{\text{grid}}$ points and a bandwidth $h$ for the Gaussian kernel function (11);
T5. PDFs estimated by KDE (11) are stored.

In diagnosis step, the models previously obtained are compared with the new data and a fault statistical index is calculated. The diagnosis steps are summarized below:

D1. Stator current signals are acquired;
D2. The matrices $P$ and $\mu$, previously computed (T3), are applied to signals;
D3. KDE is performed on the lower-dimensional principal components space (5) using the same point grid \( n_{grid} \) and bandwidth \( h \) used in the training step (T4);

D4. Symmetric K-L divergence (14) is computed between the PDF, estimated by KDE using the acquired current signals (11), and those stored in the training step (one for each condition) (T5);

D5. Diagnosis is evaluated using Eq. (15).

Faults are identified using the Eq. (14) where \( f_1 \) is the PDF in the PCs space, estimated by KDE, of the oncoming current measurements and \( f_2 \) is the PDF of the healthy motor. K-L divergence is used as an input for fault decision algorithm allowing to take decision automatically on the operating state and condition of the machine and detecting any abnormal operating conditions.

IV. Case Study

Several simulations are carried out using two benchmarks in order to verify the effectiveness of the proposed methodology. The first is a mathematical stator current signal model from which simulated current signals are generated. Given in input to the model the current signals, the model in [27] is used to estimate the fault characteristic frequencies for air gap eccentricity, such as the fault frequency and the sideband amplitude. In the present work this model is inverted to generate the current signals for air gap eccentricity, given in input to the model the characteristic frequencies, which are listed in [27] for eccentricity fault type. The second uses a Time Stepping Coupled Finite Element-State Space modeling approach to generate current signals for induction motors as described in [28]–[32]. The simulation dataset consists of twenty-one different motor conditions, which are one healthy condition, ten broken bars conditions and ten broken connectors conditions. Twenty time series are generated for each motor condition. Each signal consists of 1500 samples. The dataset can be downloaded from UCR time series data mining archive [33]. The characteristics of the three-phase induction motor dataset are: input voltage 208 V, frequency 60 Hz, number of rotor bars 34, pole number 2 and power 1.2 hp. The results are presented in two subsections, because two different benchmarks are used to generate datasets.

V. Results

The approach proposed is tested by broken rotor bars, broken connectors and air gap eccentricity faults. The faults are implemented and stator currents are retrieved at 10 kHz sampling rate for 2 s in the case of air gap eccentricity and at 33.3 kHz sampling rate for 0.1 s in the case of broken rotor bars and connectors. Then, three-phase stator currents are processed by PCA and KDE in order to model the motor faults and afterwards fault detection and diagnosis is implemented by K-L divergence as described in section III. The results are the average of 100 Monte Carlo simulations where the training and testing data are changed. Gaussian white noise with standard deviation \( \sigma = 0.4 \) and zero mean is added to current signals.

A. Air Gap Eccentricity Diagnosis

In the following, the simulation results given in Fig. 2 show the faults diagnosis for air gap eccentricity, where Fig. 2(a) shows the K-L divergence among the modeled PDFs, of all motor conditions (i.e. healthy, static eccentricity, dynamic eccentricity and mixed eccentricity) and the PDF estimated by KDE from stator current signals of healthy motor. The results show as the minimum K-L distance is exactly the healthy condition. Fig. 2(b) shows the K-L divergence among all PDFs and the PDF estimated from stator current signals affected by air gap static eccentricity. In this case the graph shows as the minimum K-L distance is exactly the static eccentricity condition. From Figs 2(c) and 2(d) can be noticed that dynamic and mixed eccentricity are correctly identified also. By Monte Carlo simulations, all the fault types are diagnosed with 100% accuracy.

B. Broken Rotor Bars and Connectors Diagnosis

In the following, the simulation results given in Fig. 3 show the faults diagnosis for broken rotor bars and connectors, where Fig. 3(a) shows the K-L divergence among the PDFs,
estimated by KDE, of all motor conditions (i.e. healthy, from one to ten broken rotor bars and from one to ten broken connectors) and the PDF estimated by KDE from stator current signals of healthy motor. The results show as the minimum K-L distance is exactly the healthy condition. Fig. 3(b) shows the K-L divergence among all PDFs and the PDF estimated from stator current signals affected by one broken rotor bar.

In this case the graph shows as the minimum K-L distance is exactly the broken bar condition. The last graph, Fig. 3(c), shows the one broken connector diagnosis. Even in this case the K-L divergence detects and identifies the fault, that is one broken connector. It can be noticed by the Fig. 3 as the K-L divergence varies in proportion to the number of broken rotor bars and broken connectors. By Monte Carlo simulations, all fault types are diagnosed with 100% accuracy hence the K-L divergence figures for the others fault types are not reported.

VI. CONCLUSION

Fault diagnosis systems for induction motors based on Motor Current Signature Analysis are intensively researched in the literature. This paper proposes a novel data-driven Fault Detection and Diagnosis algorithm that uses PCA and KDE to assess the two-dimensional PDF, in order to identify the stator current state space patterns of an healthy motor and...
of each fault type. K-L divergence is used as a fault index allowing the identification of distinct types of faults. Using this approach, it is possible to easily typify the distinct patterns for each working condition. Simulations are presented revealing the capabilities of the proposed system and as the proposed fault diagnosis algorithm can correctly and efficiently diagnose multi-class faults for air gap eccentricity, broken rotor bars and broken connectors. The fault classification is of 100% and the proposed algorithm enhances the safety of induction motor and can reduce the maintenance cost.

Authors are currently considering three possible future developments for the fault diagnosis algorithm based on KDE. The first is related to the extension of the algorithm to other electrical machines and fault types. The second deals with the algorithm testing by means of current signals measured by real induction motor. The last is related to the extension of the algorithm to on-line fault detection and diagnosis procedure to avoid one of the major drawback of the algorithm which concerns the data batch processing because it needs to acquire several current samples for the fault diagnosis procedure.

REFERENCES


