Automatic improvement of unstructured grids for coastal simulations

A.B. Fortunato†, N. Bruneau†, A. Azevedo†, M.A.V.C. Araújo‡, and A. Oliveira†
† Departamento de Hidráulica e Ambiente
Laboratório Nacional de Engenharia Civil
Avenida do Brasil, 101
1700-066 Lisboa
‡ Departamento de Engenharia Civil e Arquitectura
Instituto Superior Técnico
Av. Rovisco Pais, 1
1049-001 Lisboa

ABSTRACT

The generation of unstructured triangular grids for coastal applications requires extensive manual tuning, in order to improve the stability, accuracy and efficiency of the model simulations. As current grids reach $10^5$-$10^6$ nodes, this manual tuning is no longer feasible. This paper presents a new post-processor that aims at optimizing the local distribution of nodes and elements by adding and deleting nodes and by changing the connections between nodes. The algorithms target a smooth transition between elements sizes by making the number of connections of each internal node as close to 6 as possible. An application of the post-processor to various grids shows that the grid roughness is reduced by 10-30%, while decreasing marginally the number of nodes and maintaining the global nodal distribution. To illustrate the benefits of the post-processor, a grid with 130,000 nodes of the maritime zone under Portuguese jurisdiction is used to simulate tides with a good accuracy (root mean square errors below 10 cm) and without numerical oscillations. The model results provide an extensive database of tidal elevations that can feed boundary conditions to local estuarine models.

ADDITIONAL INDEX WORDS: Tidal model, Portuguese shelf, grid generation

INTRODUCTION
Coastal ocean models based on unstructured grids are becoming increasingly popular because they offer the possibility for local grid refinement and cross-scale simulations. For instance, the generation, propagation and landfall of a hurricane storm surge can be simulated with a single grid, with horizontal resolutions spanning 3 orders of magnitude (Tanaka et al., 2011). Taking advantage of these possibilities, we have developed a regional tidal model for the maritime zone under Portuguese jurisdiction. This model provides a detailed database of tidal constituents to specify boundary conditions for local estuarine studies, and constitutes a stepping stone towards storm surge simulations in the Portuguese coast. Relative to previous model applications to the same area (Fortunato et al., 2002), the new model provides a larger spatial coverage, finer grid resolution in the shelf, and the possibility to represent numerous islands in the Atlantic (Azores, Madeira and Canary islands).

The advantages of unstructured grids, however, come with a price: grid generation, even using semi-automatic grid generators, can still be extremely time-consuming, as small manual adjustments are necessary to improve the quality of the grids. The distribution of nodes in ocean model applications with unstructured grids follows several general guidelines. Globally, the nodal density should be related to the wave celerity, the expected frequency content of the signal, and both bathymetric and velocity gradients (e.g., Westerink et al., 1994). Several grid generators are available to build grids with appropriate nodal distributions through automated or semi-automated procedures (e.g., Hagen et al., 2006). Locally, the nodal density should vary smoothly, in order to minimize truncation errors, which affect the accuracy and stability of the simulations. For grids based on triangles, a smooth variation of the element sizes can be achieved by forcing each internal node to be connected to about 6 other nodes. Indeed, the truncation error for the 2D shallow water equations is smaller for nodes belonging to 6 elements than for nodes belonging to 4 or 8 elements (Hagen et al., 2001). Having each internal node connected to 6 other nodes leads to triangles that are approximately equilateral and allows smooth transitions between element sizes. In addition, reducing the maximum number of neighboring nodes also decreases the memory requirements, because the bandwidth of the mass matrices in finite element models is usually associated to the maximum number of connections between nodes.

However, as grids for ocean applications reach $10^5$-$10^6$ nodes, manually adjusting the elements and nodes to optimize the nodal distribution becomes unfeasible. Hence, a grid post-processor was developed to optimize the local distribution of nodes and elements by adding and deleting nodes and by changing the connections between nodes, without affecting the global nodal distribution. This paper is divided into three sections besides this introduction. Section 2 describes algorithms of the post-processor and illustrates them with simple grids. An application of the post-processor to the regional tidal model grid is described in section 3, together with selected results from the model. Applications of the post-processor to other grids confirm the generality of its performance. Section 4 summarizes the main conclusions.
**ALGORITHMS**

The post-processor changes the input grids such that each internal node is connected to about 6 other nodes, with 5 and 7 connections as the target minimum and maximum values, respectively. The grids are improved by deleting and adding interior nodes, or by changing the connections between nodes. In addition, straight boundaries are checked for the existence of nodes that belong to only two elements. These nodes are eliminated and the adjoining nodes are moved along the boundary to minimize the element size variation along the boundary. This procedure reduces the occurrence of elements with internal angles above 90º, which are unacceptable in some model formulations (e.g., Zhang et al., 2004).

Because these procedures can create elements with undesirable characteristics, such as large internal angles, the position of each node is adjusted by setting its coordinates equal to the average of the coordinates of all the nodes to which it is connected. This iterative procedure converges in about 10-20 iterations.

The general algorithm, which can be applied to the whole grid or only part of it, proceeds as follows.

1. Boundary nodes along straight boundaries (i.e., with angles between 170º and 190º) are checked for the number of elements. The nodes that belong to only 2 elements (Figure 1a) are deleted.

2. Interior nodes that belong to only 3 elements are identified. If the number of elements belonging to those nodes can be increased by changing the connection with an opposing node without generating nodes attached to more than 6 other nodes, the connections are changed; otherwise the node is deleted (Figure 1b).

3. Step 2 is repeated for interior nodes belonging to 4 elements (Figure 1c).

4. Interiors nodes belonging to 5 elements are identified, together with the opposing node (i.e., node for which one of the elements shares an edge) with the smaller number of connections.

   a. If this number is smaller than 6, and if the connection between these two nodes can be changed such that the number of connections of the other nodes affected is improved, this connection is changed (Figure 1d).

   b. If one of the adjoining nodes also has 5 connections, and the two nodes that share an element with these two nodes are connected to more than 6 nodes, the node is deleted and the elements are rearranged accordingly (Figure 1e).

5. Interiors nodes belonging to 8 elements are identified, together with the adjoining node with the larger number of connections. If this adjoining node has more than 7 connections, a new node is added between the two and the elements are rearranged accordingly. Because Step 5 can produce new nodes with fewer than 5 connections, the number of connections of the nodes is verified, and the algorithm returns to steps 2 or 3 if needed.

Nodes belonging to more than 8 elements are not explicitly addressed. However, experience indicates that these cases are addressed in steps 2-4, as nodes belonging to many elements are usually associated to other nodes with few connections (Figure 1f). Hence, steps 2-4 of the algorithm end up correcting most of the cases with nodes with more than 7 connections as well.

**TESTING AND APPLICATION**

The regional tidal model

A regional tidal model of the maritime area under Portuguese jurisdiction was developed. The model selected is SELFE, a 3D baroclinic community model developed at the Center for Coastal Margin Observation and Prediction (Zhang and Baptista, 2008). SELFE solves the shallow water equations using triangular finite elements in the horizontal and hybrid Z-S coordinates in the vertical. For regional-scale applications, as the one presented herein, the equations are solved in spherical coordinates following...
Comblen et al. (2009). SELFE is semi-implicit and fully parallelized (http://www.stccmop.org/CORIE/modeling/selfe/). The domain extends from 26ºN to 46.5ºN, and from 32ºW to the European and African coastlines (Figure 2). Part of the Mediterranean Sea (up to 3.6ºW) is included in the domain to facilitate the specification of boundary conditions. The model is forced inside the domain by the tidal potential and at the boundaries by results from the global model of Eanes and Bettadpur (1996). One-year time series of elevations from the global model were harmonically analyzed to extract the 12 major tidal constituents (4 low-frequency, 4 diurnal, 4 semi-diurnal). Similarly, one-year simulations were performed with the regional model to allow the extraction of these constituents and their major harmonics through harmonic analysis. Bathymetric data are taken from a variety of sources, including the world database of the Institute of Geophysics and Planetary Physics (Smith and Sandwell, 1997), with a resolution of 1' and hydrographic charts in the Portuguese and Moroccan shelves. Land boundaries were defined using data from the Portuguese Navy (Instituto Hidrográfico) and from the World Vector Shoreline database developed by the U.S. Geological Survey. The major parameters are set as follows: time step: 300 s; turbulence closure scheme: Kantha and Clayson; bottom stress coefficient: 0.002; minimum depth: 5 m. Horizontal diffusion was neglected, and preliminary runs showed that results are insensitive to the friction coefficient. The model was not calibrated.

**Grid generation**

The grid was initially generated with the Surface Water Modeling System (SMS), and manually improved with GREDIT (Turner and Baptista, 1993). The grid has about 130,000 nodes, with resolutions between 250 m in the Portuguese coast and 40 km in the deep Atlantic Ocean. Various features, such as islands, inlets and the Strait of Gibraltar, generate rapid element size variations. The post-processor was applied to this initial grid, resulting in 2441 operations (the elimination of 530 boundary nodes and 44 interior nodes, and the swapping of 1867 connections between nodes). No nodes were added because the manual improvement of the grid, done before the post-processor had been developed, had already corrected the parts of the grid that could benefit from the addition of nodes. Table 1 compares the characteristics of the initial and final grids. A roughness index of the grid is defined herein as the average deviation between the number of elements attached to each interior node and the optimal value of 6, i.e.:

\[
RI = \frac{1}{\text{nodes}_{i}} \sum_{j=1}^{\text{nodes}_{j}} |NE_{j} - 6|
\]

where \(\text{nodes}_{i}\) is the number of interior nodes, and \(NE_{j}\) is the number of neighbors of interior node \(j\). \(RI\) is equal to 0 for grids in which all the interior nodes belong to 6 elements, and grows as the number of interior nodes belonging to more or less than 6 elements increases.

Figure 3 shows the final grid. The post-processor reduces marginally the size of the grid, but makes it significantly smoother; the roughness index decreases by 30%, internal nodes belonging to 4 or 9 elements are eliminated, and those belonging to 8 elements are reduced from 108 to 5. Some nodes belonging to 8 elements remain, as some cases are not addressed by the code. While the grid is locally improved, the global nodal distribution remains unchanged (Figure 4).

To verify the generality of this assessment, the post-processor was applied to five grids used in previously published applications. Results confirm that the post-processor reduces marginally the number of nodes, and improves significantly the smoothness of
Improvement of unstructured grids

The grid (Table 2). This improvement is smaller than the one obtained for the regional model grid because the previous grids have fewer nodes (about $10^4$), thus their manual improvement was more thorough. In all cases, the maximum number of neighbors was maintained or reduced.

Table 1. Changes in the characteristics of the regional model grid made by the post-processor.

<table>
<thead>
<tr>
<th></th>
<th>Initial grid</th>
<th>Final Grid</th>
<th>Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nodes</td>
<td>128223</td>
<td>127649</td>
<td>-0.4</td>
</tr>
<tr>
<td>Elements</td>
<td>247909</td>
<td>247291</td>
<td>-0.2</td>
</tr>
<tr>
<td>Neighbors (min.)</td>
<td>4</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>Neighbors (max.)</td>
<td>9</td>
<td>8</td>
<td>-11</td>
</tr>
<tr>
<td>Roughness index</td>
<td>0.13</td>
<td>0.09</td>
<td>-30</td>
</tr>
</tbody>
</table>

Table 2. Changes in the characteristics of various grids made by the post-processor.

<table>
<thead>
<tr>
<th>System</th>
<th>Reference</th>
<th>Number of nodes change</th>
<th>Roughness index change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tagus estuary</td>
<td>Fortunato et al., 1999</td>
<td>-0.3 %</td>
<td>-13.6 %</td>
</tr>
<tr>
<td>Guadiana estuary</td>
<td>Oliveira et al., 2006a</td>
<td>0.0 %</td>
<td>-12.1 %</td>
</tr>
<tr>
<td>Óbidos lagoon</td>
<td>Oliveira et al., 2006b</td>
<td>-0.2 %</td>
<td>-12.5 %</td>
</tr>
<tr>
<td>Formosa lagoon</td>
<td>Dias et al., 2009</td>
<td>0.0 %</td>
<td>-21.2 %</td>
</tr>
<tr>
<td>Portuguese shelf</td>
<td>Fortunato et al., 2002</td>
<td>-0.7 %</td>
<td>-9.0 %</td>
</tr>
</tbody>
</table>

Results

Root mean square errors were computed at 25 stations (Figure 2), for time series synthesized with the five major tidal constituents ($O_1$, $K_1$, $M_2$, $S_2$ and $N_2$). The model is able to reproduce tidal propagation correctly with root mean square errors typically below 10 cm (Figure 5). A co-tidal chart of the $M_2$ indicates the model is also free of spurious oscillations (Figure 5).

CONCLUSIONS

Generating large finite element grids for coastal ocean application still entails extensive manual tuning. This tuning smoothes the grid size transition, thus reducing the truncation errors and improving the stability and accuracy of the simulations. Furthermore, decreasing the maximum number of nodal connections also reduces the memory requirements. This paper describes the implementation of an algorithm to automate the grid improvement, allowing the rapid treatment of large computational grids. The application of the algorithm to a grid of the maritime area under Portuguese jurisdiction with $10^4$ nodes resulted in 2000 operations (elimination of nodes and switching of node connections). A grid roughness index, defined herein, was reduced by 30%. Application of the post-processor to grids used in previously published applications confirms the marginal reduction of the number of nodes, and the significant improvement of the grid smoothness.

The results obtained with the regional grid treated with the grid post-processing tool showed the current ability to simulate tides in a large domain with a good accuracy and without numerical
oscillations. The model results provide an extensive database of tidal elevations that can feed boundary conditions to local estuarine models, with smaller errors than previous applications.

REFERENCES


Oliveira, A., Fortunato, A.B. and L. Pinto, L., 2006a. Modeling the hydrodynamics and the fate of passive and active tracers in the Guadiana estuary, Estuarine, Coastal and Shelf Science, 70, 76-84.

Oliveira, A., Fortunato, A.B. and Rego, J., 2006b. Effect of morphological changes on the hydrodynamics and flushing properties of the Óbidos lagoon, Continental Shelf Research, 26, 917-942.


ACKNOWLEDGEMENTS

This work was partially funded by the Fundação para a Ciência e a Tecnologia (FCT), the programa operacional “Ciência, Tecnologia, Inovação” and FEDER, project G-CAST: Application of GRID-computing in a coastal morphodynamics nowcast-forecast system (GRID/GRI/81733/2006). A. Azevedo and M.A.V.C. Araújo were also partly funded by FCT through post-doctoral fellowships (SFRH/BPD/73089/2010 and SFRH/BPD/43220/2008). We thank Profs. Joseph Zhang and António Baptista for the model SELFE and guidance on its use, and Prof. Rick Luettich for the harmonic analysis routines.