Abstract—In this study, the bit-rate and power-bandwidth (P-B) efficiency of wireless adaptive feedback communication systems (AFCS) are considered. The analysis is based on the results of AFCS optimization [1, 2]. It is shown that the capacity of optimal AFCS is equal to that of the forward channel in the initial $n^*$ cycles of the samples transmission, and decreases for $n > n^*$. Transmission in the “threshold” number of cycles $n^*$ provides “ideal” P-B trade-off and full utilization of the resources of the system.

Index Terms—Adaptive modulation, analog transmission, capacity, feedback systems, power-bandwidth efficiency.

I. INTRODUCTION

This study presents an analysis of performance and power-bandwidth (P-B) efficiency of optimal point-to-point wireless adaptive feedback communication systems (AFCS) considered as the generalized communication channel. The particularity of these systems (“analogue” AFCS) is the lack of digitizing and coding units in the peripheral transmitting unit (TU), which are replaced by the adaptive pulse-amplitude (PAM) modulator $\Sigma + M_1$ (see block-diagram in Fig. 1). The input signal $x_t$ is sampled in the sample-and-hold unit (S&H) and each sample $x$ is transmitted in $n$ cycles in the same way and independently of the previous ones. In each cycle, the base station (BS) demodulates the received signal in DM1 unit. The demodulated signal $\tilde{y}_k$, $(k = 1, \ldots, n)$ is routed to the digital signal processing unit (DSPU), which computes the corrected estimate $\hat{x}_k$ of the input sample $x$. Simultaneously, the DSPU computes new values of the control signals $B_k, M_k$ transmitted to the TU via feedback channel T2-Ch2-R2. Both forward and feedback channels Ch1, Ch2 are memoryless channels with additive white Gaussian noises (AWGN).

The method of feedback transmission (digital or analog) and the architecture of transmitter T2 and receiver R2 are not specified, and their choice depends on the AFCS performance and deployment requirements. The controls $B_k, M_k$ reset the adjusted parameters of the modulator. Simultaneously, parameters of the signal processing algorithm in DSPU are set to new values, and the next cycle of the sample transmission begins. After $n$ cycles, the final estimate of the sample $\hat{x}_n$ is routed to the addressee, and the AFCS begins transmission of the next sample.

The described transmission scheme permits to solve a number of actual tasks of the AFCS theory and design. First, it permits to define the MSE of transmission and perform concurrent optimization of the transmitting and receiving parts of AFCS.

This task was solved in [1, 2] using the approach presented in [3], under assumption that the signals are Gaussian with limited base-band and that the overmodulation probability does not exceed a given small value $\mu$.

The obtained relationships contain the information permitting to design the optimal AFCS which could transmit the signals from the TU to BS with minimal mean square errors (MMSE). These relationships include the explicit expression for MMSE as a function of main factors influencing the quality of transmission. Analysis of MMSE has allowed obtaining a series of new general results important for the wireless AFCS optimal design and applications [1, 2]. It has been established that the bit-rate $R_{\text{Ch1}}$ in the forward channel of the optimal AFCS attains a value equal to the channel capacity $C$, and both $R_{\text{Ch1}}$ and $C$ do not depend on the feedback transmission errors and number of cycles of the sample transmission. The P-B efficiency of transmission through the forward channel $M_1$-Ch1-DM1, being assessed as the point $(E_{\text{bit}}/N_\xi, R_{\text{Ch1}}/F_0)$, attains Shannon boundary determined by the relationship ([4], see also Fig. 4):

$$
\frac{E_{\text{bit}}}{N_\xi} = \frac{F_0}{C} \left( 2^\frac{C}{F_0} - 1 \right)
$$

which expresses the ideal P-B trade-off in communication systems, $E_{\text{bit}}$ is the energy per bit of the signal received by BS, $N_\xi/2$ is the double-side spectral power density of forward channel noise, and $2F_0$ is the channel bandwidth. This result suggests that the AFCS designed according to the solution of the optimization task ensures full utilization of the resources of the TU and forward channel, and the system transmits the signals with theoretically achievable accuracy, bit-rate, and efficiency.

However, further research shows that the bit-rate at the optimal AFCS output is equal to the forward channel capacity only during the definite “threshold” number $n^*$ of the initial cycles of sample transmission and quickly diminishes when this number is exceeded. In a similar way, the P-B efficiency of transmission for AFCS as a whole (as the generalized channel) attains Shannon’s boundary (1) only for $1 \leq n \leq n^*$. For $n > n^*$, the P-B efficiency of AFCS monotonically decreases. This effect was not studied earlier, and should be taken into account in practical design of optimal or close-to-optimal...
AFCS. To explain its origin, brief description, formulation, and solution of the optimization task are given in Section 2. In Sections 3 and 4, the bit-rate and capacity of the forward channel and AFCS as a whole are discussed. The concluding remarks are given in Section 5.

II. FORMULATION AND SOLUTION OF OPTIMIZATION TASK

We assume that the input signal $x_t$ is a stationary Gaussian process with mean value $x_0$, variance $\sigma_0^2$, and base-band 2F. Each sample $x^{(m)} = x(m/2F)$ formed by the S&H unit $(m = 1, 2, \ldots)$ is transmitted independently of the previous samples in $n = T/\Delta t_0 = F_0/F$ cycles, each of duration $\Delta t_0 = 1/2F_0$. Under these conditions, analysis of AFCS can be reduced to the consideration of a single sample transmission.

To simplify further calculations, we consider the double-side band suppressed carrier (DSB-SC) adaptive modulator $\Sigma + M1$. Taking into account its possible overmodulation or saturation of the transmitter) during the sample transmission, we use the nonlinear model of modulator presented in Fig. 2. This model enables direct consideration of abnormal errors caused by overmodulation and permits to find the method of their elimination. Adaptive adjustment of modulator is realized by setting the parameters, in each cycle of the sample transmission, to the values $\hat{B}_k, \hat{M}_k$ referring to the control signals $B_{k-1}, M_{k-1}$ computed in the DSPU of the BS and delivered to TU through the feedback channel. The controls $B_{k-1} = B(y_{k-1}^1)$; $M_{k-1} = M_{k-1}(y_{k-1}^1)$ are computed using sequences of the signals $\hat{y}_{k-1}^1 = \hat{y}_{k-1}$ formed by demodulator DM1 in previous cycles. Under these assumptions, the signal emitted by TU in cycle $k$ can be written in the form:

$$s_{t,k} = A_0 \begin{cases} M_k(x - \hat{B}_k) & \text{if } M_k(x - \hat{B}_k) \leq 1 \\ \text{sign}(x - \hat{B}_k) & \text{if } M_k(x - \hat{B}_k) > 1 \end{cases} \cos(2\pi f_0 t + \phi_k) + \xi_t,$$  

where $(k-1)\Delta t_0 \leq t \leq k\Delta t_0$ and $A_0, f_0, \phi_k$ are parameters of the carrier. The initial values $B_0, M_0$ depend on the distribution of input signals and noise, and are computed and set independently.

Overmodulation and abnormal errors cause loss of information about the sample and crucially degrade the quality of transmission. However, these errors can be practically eliminated if the samples are transmitted under the statistical fitting condition (1–3) formulated as follows:

**Statistical fitting condition:** For each cycle of transmission $k = 1, \ldots, n$, parameters $\hat{B}_k, \hat{M}_k$ of the modulator should have the values guaranteeing that the probability of overmodulation $P_r^{over}$ will not exceed a given small value $\mu$:

$$P_r^{over} = \Pr(\hat{M}_k | x - \hat{B}_k | > 1 | y_{k-1}^1, B_0^{k-1}, M_0^{k-1}) < \mu \quad (3)$$

where $B_0^{k-1}, M_0^{k-1}$ are the controls computed by DSPU in previous cycles, and their values do not violate (3) for each $k = 1, \ldots, n$. The value $\mu \ll 1$ is a permissible probability of overmodulation specified according to the requirements for the quality of transmission (in practice, $10^{-12} \leq \mu \leq 10^{-4}$).

Formula (3) determines the conditional sets $\Omega^{k-1}_k(y_{k-1}^1)$ of the controls $B_0^{k-1}, M_0^{k-1}$ (and “permissible” parameters $\hat{B}_k, \hat{M}_k$), which exclude, for each $k = 1, \ldots, n$, violation of the inequality $\hat{M}_k | x - \hat{B}_k | \leq 1$ with a probability not smaller than $1 - \mu$. In this case, the probability of distorted sample emission is $n\mu(1-\mu)^n - n\mu$, and the overwhelming part of the samples in the transmitted sequences will be transmitted without overmodulation. This indicates that the statistically fitted modulator works practically all the time as the linear unit, model (2) can be replaced by the linear one, and signal $\hat{s}_{t,k}$ delivered to BS can be written in the form:

$$\hat{s}_{t,k} = A\hat{M}_k(x - \hat{B}_k) \cos(2\pi f_0 t + \phi_k) + \xi_t.$$  

The demodulated signal at the DM1 output can be written as:

$$\tilde{y}_k = A\hat{M}_k\varepsilon_k + \xi_k = A\hat{M}_k(x - \hat{B}_k) + \xi_k,$$

where $A = A_0\gamma_0/r$; $\gamma_0$ is the channel gain and $r$ is the distance between the TU and BS (numerical coefficient appearing in demodulated signal is included in $\gamma_0$). Noise $\xi_k$ in (5) is AWGN with the power $\sigma_0^2 = N_0 F_0$. Transition from the nonlinear to the linear model of (statistically fitted) modulator changes the MSE of transmission minimally - the differences being of $O(\mu)$ order.

Apart from the controls $B_k, M_k$, in each cycle, the digital unit of the BS computes the estimates $\hat{x}_k = \hat{x}_k(y_{k}^1)$ of the transmitted samples according to the Kalman equation:

$$\hat{x}_k = \hat{x}_{k-1} + L_k[\hat{y}_k - E(\hat{y}_k | y_{k-1}^1)], \quad (k = 1, \ldots, n)$$

where $L_k$ determines the convergence rate of the algorithm.

The controls received by the TU can be written in the form:

$$\hat{B}_k = B_{k-1} + v_{k}^{(1)}, \hat{M}_k = M_{k-1} + v_{k}^{(2)}, \quad \text{where } v_k^{(1)}, v_k^{(2)} \text{ are transmission errors.}$$

As shown in [1]–[3], for the Gaussian case, optimal values of the gains $M_{k-1}$ do not depend on the signals delivered to the BS and can be computed independently. The latter permits to set the gains $M_k$ directly to the computed values $M_{k-1}$, without their transmission and we further assume $\hat{M}_k = M_{k-1}$. In these conditions and in the absence of abnormal errors, the controls $B_{k-1} = B(y_{k}^1)$, computed by the DSPU are Gaussian random values. For this reason, both the signals $\hat{B}_k$ delivered through the Gaussian feedback channel T2-Ch2-R2 and transmission errors $\nu_k = \hat{B}_k - B_{k-1}$ will also be Gaussian, regardless of the chosen method of transmission. This permits to consider dependence of the AFCS performance on the variance of feedback transmission errors $\sigma^2 = E(\nu_k^2)$, omitting analysis of the transmission scheme.

**Formulation of optimization task:** One should find the values $B_{k-1, M_{k-1}, L_k}$ which minimize, for each $k = 1, \ldots, n$, the mean square error (MSE) of estimates $P_k = E[(x - \hat{x}_k)^2]$ under fulfilled condition (3).
Solution: By substituting (5) into (6) and taking into account that $E(\hat{y}_k | y_k^{-1}) = AM_{k-1}(\hat{x}_{k-1} - B_k)$, where $\hat{x}_{k-1} = E(x | y_k^{-1})$, one may obtain the relationship:

$$\hat{x}_k - x = (1 - AM_{k-1}L_k)(\hat{x}_{k-1} - x) + L_k(AM_{k-1}\nu_k + \xi_k).$$

(7)

Averaging the squared equation (7) under arbitrary parameters $B_0, M_0$ satisfying condition (3) permits to find the optimal gains $L_k$, minimizing MSE for each $k = 1, \ldots, n$ and corresponding values of MMSE $P_k$ (index “min” is omitted):

$$L_k = \frac{AM_{k-1}P_{k-1}}{\sigma^2_z + A^2M_{k-1}^2(\sigma^2_z + \nu^2_k + P_{k-1})};$$

$$P_k = \frac{(\sigma^2_z + A^2M_{k-1}^2\nu^2_k)P_{k-1}}{\sigma^2_z + A^2M_{k-1}^2(\sigma^2_z + P_{k-1})}.$$  

(8)

As follows from (8), greater values of $M_k$ (not violating (3)) decrease MSE independently of the controls $B_k$. Taking into account (5, 6), as well as $B_k = B_{k-1} + \nu_k$ and $M_k = M_{k-1}$, condition (3) can be written in the form:

$$P_k^{\text{over}} = 1 - \frac{1}{\sigma^2_z + A^2M_{k-1}^2} \exp \left\{ -\frac{[z - M_{k-1}(\hat{x}_{k-1} - B_k)]^2}{2M_{k-1}^2(\sigma^2_z + P_{k-1})} \right\} \leq \mu.$$  

(9)

According to (9), the upper boundary of permissible values $M_{k-1}$ depends on the values $B_{k-1}$, $M_{k-1}$. Optimal controls $B_{k-1}$, $M_{k-1}$, ensuring, for each $k = 1, \ldots, n$, achievement of the absolute minimum of MSE are determined by the relationships:

$$B_{k-1} = \hat{x}_{k-1} - E(x | y_k^{-1}); B_k = B_{k-1} + \nu_k;$$

$$M_{k-1} = \hat{M}_k = \frac{1}{\alpha \sigma^2_z + P_{k-1}}; \alpha = \left\{ \begin{array}{l} \alpha : \Phi(\alpha) = \left( 1 - \mu \right) / 2 \end{array} \right\}.$$  

(10)

where $\Phi(\alpha)$ is the Gaussian error function and $\hat{M}_k = (\alpha \sigma^2_z)^{-1}, B_1 = x_0$. Substitution of (10, 11) into (6, 8) after non-complex transformations gives the following optimal estimation algorithm:

$$\hat{x}_k = \hat{x}_{k-1} + L_k\hat{y}_k; L_k = \frac{1}{AM_{k-1}}(1 - P_kP_{k-1}^{-1});$$

$$P_k = (1 + Q^2)^{-1} \left[ \frac{(1 + Q^2)\sigma^2_z + P_{k-1}}{\sigma^2_z + P_{k-1}} \right] P_{k-1}.$$  

(13)

where

$$Q^2 = \frac{W_{\text{sign}}}{W_{\text{noise}}} = \frac{A^2M_{k-1}^2E(e_k^2)}{\nu^2_k} = \left( \frac{A}{\alpha} \right)^2 \frac{1}{\sigma^2_z + P_{k-1}}.$$  

(14)

is the SNR at the demodulator DM1 output, $W_{\text{sign}} = (A/\alpha)^2$ is the mean power of the information component of the received signal, $e_k = x - \hat{x}_{k-1} + \nu_k$, $e_1 = x - x_0$, and $\hat{x}_0 = x_0; P_0 = \sigma^2_z$.

III. CAPACITY AND EFFICIENCY OF OPTIMAL AFCs

The bit-rate in the forward channel M1-Ch1-DM1 of the optimal AFCs designed according to (2, 10–14) can be found by taking into account that signals $e_k = x - \hat{x}_{k-1} + \nu_k$ at the input of the channel are Gaussian and orthogonal. In this case, the amount of mutual information $I(\hat{Y}^n; e^n)$ in the sequences $\hat{y}_k^n, e_k^n$ is determined by the expression $I(\hat{Y}^n; e^n) = n[I(\hat{Y}; e)]$, where $H(\hat{Y}) = 1/2 \log_2[2\pi \sigma^2_{\hat{Y}}(1 + Q^2)]$ and $H(e) = 1/2 \log_2(2\pi \sigma^2_e)$. The non-conditional and conditional entropies of observations, respectively. In this case, the mean bit-rate of transmission in the channel M1-Ch1-DM1 $R_{\text{Ch}1}$ is equal to the instant bit-rate $R_{\text{Ch}1}^t = I(\hat{Y}; e)/\Delta t_0$, and the following equalities hold:

$$R_{\text{Ch}1} = \lim_{n \to \infty} \frac{I(\hat{Y}^n; e^n)}{n\Delta t_0} = R_{\text{Ch}1} = F_0 \log_2(1 + Q^2) =$$

$$= F_0 \log_2 \left( 1 + \frac{W_{\text{sign}}}{N_f F_0} \right) = C.$$  

(15)

The right side of (15) is Shannon’s formula for the capacity $C$ of Gaussian channel. The novel element is that the bit-rate (capacity of the channel (15)) is now connected with the overmodulation probability $\mu$ through the power of the received signal $W_{\text{sign}} = (A/\alpha)^2$, where $\alpha = \alpha(\mu)$. This dependence is the result of fitting condition (9). Formula (15) also confirms another Shannon’s result of ([5], Theorem 6) that the forward capacity is equal to the ordinary capacity $C$ (without feedback) and does not depend on the feedback noise.

Let us consider the work of AFCs as a whole. One may see that each statistically fitted AFCs can be represented as the linear “macro” channel described by the relationship:

$$\hat{x}_n^{(m)} = x^{(m)} + \chi_n, \quad (m = 1, 2, \ldots)$$  

(16)

where $\hat{x}_n^{(m)}$ are the output estimates of the samples $x^{(m)}$ transmitted by the system and $\chi_n$ is the additive Gaussian noise describing the errors of the sample transmission. Unlike the forward channel noise $\xi_k$, noise $\chi_n$ in (16) is not stationary and its variance $E(\chi^2_n) = E[(x - \hat{x}_n)^2] = P_n$ depends on the transmission method and number of cycles, parameters of transmitters and receivers, as well as characteristics of input signal and channel noise. For the samples transmitted independently, the mean bit-rate at the AFCs output is determined by the following relationship:

$$R^\text{AFCS} = \frac{I(X; \hat{X}_n)}{T} = F_0 \log_2 \frac{\sigma^2_{\hat{X}_n}}{P_n} = \frac{F_0}{n} \log_2 \frac{\sigma^2_{\hat{X}_n}}{P_n}.$$  

(17)

where $I(X; \hat{X}_n) = H(X) - H(X | \hat{X}_n)$ is the amount of information in the output estimates $\hat{x}_n$ about the input samples $x$, and $H(X), H(X | \hat{X}_n)$ are the prior and posterior entropies, respectively. Formula (17) describes the mean bit-rate for arbitrary, not necessarily optimal (but statistically fitted) Gaussian AFCs.

In the optimal AFCs, for each $k = 1, \ldots, n$, the MSE of transmission (13) takes minimal values, and $I(X, \hat{X}_n)$ is the maximal amount of information in the estimates $\hat{x}_n$. The latter means that the bit-rate (17), for $P_n$ (13), determines the upper boundary of mean bit-rate at the AFCs output. It follows that we can interpret this bit-rate as the capacity of AFCs considered as a generalized communication channel. Thereby, the capacity of AFCs as a whole can be defined as
the conditional maximum of the mean bit-rate $R_n^{AFCS}$ (17):

$$C_n^{AFCS} = \max_{L_n^{-1}, M_n^{-1}} F_n^{AFCS} = \frac{F_0}{n} \min_{L_n^{-1}, M_n^{-1}} \left( \frac{\log_2 \sigma_v^2}{P_n} \right)$$

under the condition that, for each $k = 1, \ldots, n$,

$$Pr(\hat{X}_k|X^k > \hat{B}_k) > 1 \left| g_{k^{-1}} B_{k^{-1}} L_k^{-1}, M_k^{-1}, X < \mu \right.$$.

Let us assume that the signals received by BS are weak, feedback channel is sufficiently good ($\sigma_v^2 \ll \sigma_p^2$), and

$$SNR_{inp}^{Ch1} = \frac{\sigma_v^2}{\sigma_p^2} \gg 1 + Q^2 = 1 + SNR_{out}^{Ch1}.$$ (19)

In this case, the MMSE (13) can be approximated by the relationship [1–3]:

$$P_n = \begin{cases} \sigma_v^2(1 + Q^2)^{-n} & \text{for } 1 \leq n \leq n^* \\ \sigma_v^2(n - n^* + 1)^{-1} & \text{for } n > n^* \end{cases}$$

for the threshold point $n^*$ is a solution of the equation $P_{n^*} = \sigma_v^2$ and has the form:

$$n^* = \frac{1}{\log_2(1 + Q^2)} \log_2 \left( \frac{\sigma_v^2}{\sigma_p^2} \right) = \frac{\log_2(SNR_{inp}^{Ch1})}{\log_2(1 + SNR_{out})} = \frac{\log_2 \sigma_v^2}{\log_2 \left( \frac{\sigma_v^2}{\sigma_p^2} \right)} = \frac{\log_2 \sigma_v^2}{\log_2 F}$$. (21)

Substitution of (20) into (17) gives the following relation between the bit-rate at the optimal AFCS output and the capacity of the forward channel (see also Fig. 3):

$$R_n^{AFCS} = C_n^{AFCS} = \left( \frac{C}{n} \right) \left[ C + \frac{\log_2(n - n^* + 1)}{n^*} \right]$$

for $1 \leq n \leq n^*$

$$R_n^{AFCS} = \left( \frac{C}{n} \right) \left[ C + \frac{\log_2(n - n^* + 1)}{n^*} \right]$$

for $n > n^*$.

One may easily check that the right side of (15) rewritten in terms $E^{bit}/N_0 = W_{signal}/T_{bit}/N_0 = W_{signal}/C/N_0$ and $C/F_0$, takes the form of (1). Similar transition in (22) shows that the P-B efficiency of the optimal AFCS preserves maximal value (1) only in the interval $1 \leq n \leq n^*$ and diminishes for $n > n^*$. In Fig. 4, the results of simulations carried out using algorithm (12–14) and relationship (5) (with the added levels of saturation) are presented. The experiments show that for $1 \leq n \leq n^*$, the efficiency of AFCS preservation triangles preserve a fixed position at the Shannon boundary and decline from this boundary down-right for $n > n^*$.

**Remark:** Probability $n\mu$ of the appearance of overmodulation determines the mean percentage of distorted estimates at the AFCS output. Assuming that each distortion causes a loss of $I(X; X_n)$ bits of information, the mean percent of the lost (erroneous) bits in the output sequences of bits is also equal to $n\mu$. This permits to consider $\mu$ as a direct analog of the bit error rate (BER) used in digital communications.

**IV. ANALYSIS OF THE RESULTS**

1. The presented results show that the bit-rate of transmission in the forward channel of the optimal AFCS realized according to (2), (10)–(14) is equal to its capacity $C$, independent of the number of cycles of the sample transmission. In turn, the capacity of AFCS is equal to $C$ only for $1 \leq n \leq n^*$. Transmission in $n > n^*$ cycles reduces the capacity of AFCS.

2. The right side of (18) allows us to interpret the capacity of the AFCS as the minimal mean bit-rate at its output, which permits to restore the samples with given MMSE $P_{n}^{min}$. In this interpretation, capacity (18) is equivalent to the rate distortion function $R(\varepsilon)$ defined as the minimal number of bits per second necessary to reproduce the input samples with distortion (MSE) not greater than given $\varepsilon$ (e.g. [6, 7–9]; in our case, $\varepsilon = P_{n}^{min}$). Assuming that the forward channel capacity $C$ (in nats) is known, formulas (18, 22) determine the MMSE of transmission, i.e. low boundary of distortions achievable in $n$ cycles of the sample transmission:

$$P_{n}^{min} = \sigma_v^2 \left( \frac{C_{AFCS}^{n}}{n} \right) = \sigma_v^2 \left( \frac{n \sigma_v^2}{C_{AFCS}^{n}} \right)$$

(23)

for $n = F_0/F$ is taken into account. For $1 \leq n \leq n^*$, (23) coincides with formula (2) in [7], where the equivalence of the capacity and rate distortion function was also discussed.

3. The threshold number of cycles $n^*$ (21) determines the optimal number of cycles of sample transmission. Transmission in $n > n^*$ cycles ensures further diminution of MMSE, but diminishes the capacity and P-B efficiency of AFCS (the smaller $C_{AFCS}^{n}$, the greater $E^{bit}$). Shorter transmission (in $1 \leq n < n^*$ cycles) preserves the ideal mode of transmission, i.e. relationship (1), but the interval of the fastest decrease in MMSE is not utilized completely. Thus, full utilization of AFCS resources can be achieved only if the system is used in the threshold mode ($n = n^*$).

4. The results of Sect. 3 show that the capacity and P-B efficiency of the optimal AFCS are constant in the interval $1 \leq n < n^*$, while the MMSE $P_n$ changes according to (20). For this reason, closeness of the bit-rate and P-B efficiency of real AFCS to their upper boundaries does not indicate full utilization of the resources of the system. In turn, the closer the MSE is to the values (20), the closer are the bit-rate and efficiency of AFCS to their limit values. This means that the bit-rate and P-B efficiency are necessary but not sufficient...
characteristics of the system performance, and that the MSE of the transmission is a more adequate criterion.

V. CONCLUDING REMARKS

Optimization of AFCS with analog TU was a subject of intensive and promising research in the 1950s–1960s, which gave a series of important theoretical results ([7–10], etc.). However, rapid development of digital communications had radically decreased the interest in the analog AFCS now considered as passé. The cost of transition to digital transmission radically decreased the interest in the analog AFCS now considered as passé. The cost of transition to digital transmission was the impossibility to formulate MSE and find its minimal value over the set of possible codes and methods of digitizing. This resulted in the appearance of a number of mutually dependent performance criteria in digital communications theory (P-B efficiency, bit-rate, BER), which significantly complicated the design of highly efficient communication systems. As shown above, analog AFCS allows optimization according to MMSE criterion and the obtained results permit to design systems transmitting the signals at (or close to) the theoretically achievable performance. Additional arguments to restore attention to AFCS are the simplicity, low energy consumption, and cost of analog TU.

The results of our research also address the following question: Why such important results obtained in the 1960s got no practical implementation before the digital revolution or later? The main reasons were application of the strictly linear model of TU and solution of the optimization task under constraints on the mean power or peak values of the signals emitted by the TU. These assumptions do not allow the analysis and elimination of overmodulation (saturation of the transmitter), which substantially degrades the performance of AFCS. Fitting condition (3) enables direct consideration of these factors. Also, it naturally introduces a small parameter $\mu$ that enables derivation of optimal transmission/reception algorithm (2, 12–14), as well as the analysis of particularities of optimal AFCS functioning, performance, and application.

Comparison of the main results of [7]–[10] and those presented in the previous sections shows their full agreement for $1 < n < n^*$. In particular, (20) is analogous to (1) in [7], to (29) in [8], and to (21) in [9], while (21) is analogous to (15) in [10], etc. The only difference is that the power $W_k$ of the received signals should be computed taking into account the saturation factor $\alpha$ which depends on the probability $\mu$.

We believe that the presented results give a new outlook to the potential possibilities of the analog AFCS.

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