ANALYSIS OF PHASE NOISE EFFECTS IN MULTI-USER OFDM

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ABSTRACT

This paper analyzes the effects of phase noise in multi-user OFDM systems. The impact on system performance of different phase noise characteristics and received power among users is considered. The main conclusion that can be achieved is the importance of power control in order to avoid a near-fact effect. Besides, the presence of transceivers with bad phase noise characteristics can ruin the performance of better (more expensive) ones.

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a candidate technique to provide broadband wireless communications in hostile multipath environments. With this goal in mind, OFDM performance in multi-user environments is being examined in PACWOMAN IST project funded by European Commission [1].

Phase noise must be carefully considered when designing an OFDM system since an accurate prediction of the tolerable phase noise can allow the system and RF designers to relax specifications.

Phase noise effects in single-user OFDM have been analyzed by several authors and the degradation introduced in the system by phase noise has been characterized for some particular cases [2-6]. It is demonstrated in [3] that the degradation caused in OFDM by phase noise is the same as in a single carrier system when phase noise effects are considered without attempting to correct them. This does not hold if phase noise effects are corrected as further shown in [6]. Indeed, the performance after correction depends on the spectral characteristics of the phase noise source.

When dealing with multi-user systems, phase noise issues are potentially more critical, since different transceivers with different phase noise characteristics must coexist in the same environment.

In [7] the impact of phase noise in multi-user OFDM systems is considered and some analytical expressions are given for the case when all users’ signals are received with equal power and all transceivers exhibit the same phase noise characteristics. In this paper, we consider a more general case and compare the results with those of [7].

The paper is structured as follows. After this introduction, section 2 shows the multi-user OFDM system model. In section 3, phase noise performance in multi-user OFDM systems is reviewed and section 4 discusses some simulation results. The paper finishes with some concluding remarks.

2. MULTI-USER OFDM

OFDM-based multiple access (OFDMA) or multi-user OFDM has been proposed for several applications in the literature. It constitutes one of the most feasible approaches to maximize the number of terminals and the aggregate data rate [1].

In OFDM the total available bandwidth is divided in $N$ sub-channels. Each of them is modulated with one symbol from a digital constellation (for our results we will use QPSK), so that a set of $N$ symbols are sent modulating $N$ sub-carriers and they are all multiplexed in frequency. The signals of the different sub-channels are allowed to overlap, and the fact that they are kept orthogonal facilitates their separation in the receiver.

Let $T$ be the duration of the OFDM symbol and $B$ the total bandwidth. The separation between sub-channels is chosen so that $B = N/T$. The complex envelope of the OFDM signal sampled with sampling frequency equal to $B$ can be obtained by an Inverse Discrete Fourier Transform of the complex input symbol sequence $s_{in}$ (where $i$ is the time index and $n$ is the sub-carrier index) as [8]:
\[
x(k) = \sum_{j=-\infty}^{\infty} \text{DFT}^{-1}\{s_{in}\} \prod_{j=N}^{\infty} (k-jN) \quad (1)
\]

Here \( \Pi_N(k) \) represents a rectangular pulse with \( N \) samples of duration. The frequencies of the \( N \) subchannels are:

\[
f_n = f_0 + n/T, \quad n = 0, 1, ..., N-1 \quad (2)
\]

Several possibilities arise when an OFDM signal must be shared between several users in a multi-user environment. Given the inherent frequency-division nature of the signal, an FDMA approach is an immediate choice. When an OFDM signal is shared in an FDMA fashion, that is, different users transmit in different sub-carriers, the OFDM-based multiple access scheme is commonly named OFDMA (Orthogonal Frequency Division Multiple Access).

In its simplest form, each user sharing an OFDMA channel will transmit a single-carrier signal and the signals from different users will be separated with a carrier spacing of \( 1/T \), \( T \) being the OFDM symbol time. In this way, the composite signal with the contributions of \( N \) users will be an OFDM signal that may be demodulated with a Discrete Fourier Transform, which is much simpler than a set of \( N \) traditional FDMA demodulators. This idea is easy to generalize to the case in which users may transmit a different number of sub-carriers depending on their needs.

If we denote by \( n \) the user number an \( s_n \) the information symbol to be transmitted by that user at a given time, each OFDM symbol formed by the composite signal with the contribution of each of the \( N \) users is:

\[
x(k) = \sum_{n=0}^{N-1} s_n e^{\frac{2\pi}{N} nk} \quad (3)
\]

Table 1 shows the parameters of the OFDM signal that have been chosen for PACWOMAN project [1].

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of sub-carriers ((N))</td>
<td>64</td>
</tr>
<tr>
<td>Number and distribution frequency guards</td>
<td>8 at each boundary</td>
</tr>
<tr>
<td>Number and distribution of pilots</td>
<td>Full-pilot preamble; 4 scattered in every symbol</td>
</tr>
<tr>
<td>Number of useful sub-carriers</td>
<td>44</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>10 MHz</td>
</tr>
<tr>
<td>Duration of cyclic prefix</td>
<td>400 nsec</td>
</tr>
<tr>
<td>Duration of useful OFDM symbol</td>
<td>6.4 ( \mu )sec</td>
</tr>
</tbody>
</table>

Multi-user scenarios can be divided into two separate cases: broadcast and multiple access scenarios. Figure 1 illustrates both types of communications. While in broadcast systems the composite signal received by every user is affected by the same phase noise and arrives with equal power in each component, in multiple access systems signals arrive with (generally) different power and come from different sources with (possibly) different phase noise.

### 3. PHASE NOISE IN MULTI-USER OFDM

In [7] the impact of phase noise in multi-user OFDM systems is considered. It is shown that the degradation of the signal-to-noise ratio (SNR) of the \( n \)-th user when phase noise is present is given by:

\[
D_n = 10 \log \left( 1 + \frac{E_{s,n}}{E_W(0)} \left[ I_{n,0} - E[I_{n,0}] \right]^2 \right) \]

\[
+ \sum_{m=0, m\neq n}^{N-1} \frac{E_{s,m}}{E_W(0)} \left[ I_{m,n-m} \right]^2 \right)
\]

where \( E_{s,n} \) is the energy per symbol of user \( n \) and the terms:

\( I_{n,0} \) represents the phase noise contribution associated to self-interference of user \( n \) caused by its own phase noise.

\( I_{m,n} \) \((m\neq n)\) represents the phase noise contribution associated to the interference that \( m \)-th user’s phase noise causes to user \( n \).
$W_n(0)$ is the AWGN contribution to $n$-th user’s received signal.

Equation (4) can be simplified when the energies and phase noise spectra of all users are the same. In this case the SNR degradation is:

$$
D_n = 10 \log \left( 1 + \frac{E_s \sigma_\phi^2}{E[W_n(0)]} \right)
$$

(5)

According to (5), the degradation of SNR of multi-user OFDM when the above mentioned conditions are satisfied does not depend on phase noise spectrum but just on phase noise variance. However, it is interesting to find out what happens when different phase noise sources coexist or the signals of each user are received with different power.

4. SIMULATION RESULTS AND DISCUSSION

In order to find out the influence of different phase noise sources or received powers, simulations have been run with phase noise modeled as a phase modulation of the carrier. The modulating signal is a zero mean white gaussian random process $\phi(k)$, with variance $\sigma_\phi^2$.

Figure 2 shows a comparison of simulation results and the theoretical degradation of (5) for different phase noise variances, when received power and phase noise variances are identical for all users. The number of sub-carriers is 64 and users share the bandwidth by groups of 6 sub-carriers. It can be seen that simulations and theory agree for low phase noise variances whereas some differences are found for higher phase noise variances. This disagreement, due to the fact that (4) and (5) are found assuming small phase noise variance, is further explained in [6].

It is important to note that there is a near-far effect associated to phase noise in multi-user OFDM. The user whose signal arrives with low power suffers a higher degradation due to phase noise than what can be expected from (5) due to the fact that there are other users whose signals arrive with higher power.

In order to see the impact of the phase noise when users’ signals arrive with different powers, we have simulated the following situation:

- 2 users with received power $P$
- 2 users with received power $90 \%$ of $P$
- 2 users with received power $80 \%$ of $P$
- 2 users with received power $70 \%$ of $P$

SNR degradation is shown in Figure 3. It can be seen that the powerful the user is the lower degradation it experiences. The explanation can be found examining (4): users with more power will interfere severely on those other users with lower power because the phase noise degradation will also depend on the value of $E_s$ $\sigma_\phi$. On the other hand, the contribution of phase noise of users with lower power will be lower and the interference could be neglected to high power users as it is shown in Figure 3. For this reason, the approximation (5) is still valid for users with very similar powers but it is not valid when there are differences in power larger than 80 %.

![Figure 2. Performance with equal power and phase noise](image-url)

![Figure 3. Performance with different received powers](image-url)
what they would achieve if all sub-carriers were affected by their same phase noise. This means that there is an averaging of the performance degradation between all users, that benefits those users with the worst oscillators.

The main conclusion that can be achieved is the importance of power control in order to avoid a near-fact effect. Besides, the presence of transceivers with bad phase noise characteristics can ruin the performance of better (more expensive) ones.

6. REFERENCES