Downscaling of remotely sensed soil moisture with a modified fractal interpolation method using contraction mapping and ancillary data

Gwangseob Kim¹, Ana P. Barros *

Division of Engineering and Applied Sciences, Harvard University, 118 Pierce Hall, 29 Oxford Street, Cambridge, MA 02138, USA

Received 29 June 2001; received in revised form 1 April 2002; accepted 6 April 2002

Abstract

Previous work showed that remotely sensed soil moisture fields exhibit multiscaling and multifractal behavior varying with the scales of observations and hydrometeorological forcing (Remote Sens. Environ. 81 (2002) 1). Specifically, it was determined that this multiscaling behavior is consistent with the scaling of soil hydraulic properties and vegetation cover, while the multifractal behavior is associated with the temporal evolution of soil moisture fields. Here, we apply these findings by directly incorporating information on the spatial structure of soil texture and vegetation water content to the spatial interpolation of remotely sensed soil moisture data.

A downscaling model is presented which consists of a modified fractal interpolation method based on contraction mapping. This methodology is different from other fractal interpolation schemes because it generates unique fractal surfaces. It is different from other contraction mapping models because it includes spatially and temporally varying scaling functions as opposed to single-valued scaling factors. The scaling functions are linear combinations of the spatial distributions of ancillary data. The model is demonstrated by downscaling soil moisture fields from 10 to 1 km resolution using remote-sensing data from the Southern Great Plains 1997 (SGP'97) field experiment.

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1. Introduction

In remote-sensing applications, as in physically based modeling of land-surface processes, the representation (inclusion) of subgrid-scale variability in coarse resolution data remains an elusive challenge. The problem is one of spatial interpolation, or downscaling. The challenge results from the discrepancy between the coarse spatial scales (and often temporal scales) of available data and the fine scales necessary for meaningful research and applications. In this paper, we focus specifically on the downscaling of remotely sensed soil moisture fields.

The statistical characteristics of remotely sensed soil moisture images were analyzed by Rodriguez-Iturbe et al. (1995), Hu, Islam, & Cheng (1997), and Kim & Barros (2002) among others. Using soil moisture data from the Little Washita ’92 experiment, Rodriguez-Iturbe et al. (1995) concluded that the spatial variance of soil moisture fields follows a power law decay as a function of the spatial scale. They also suggested that the scaling behavior of soil moisture could be related to the scaling behavior of soil porosity. Hu et al. (1997) estimated the Hurst exponent of soil moisture fields at about 0.1 using the same data. This reflects the lack of persistence in the soil moisture images because of the succession of wetting and drying cycles in response to rainfall, and due to the subgrid-scale heterogeneity of soils. Kim & Barros (2002) showed that large-scale soil moisture images from SGP’97 (Southern Great Plains 1997 field experiment) do not exhibit scale invariance at spatial scales larger than 10 km (r-scale range) (Fig. 1). They also found that the multiscaling properties of soil moisture imagery could be explained through the scaling characteristics of land-surface attributes such as soil texture and vegetation water content, and that this scaling behavior changed with time as a function of changes in soil moisture level (multifractal scaling). The immediate implication of these results is that any downscaling model designed to improve upon the spatial resolution of remotely sensed soil moisture data must capture this multifractal and multiscaling behavior. That is, a robust downscaling algorithm should reflect the association between the spatial structure of soil moisture and relevant ancillary data. For this purpose, we introduce
here a modified fractal interpolation model based on contraction mapping principles. The model includes spatially and temporally varying scaling functions derived from the spatial distributions of soil texture and vegetation indices. We examined and tested model performance using data from the Southern Great Plains experiment in the summer of 1997 (SGP’97). The model formulation is presented in Section 2. Section 3 describes the data, while results and error analysis are discussed in Section 4. The manuscript concludes in Section 5 with a brief summary and assessment of further research needs.

2. Downscaling with fractal interpolation functions

2.1. Rationale

Typical interpolation such as linear averaging, kriging and polynomial interpolation produce smoothed versions of the original data at a different (and finer) spatial resolution. Because the complex spatial structure of environmental data is often charged with physical meaning, efforts to preserve the spatial variability of original data in transferring information across scales have been many (e.g., Mandelbrot, 1982 among others). For example, Bindlish & Barros (1996) used a fractal interpolation method to map digital elevation data at different spatial resolutions. With a fractional Brownian surface as the interpolating basis function, they found that the fractal interpolation approach preserved well the spatial structure and the vertical scale of the data. Subsequently, Bindlish & Barros (2000) proposed a dynamic fractal interpolation method to downscale rainfall fields generated by a numerical weather prediction (NWP) model. The downscaling model relied on spatially and temporally varying wind fields simulated by the numerical model to modify the fractional Brownian surface used as interpolation function. However, fractal interpolation based on the use of random fields (e.g., fractional Brownian surfaces) as interpolating functions relies on two implicit assumptions: (1) all data with scaling behavior consistent with an exponential power law are simple fractals (i.e., do not vary with time); and (2) the power spectrum is the unique determinant of the fractal surface. Generally, these assumptions are not valid, and therefore each interpolated field is only one of many possible realizations of a random process.

In summary, the construction of fractal interpolation surfaces using random grids is lacking in two ways: (1) it cannot be applied to multifractals; and (2) the downscaled surfaces are not unique solutions, but merely one of many possible realizations of a random function. The second is only a weak limitation when physically based constraints can be specified as shown in Bindlish & Barros (1996, 2000), but it is a critical shortcoming for data with complex non-linear physical underpinnings (e.g., soil moisture).

Recently, Bindlish & Barros (2002) used fractal interpolation to downscale L-band passive microwave data using active microwave data at the same wavelength as interpolating surface to improve the resolution of brightness temperature fields prior to soil moisture retrieval. This strategy eliminated the need to use a Brownian surface as interpolating surface. However, this approach is limited by the resolution of the active data, and thus even in the case of combined passive–active sensors operating at scales of the order of tens of kilometers, further downscaling is still necessary for meaningful hydrological studies.

Barnsley (1986) explored the application of Iterated Function Systems (IFS), which are both not differentiable and deterministic, as fractal interpolation functions of self-affine data. IFSs retain the irregularity of the original data in transferring information across scales, and are determined uniquely by the existing data. It follows that the interpolated surfaces are unique spatial distributions of the original data at higher resolutions.

The application of iterated function systems as fractal interpolation functions was further investigated by Xie & Sun (1997) who proposed a mathematical model for contraction mapping based on a system of bivariate fractal interpolation functions, the roughness and fractal dimensions of which are determined by a specified vertical scaling factor. In this work, we use their findings, and modify their
model in two fundamental ways: (1) effects associated with spatial non-stationarity are included by repeatedly applying the contraction mapping on a sliding window rather than globally to the entire image; and (2) the bivariate fractal interpolation functions are modified to include vertical scaling factors that vary spatially (and, or temporally) with changes in the land-surface such as soil moisture level and

Fig. 2. (a) Schematic diagram of the downscaling algorithm by sliding the mapping window from central location \((x_i, y_j)\) to \((x_{i+1}, y_j)\); (b) mapping window centered at grid-cell \(O|_{i,j}(x_i, y_i)\) and contraction mapping operation \(T_{i,j}^{n}_{m,n} = \cdots\) along the \(x\) and \(y\) directions.

Fig. 3. Topography and geographical setting of the soil moisture mapping region of the SGP’97 experiment.
Fig. 4. Time-series of remotely sensed soil moisture during SGP’97.
vegetation. A detailed description of the model formulation is presented next.

2.2. Mathematical model

Let us consider

\[ R^h = \mathbb{R}^h \times \mathbb{R}^h \times \mathbb{R}^h = \{(x, y, s) : -\infty \leq x \leq +\infty, -\infty \leq y \leq +\infty, 0 \leq s \leq S_{\text{max}}\}, \]

a three-dimensional Euclidean space where \( \{x, y\} \) are spatial coordinates corresponding to a grid of resolution \( (\Delta x \times \Delta y = h \times h) \), and the vertical dimension \( s \) is the local value of a bounded state variable (e.g., volumetric soil moisture). A remotely sensed soil moisture image is a finite-dimensional \( C^h \subset R^h \) such that

\[ C^h = \mathbb{R}^h \times \mathbb{R}^h \times \mathbb{R}^h = \{(x, y, s) : A \leq x \leq B, C \leq y \leq D, 0 \leq s \leq S_{\text{max}}\}, \]

where \( A, B, C \) and \( D \) are the spatial dimensions, and \( S_{\text{max}} \) is the maximum volumetric soil moisture (i.e., soil porosity). The objective of downsampling is to derive soil moisture images \( C^{ah} \) at resolution \( ah \) \((0 < a < 1)\) from the existing images \( C^h \) at resolution \( h \) (Fig. 2a). For consistency in the algebraic implementation of the model, the pixels in the soil moisture images are mapped into a grid with nodes at the central point of each pixel. Each grid-cell has four nodes, each corresponding to a different pixel.

The downsampling model relies on the application of the principles of contraction mapping using bivariate fractal interpolation functions within a limited region, the mapping window, centered at grid-cell \( O^{h}_{i,j} \) (Fig. 2a).

Fig. 5. Normalized field parameters for the SGP'97 domain: (a) vegetation water content [VWC]; and (b) sand content in the soil [PS].

Fig. 6. Functional parameter estimation: (a) optimization of test statistics such as maximum (top panel); (b) minimum (middle panel); and (c) standard deviation (bottom panel) values.
The mapping window comprises \( [N \times M] \) grid-cells distributed as follows:

\[
d^l = x_0 < x_1 < \ldots < x_N = b^l
\]
\[
e^l = y_0 < y_1 < \ldots < y_M = d^l
\]

(1)

The size of the window determines the number of neighbor grid-cells that contribute information to downscaling at the central grid-cell [a minimum of \( (\frac{1}{2} \times \frac{1}{2}) \)] grid-cells are required when \( \alpha \leq 1/3 \). Note that the mapping must be designed such that the window is small enough to preserve the spatial structure of the data at resolution \( h \), and thus, more than one iteration may be required to go from resolution \( h \) to resolution \( zh \). That is, when the value of the ratio between the original and desired fine spatial resolutions is small, the methodology should be applied recursively between intermediate resolution levels to control the size required for the mapping window, and thus the areal extent of the region that contributes to the downscaling of an individual pixel within the image (Fig. 2a and b).

To make use of the relationships between the vertical dimension (soil moisture) and ancillary data, the three-dimensional contraction mapping \( Q_{n,m} \upharpoonright_{h \rightarrow zh} \); \( O^h \upharpoonright_{l,j} \rightarrow O^{zh} \upharpoonright_{l,j} \) is separated into two independent transformations: (1) a linear horizontal projection \( T_{ij} \upharpoonright_{h \rightarrow zh} \) for the spatial coordinates, which generates the support grid within the central pixel of the mapping window (Fig. 2a); and (2) a non-linear vertical projection \( P_{ij} \upharpoonright_{h \rightarrow zh} \) for the third dimension, which generates the soil moisture distribution at resolution \( zh \).

The horizontal contraction mapping function \( T_{ij} \upharpoonright_{h \rightarrow zh} = \{ p_n(x)_n \upharpoonright_{i,j} \}, (y)_n \upharpoonright_{i,j} \} \) generates a two-dimensional subspace \( L^{zh} \upharpoonright_{i,j} \); \( X^{zh} \upharpoonright_{i,j} \) corresponding to a grid of resolution \( zh \) within \( O^h \upharpoonright_{i,j} \), where \( X^{zh} \upharpoonright_{i,j} = [x_{i-1}^h, x_i^h] \) and \( Y^{zh} \upharpoonright_{i,j,m} = [y_{j-1}^h, y_j^h] \). The mapping constraints imposed on \( T_{ij} \upharpoonright_{h \rightarrow zh} \) are illustrated in Fig. 2b:

\[
p_n(x_n^l) = x_n^{l-1}; \quad p_n(x_N^l) = x_N^l
\]
\[
q_m(y_m^l) = y_m^l; \quad q_m(y_M^l) = y_M^l
\]

\[
|p_n(c_1^l) - p_n(c_2^l)| < k_1 |c_1^l - c_2^l| \quad 0 < k_1 < 1
\]

\[
|q_m(d_1^l) - q_m(d_2^l)| < k_2 |d_1^l - d_2^l| \quad 0 < k_2 < 1
\]

(2)

where \( (c_1^l \land c_2^l) \) and \( (d_1^l \land d_2^l) \) are \( \in [0,1] \). Following Xie & Sun (1997), the linear, one-dimensional contraction mapping functions in \( T_{ij} \upharpoonright_{h \rightarrow zh} \) have the form:

\[
p_n(x_n^l) = x_n^{l-1} + \alpha(x_n - x_0^l)
\]
\[
q_m(y_m^l) = y_m^{l-1} + \alpha(y_m - y_0^l)
\]

(3)

Next, let us take a vertical downscaling function \( H_{n,m}(x^l, y^l, s^l) \) with the general form

\[
H_{n,m}(x_n^l, y_n^l, s_n^l) = e_{n,m}x_n + f_{n,m}y_n + g_{n,m}x_n y_n + r_1(x_n, y_n)s_n^l
\]

(4)

where \( r_1(x_n, y_n) \) is a real-valued bivariate function \( 0 \leq r_1(x_n, y_n) \leq 1 \), the vertical downscaling function. \( H_{n,m}(x^l, y^l, s^l) \) is a continuous function and for any pair of grid-cells \( (x_1^l, y_1^l, s_1^l); (x_2^l, y_2^l, s_2^l) \) \( \in C^h \upharpoonright_{l,j} \):

\[
\begin{align*}
|H_{n,m}(x_1^l, y_1^l, s_1^l) - H_{n,m}(x_2^l, y_2^l, s_2^l)| & \leq k_3 |s_1^l - s_2^l| \\
0 & \leq k_3 \leq 1
\end{align*}
\]

(5)

The vertical transformation \( P_{ij} \upharpoonright_{h \rightarrow zh} = \{ H_{n,m}(x_i^l, y_j^l, s_i^l) \}; S_{l,j} \upharpoonright_{h \rightarrow zh} \) generates a downscaled soil moisture field at resolution \( zh \) such \( s_{ij} \upharpoonright_{h \rightarrow zh} \) as follows:

\[
\begin{align*}
s_{ij}^{n-1,m-1} & = H_{n,m}(x_i^l, y_j^l, s_{ij}^{n-1,m-1}) \\
s_{ij}^{n,m-1} & = H_{n,m}(x_i^l, y_{j-1}^l, s_{ij}^{n,m-1}) \\
s_{ij}^{n-1,m} & = H_{n,m}(x_{i-1}^l, y_j^l, s_{ij}^{n-1,m})
\end{align*}
\]

(6)

By replacing Eq. (4) in Eq. (6) above, we obtain:

\[
\begin{align*}
s_{ij}^{n-1,m-1} & = e_{n,m}x_i^l + f_{n,m}y_j^l + g_{n,m}x_i^l y_j^l + r_1(x_i^l, y_j^l)^{s_{ij}^{n-1,m-1}}
\end{align*}
\]

(7)

Through further manipulation of Eq. (7), the terms \( g_{n,m} \), \( e_{n,m} \), \( f_{n,m} \), and \( k_{n,m} \) can be expressed in terms of the spatial coordinates of the mapping window \( x \) and \( y \), and the interpolated (downscaled) soil moisture data \( s \), and a vertical scaling surface \( R \):

\[
\begin{align*}
& e_{n,m} = \frac{s_{n-1,m-1} - s_{n-1,m} - s_{n,m-1} + s_{n,m} - R_1^e}{x_0^l y_0^l - x_N y_0^l - x_0^l y_M^l + x_N y_M^l}
\end{align*}
\]

(8)
Fig. 7. Evaluation of the scaling behavior for spatial interpolations corresponding to the six distinct parameter combinations (A–F) marked in the middle panel of Fig. 6b.
where

\[ R^c_1 = r_1(x_{n-1}^i, y_{m-1}^j) s_{o,o}^{ij} - r_1(x_{n-1}^i, y_{m-1}^j) s_{N,o}^{ij} \]

\[ - r_1(x_{n-1}^i, y_{m}^j) s_{o,M}^{ij} + r_1(x_{n}^i, y_{m}^j) s_{N,M}^{ij} \]  \hspace{1cm} (9)

Table 1

<table>
<thead>
<tr>
<th></th>
<th>(\gamma_1)</th>
<th>(\beta_1)</th>
<th>(\delta_1)</th>
<th>(\gamma_2)</th>
<th>(\beta_2)</th>
<th>(\delta_2)</th>
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<td>Wet</td>
<td>0.01</td>
<td>0.04</td>
<td>0.01</td>
<td>0.06</td>
<td>0.08</td>
<td>0.02</td>
</tr>
<tr>
<td>Dry</td>
<td>0.01</td>
<td>0.04</td>
<td>0.01</td>
<td>0.08</td>
<td>0.09</td>
<td>0.02</td>
</tr>
</tbody>
</table>

The vertical scaling surface \(r_1(x,y)\) modulates the relative roughness (i.e., normalized spatial variance) of the down-scaled soil moisture fields. It is expressed as a linear combination of the normalized spatial distributions of the ancillary data \[ r_1(x,y) = \gamma_1 F(x,y) + \beta_1 G(x,y) + \delta_1 \] where \(\gamma_1\), \(\beta_1\), and \(\delta_1\) are functional parameters, and \(F(x,y)\) and \(G(x,y)\)

Fig. 8. Comparison between the spatial distributions of soil moisture using two alternative fractal interpolation methods against the observations for wet soil conditions (June 30) and dry soil conditions (July 3). [BB—Bindlish & Barros (1996); KB—this work].
represent soil texture (e.g., sand content) and vegetation water content, respectively.

As stated earlier, the approach proposed here differs from Xie & Sun (1997) in that the fractal interpolation functions reflect the physical controls exerted by soils and vegetation on the spatial structure of soil moisture (Kim & Barros, 2002). This is accomplished by incorporating explicit relationships between soil moisture and existing soil texture and vegetation water content indices at resolution $\eta h$ ($\eta \leq x$), and thus vary from grid-cell to grid-cell within $C^h$. Note that in Xie & Sun (1997), the vertical scaling surface is a uniform surface [i.e., $r_1(x,y) = \text{const, } \forall \{x,y\} \in C^h$].

A more general contraction mapping function $I_{n,m}(x,y,s)$ is proposed here for the vertical dimension $P_{i,j}^{n \rightarrow 2h} = (I_{n,m} (x,y,s); S^h_{i,j} \rightarrow S^{2h}_{(i,n \cdot j,m)}):

$$I_{n,m}(x',y',s') = H_{n,m}(x',y',s') \times r_2(x',y')$$

(10)

where the weighting surface $r_2(x,y)$ is an amplification function used to adjust the magnitude of the roughness of

Fig. 9. Comparison of spatial statistics of observed and downscaled soil moisture fields for three different days using two-dimensional correlograms: (a) June 30; (b) July 3; and (c) July 12.
the downscaled soil moisture fields \( r_2(x,y) = \gamma_2 \cdot F(x,y) + \beta_2 \cdot G(x,y) + \delta_2 \), \( F(x,y) \) and \( G(x,y) \) are the normalized distributions of field parameters as described above, and \( \gamma_2, \beta_2, \) and \( \delta_2 \) are functional parameters. The vertical contraction mapping must preserve the mean of the original data, that is, the original data at coarse resolution can always be recovered exactly (reversible mapping). Finally, the system of equations defined in Eq. (11) below is solved to obtain the downscaled soil moisture field \( O^{bh}_{i,n,j,m} = L^{bh}_{i,n,j,m} \cdot X^{bh}_{i,n,j,m} \cdot Y^{bh}_{i,n,j,m} \cdot S^{bh}_{i,n,j,m}, \) and subsequently repeated for \( \forall \Omega^i_{i,j} \subseteq \Omega^h \) to produce the downscaled image \( \Omega^h \).

\[
\begin{align*}
\frac{s_{n-1,m-1}^{ij}}{s_{n,m-1}^{ij}} &= I_{n,m}(x_{i,j}^{N}N_{a,b}^{ij} s_{n,m}^{ij}) \\
\frac{s_{n-1,m-1}^{ij}}{s_{n,m}^{ij}} &= I_{n,m}(x_{i,j}^{N}M_{a,b}^{ij} s_{n,m}^{ij}) \\
\frac{s_{n-1,m-1}^{ij}}{s_{n,m}^{ij}} &= I_{n,m}(x_{i,j}^{N}M_{a,b}^{ij} s_{n,m}^{ij}) \\
\frac{s_{n,m}^{ij}}{s_{n,m}^{ij}} &= I_{n,m}(x_{i,j}^{N}M_{a,b}^{ij} s_{n,m}^{ij})
\end{align*}
\]

(11)

The essence of the downscaling algorithm is therefore to repeat the contraction mapping defined by the Iterated Function System (IFS) on the mapping window \( \Omega^h \mid i,j; \)

\[
IFS^{ij}_{i,n,m}(x_{i,j}^{M},y_{i,j}^{M},s_{i,j}^{bh}) = (p_{m}(x_{i,j}^{M}), q_{m}(y_{i,j}^{M}), I_{n,m}(x_{i,j}^{M},y_{i,j}^{M},s_{i,j}^{bh}))
\]

(12)

According to Barnsley (1986), Xie & Sun (1997) proved that the fractal surface generated by the IFS in Eq. (12) is unique. The implication of this result is that for any \( r_1(x,y) \) and \( r_2(x,y) \), the downscaled image \( \Omega^h \) is unique, and thus, the downscaling problem is posed as follows:

\[
\Omega^h = \{ [IFS^{ij}_{i,j}(x_{i,j}^{M},y_{i,j}^{M},s_{i,j}^{bh}) : O^h_{i,j} ; \forall C^h \}
\]

(13)

The six functional parameters \( \{ (\gamma_1, \beta_1, \delta_1), (\gamma_2, \beta_2, \delta_2) \} \) can be obtained by calibration against high-resolution data. Adaptive calibration using more than one pixel or image may be required if only a reduced number of observations are available within a given pixel, and thus if there is large uncertainty in characterizing the spatial variance of soil moisture at the desired resolution. Because both scaling functions \( r_1(x,y) \) and \( r_2(x,y) \) reflect the relative contribution of soil and vegetation controls on soil moisture distribution, the parameters may need to be recalibrated when the spatial structure of vegetation change significantly with time (e.g., harvest, clear-cutting, etc.). Furthermore, in the case of multifractal sets such as soil moisture, the functional parameters should be a function of soil moisture level to capture temporal changes in the relationships with ancillary data according to the prevailing soil moisture regime (Kim & Barros, 2002). This issue is revisited in Section 4 for the downscaling application to the SGP’97 data.

3. Description of data

The Southern Great Plains 1997 (SGP’97) Hydrology Experiment was a cooperative experiment between NASA, USDA, and several other government agencies and universities, and it was conducted between June 18 and July 17, 1997 in Oklahoma over a 10,000 km² area (Fig. 3). The Electronically Scanned Thinned Array Radiometer (ESTAR) passive imagery corresponds to a mapping area of \( 40 \times 250 \) km² at a resolution of \( 0.8 \times 0.8 \) km². Because of weather and calibration problems, only 16 of the 29 days of the experimental period are available (Fig. 4). For July 11, a large portion of the retrieved soil moisture image is missing, and thus it is not used in our analysis. The spatial distributions of soil texture, specifically vegetation water content derived from AVHRR data and sand content in the soil are shown in Fig. 5a and b, respectively. These data will be used here to estimate the vertical scaling functions \( r_1(x,y) \) and \( r_2(x,y) \). The data and details on the SGP’97 experiment can be found at the official USDA and NASA web sites, respectively: http://hydrolab.arsusda/sgp97, and http://daac.gsfc.nasa.gov/CAMPAIGN_DOCS/SGP97.

4. Downscaling application and discussion of model performance

4.1. Fractal interpolation design

To investigate the model performance, we used 15 soil moisture images from SGP’97. The application was designed to downscale soil moisture fields from 10 km (the best anticipated resolution of a future soil moisture mission) to a target resolution of 825 m (the actual resolution of the SGP’97 data). The coarse images were obtained by averaging the original data projecting an area corresponding to \( 12 \times 12 \) pixels at 825 m resolution to 1

![Fig. 10. Comparison of the observed and downscaled log–log relationships between aggregation area and the variance for the same dates in Fig. 9.](Image 309x89 to 541x256)
pixel at 10 km resolution. This process is consistent with the functional kernel of the passive microwave sensor for decreasing spatial resolution (Bindlish & Barros, 2002). The resolution of the ancillary data (Figs. 3 and 5a–b) is the same as the target resolution. Normalized vegetation water content and sand percentage were used as vertical scaling functions. The interpolation was repeated recursively with two target resolutions: (1) $\alpha = 1/4$, and (2) $\alpha = 1/3$. The mapping window was designed with $(4 \times 4)$ grid-cells for the first interpolation, and with $(3 \times 3)$ for the second. Data for June 27 and July 16, respectively, for wet and dry soil conditions, were used to estimate the functional parameters $\{(c_1, b_1, d_1), (c_2, b_2, d_2)\}$. The calibrated parameters were subsequently used for downscaling the remaining 13 soil moisture images.

To estimate the parameters in the vertical scaling functions, a multi-objective optimization procedure was implemented to minimize the downscaling errors simultaneously, hereafter understood as the differences between statistical and deterministic measures of the observed and downscaled fields: (1) errors in replicating the maximum, minimum, mean and standard deviations of the observed soil moisture distribution; and (2) errors in the scaling behaviors as described by the log–log plot of the variance vis-a-vis spatial resolution expressed in terms of area. Examples of error surfaces generated for a wide range of values of the functional parameters are shown in Fig. 6a–c, respectively, for the maximum (top panel), minimum (middle panel) and standard deviation (bottom panel) differences between the SGP’97 observations and the downscaled fields for June 27.

Fig. 11. Variation of the slope of the log–log relationships between aggregation area and the variance as a function of the order of the statistical moments of the observed and downscaled data for selected dates: (a) July 1; (b) July 2; (c) July 3; and (d) July 12.
The scaling behavior for parameter combinations corresponding to five arbitrary combinations of parameters marked as A, B, C, D, E and F in Fig. 6b is illustrated in Fig. 7 using the relationship between the log–log relationship between the power spectral density and the wavelength for the observed and downscaled data. The optimal estimates of the functional parameters for wet (June 27) and dry (July 16) conditions are reported in Table 1. Despite the multifractal nature of soil moisture, we do not see in this case a large difference in the functional parameters for two distinct soil moisture regimes. We attribute this weak sensitivity to the fact that the same normalized field of vegetation water content was used for both dates (Fig. 5a).

4.2. Analysis of results

Fig. 8a and b show images corresponding to the averaged data at coarse resolution (input), original SGP’97 data (observed), and downscaled images for two different dates. In particular, the results obtained using the downsampling method presented in this paper, denoted as [KB], are compared with the results obtained using a standard fractal interpolation method (Bindlish & Barros, 1996), denoted as [BB]. The two dates in Fig. 8a and b were selected to show the skill of the interpolation algorithm for a wide range of hydrometeorological conditions including wet (June 30) and dry (July 3) soils. The KB soil moisture fields exhibit much less spurious spatial variability than BB, and capture well the spatial structure and wet–dry gradients of the original data. Note that the functional parameters were estimated in a global sense, but calibration performed on a regional basis using spatially varying functional parameters should eliminate some of the problems in Fig. 8a and b, namely, the excessive influence of soil texture in the northern and central portions of the image.

The two-dimensional correlograms and the log–log plot of the variance vis-a-vis aggregation area compared in Fig. 9a–c and in Fig. 10 for three different days. Furthermore, Fig. 11a–d illustrate the relationship between order of moment and slope of the log–log relationship of variance and aggregation area for four sample days in the $a$- and $f$-scale ranges (see Fig. 1a), for areas below and above 100 km$^2$, respectively. The results show that the downscaled surfaces replicate the scaling behavior of soil moisture up to the third-order moment with small errors, and in some cases, even up to the fourth-order moment in both scale ranges.

In summary, analysis of the results shows that the downscaled fields reproduce both the spatial and temporal variability of the spatial structure of soil moisture observations. One possible inference of these results is that the downscaled fields reflect the control mechanisms by which the landscape governs soil moisture dynamics. In other words, by introducing the vertical scaling functions, which are shaped according to the spatial structures of vegetation and soil texture, these results suggest that we explicitly introduced physically based constraints in the standard contraction mapping model as per the hypothesis proposed by Kim & Barros (2002). This interpretation was tested by replicating with the downscaled fields the empirical orthogonal function (EOF) analysis presented by the same authors. The goal was to investigate whether the downscaled fields exhibited similar connections between the spatial variability of retrieved soil moisture and that of ancillary data. Topographic data (Fig. 3), and normalized volumetric water content (VWC), and percentage of sand in soils (PS) were used as the representative field parameters for the EOF analysis of soil moisture (Fig. 5a–b). Table 2 summarizes the results of the EOF analysis for June 30, and points to a close agreement between the percentage variance of the four EOF coefficients (principal components) and the corresponding EOFs for the original and downscaled fields. Fig. 12a and b further illustrate the agreement in the spatial patterns of the images for each of the principal components, thus suggesting that calibration of the functional parameters did not distort the natural modes of variability of soil moisture in the landscape suggested by Kim & Barros (2002).

Although topography does not play an important role in this particular data set, one expects that this will change for different regions of the world, and even within the same region depending on the temporal and spatial scales of interest. This research suggests, however, that as long as we can use high-resolution ancillary data as proxy indicators of relevant physical controls on soil moisture dynamics, the methodology proposed here will capture the physical modes of variability of soil moisture across spatial scales.

5. Summary

A downsampling model is presented which consists of a modified fractal interpolation method with contraction mapping. The model is different from other fractal interpolation schemes because it generates unique fractal surfaces. The model is different from other contraction mapping models
because it includes spatially and temporally varying scaling functions as opposed to single-valued scaling factors. These scaling functions are linear combinations of the spatial distributions of ancillary data.

The use of the model was demonstrated here using retrieved soil moisture from the SGP'97 field experiment. The data were first aggregated from 825 m to 10 km, and subsequently downscaled using the proposed model to 825 m. The results demonstrate that the downscaling model can capture all the basic statistics and scaling behavior of soil moisture at higher resolution for a wide range of environmental conditions including very wet and very dry soils.

Fig. 12. Empirical Orthogonal Function (EOF) coefficients describing the spatial modes of variability of (a) observed [SM$_{obs}$] and (b) downscaled [SM$_{dnsc}$] soil moisture fields with respect to the spatial variability of ancillary data for June 30. (EC1—normalized sand content in soils; EC2—input soil moisture field; EC3—normalized vegetation water content; EC4—topography as described by the digital elevation map). The ranges of variability of each principal component are shown at the bottom of each panel.
The significance of this contribution is that it makes the operational production of soil moisture fields possible at spatial resolutions useful for environmental applications from passive microwave remote-sensing products at coarser resolutions. Nevertheless, the practical application of this methodology hinges on the availability of soils, vegetation and terrain data at desired spatial resolutions globally, or at least over landscapes of interest.

Further testing and validation of the proposed downscaling model should be conducted in regions of more complex topography and more dense vegetation. Currently, we are working on developing a framework for mountainous terrain including explicit contributions of digital elevation data and precipitation fields in the vertical scaling functions, and adaptive estimation of functional parameters in both space and time.

Acknowledgements

This work was supported in part by NASA under Contract NAG5-7547 and by a Merck Faculty Fellowship to the second author.

References