A newsboy problem for an inventory system under an emergency order: a modified invasive weed optimization algorithm

Marziyeh Karimi, Seyed Hamid Reza Pasandideh & Amir Hossein Niknamfar


To link to this article: http://dx.doi.org/10.1080/17509653.2016.1151839

Published online: 16 Mar 2016.

Submit your article to this journal

Article views: 13

View related articles

View Crossmark data
A newsboy problem for an inventory system under an emergency order: a modified invasive weed optimization algorithm

Marziyeh Karimi, Seyed Hamid Reza Pasandideh and Amir Hossein Niknamfar

© 2016 International Society of Management Science and Engineering Management

Abstract
The newsboy problem has numerous applications for decision making in manufacturing and industrial environments. This paper presents a practical newsboy problem for an industrial system under both supplier quantity discounts and budget constraints where the storage space is stochastic and described using a normal distribution function. In this industrial system, when a shortage occurs, two strategies of the lost sale condition and the emergency order are available for the vendor as an option to fill the occurred shortage. This paper seeks to find the optimal order quantity for each product as well as to choose either the lost sale condition or the emergency order. The objective is to maximize the expected total profit of the vendor under uncertain demand and the stochastic storage space. As the proposed problem is NP-hard, a modified invasive weed optimization algorithm (IWO) is developed. The advantage of the proposed IWO is that it is capable of solving the proposed problem with both binary and continuous decision variables. As there is no benchmark available in the literature, an efficient genetic algorithm is designed to solve the problem and to compare the results obtained using IWO. Then, the algorithms are tuned using the response surface methodology and their performances are analyzed statistically. Finally, the applicability of the proposed approach and the solution methodologies are demonstrated. A sensitivity analysis on the number of products and discount segments indicates they have a significant impact on the vendor’s tendency in choosing either the emergency order or the lost sale condition.

1. Introduction
The stochastic single period inventory model so-called newsboy problem (NP) is a well-known stochastic problem in inventory control theory. There are numerous applications of the NP such as setting cash reserves by a bank or an individual for a checking account, college admissions, water reservoir management, staff sizing in a service business (Chhajed & Lowe, 2008), sport, apparel, service industries such as hotels and airlines (Alfares & Elmorra, 2005), scheduling, and evaluating advance orders in manufacturing environments (Kogan, 2004). NP has received extensive attention from both researchers and practitioners in terms of both theoretical and practical considerations over the past few years. The classical NP assumes that if the ordered items or products remain unsold at the end of the period, they are sold at a salvage value or are disposed of. An NP seeks to determine the quantity to be ordered for a given product in order either to minimize the costs or to maximize the profit in a single period (Taleizadeh, Niaki, & Hoseini, 2009). NPs have always been an important issue in inventory management (Chen & Chen, 2009), and also have wide applications in solving real-world inventory problems (Chen & Ho, 2013). Khouja (1999) classified published NP models into ten categories based on their degrees of complexity and capability in addressing real-life scenarios. Five out of them are as follows.

1. Extensions to different objectives and utility functions.
2. Extensions to different pricing policies and discounting structures.
3. Extensions to constrained multiple products.
4. Extensions to multiple products with substitution.
5. Other extensions.

Literature relating to current research can be viewed as limited to the second category, but also including multiple products and budget constraints.

Abdel-Malek and Montanari (2005) investigated the solution space for a multi-product NP under a budget constraint. Abdel-Malek, Montanari, and Meneghetti (2008) extended their previous work with a budget constraint to cover random yield scenarios, represented as the Gardener problem. A bi-level NP with fuzzy demands and discounts was developed by Ji and Shao (2006) in which the manufacturer gives the retailers either all-unit discounts or incremental discounts. However, they did not consider important issues in inventory control such as budget, storage space, and other constraints in their research. Zhang, Xu, and Hua (2009) presented a multi-product NP under a budget constraint in a classical inventory control, and then solved it using a binary solution method. An NP mathematical model with a simple reservation arrangement was presented by Chen and Chen (2009) along with a function
of the discount rate, represented as a willingness rate. A portfolio approach for a multi-product NP was described by Zhang and Hua (2010) to maximize profit in which a procurement strategy is established as a portfolio contract that contains a fixed-price and an option contract.

More recently, Qi, Ni, and Shi (2015) developed an NP for a one-manufacturer two-retailers supply chain taking into consideration customer market search behavior. In addition, they utilized game theoretic models to investigate the implications of this phenomenon in several scenarios where wholesale price is exogenous or endogenous. Table 1 presents some relevant studies on the proposed problem in the literature.

### Table 1. A summary of some studies of the newsboy problem.

<table>
<thead>
<tr>
<th>Authors</th>
<th>Method</th>
<th>Main contributions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abdel-Malek et al. (2008)</td>
<td>Nonlinear programming</td>
<td>Developing the model of Abdel-Malek and Montanari (2005) with a budget constraint to cover random yield scenarios</td>
</tr>
<tr>
<td>Ji and Shao (2006)</td>
<td>Bi-level NP with fuzzy</td>
<td>Considering both all-unit discounts and incremental discount</td>
</tr>
<tr>
<td>Chen and Chen (2009)</td>
<td>NP mathematical model</td>
<td>Developing a simple reservation arrangement along with a function of the discount rate</td>
</tr>
<tr>
<td>Zhang and Hua (2010)</td>
<td>Portfolio approach</td>
<td>Maximizing the profit in which a procurement strategy is established as a portfolio contract</td>
</tr>
<tr>
<td>Chen and Ho (2011)</td>
<td>Fuzzy NP with closed-form</td>
<td>Considering fuzzy demands and incremental discounts</td>
</tr>
<tr>
<td>Huang, Zhou, and Zhao (2011)</td>
<td>Nash equilibrium with an iterative algorithm</td>
<td>Developing a multi-product competitive NP associated with both partial product substitution and shortage penalty cost</td>
</tr>
<tr>
<td>Pando, San-José, Garcia-Laguna, and Sicilia (2013)</td>
<td>Nonlinear programming</td>
<td>Considering a certain fraction of shortage by describing a general backorder rate function</td>
</tr>
<tr>
<td>Chen and Ho (2013)</td>
<td>Fuzzy nonlinear programming</td>
<td>Applying fuzzy demands and quantity discounts in the NP</td>
</tr>
<tr>
<td>Abdel-Malek and Otegbeye (2013)</td>
<td>Duality approaches and separable programming</td>
<td>Developing a newsboy/gardener problem with uniform distribution for the supply and demand</td>
</tr>
<tr>
<td>Ding (2013)</td>
<td>Nonlinear programming with uncertainty theory</td>
<td>Developing a warehousing chance constraint based on uncertainty theory</td>
</tr>
<tr>
<td>Ding and Gao (2014)</td>
<td>Nonlinear programming with uncertainty theory</td>
<td>Applying worst-case and best-case demand scenarios</td>
</tr>
<tr>
<td>Kamburovski (2014)</td>
<td>Distribution-free NP</td>
<td>Demand only known to be non-skewed with given support, mean and variance</td>
</tr>
<tr>
<td>Kamburovski (2015)</td>
<td>Three cases of the risk-neutral NP</td>
<td>Developing an NP for a one-manufacturer two-retailers supply chain considering customer market search behavior</td>
</tr>
</tbody>
</table>
| Qi et al. (2015)             | Game theoretic models                            | Developing a new portfolio approach that considers both binary and continuous decision variables.

The objective is to maximize the expected total profit of the vendor under uncertainty as well as to choose either the lost sale condition or the emergency order condition subject to uncertain demand and stochastic storage space. As the NP is shown to be NP-hard (Abdel-Malek & Otegbeye, 2013), exact methods are not appropriate for solving it within a reasonable computational time. Therefore, a well-known meta-heuristic algorithm called a genetic algorithm is designed to solve the problem. Furthermore, this paper utilizes an invasive weed optimization (IWO) algorithm to solve the proposed problem. Unfortunately, the original IWO and most of its improved variants are implemented in continuous space, and hence are not suitable for solving binary optimization problems. To handle this challenge, this paper intends to develop a novel IWO. The advantage of the proposed IWO algorithm is that it can solve problems associated with both binary and continuous decision variables. Next, response surface methodology is utilized to tune the algorithm parameters, and then their results are compared on several numerical experiments. Moreover, the performance of the proposed algorithms is statistically analyzed. Ultimately, the impact of the number of products and discount segments on the objective function is surveyed.

In short, the contributions of the paper are as follows: (i) an emergency order strategy within the NP is proposed; and (ii) an efficient invasive weed optimization algorithm is developed. The application of the study is to generate additional opportunities for system-wide operational efficiency and cost effectiveness in manufacturing systems that utilize the NP approach in their inventory control system.

The remainder of the paper is organized as follows. Section 2 contains a problem description and a mathematical model of the problem. Section 3 discusses solving methodologies. In order to demonstrate the application of the proposed model and solution methodologies, several numerical experiments...
are investigated in Section 4. Finally, conclusions are provided in Section 5.

2. Problem description and mathematical formulation

As mentioned before, the proposed problem focuses on finding the optimal order quantity as well as choosing either the lost sale condition or an emergency order. Consider an industrial system where there is a vendor who orders several products \(i = 1, \ldots, n\) to a supplier with the following rules for placing the orders, which take place only once at the end of the period. In this system, the storage space for the purchased products is stochastic and follows a normal distribution function. It is assumed that the supplier provides quantity discounts on the purchasing prices, and that the vendor has a budget constraint for each product. As the order-processing times are very small compared with the cycle length, we assume that the lead time is equal to zero, which is common practice with NPs.

Additionally, when the inventory for each product exceeds the demand rate, the inventory holding cost at the end of period is calculated based on the holding cost and its scrap value.

In addition, the shortage occurs when the demand rate of each product is more than its inventory. However, two strategies – the lost sale condition and emergency orders – are available to the vendor, representing options for dealing with the occurrence of shortages. When a shortage occurs, some customers are given precedence to place an emergency order if they are willing to wait, whereas others will seek the product elsewhere. In this sense, a shortage is allowable and incurs the lost sale condition. Additionally, as customers are waiting for an emergency order to be fulfilled, the cost of customer waiting time is added to the total cost of the vendor. Note that, without loss of generality, it is assumed that the value of an emergency order for each product is equal to the value of the shortage that has occurred. In this way, the emergency order can be dispatched to the original or a different supplier. Therefore, the proposed problem considers the case whereby a shortage of each product can be compensated through an emergency order or incurs the lost sale condition. Under the emergency order strategy, the purchased quantity of the product does not include supplier quantity discounts. Hence, the cost of an emergency order for each product consists of the cost of purchasing the product without a discount plus the customer waiting cost.

This study assumes that the unit purchasing cost for emergency orders is more than the initial unit purchasing cost. This is because emergency orders involve higher administrative and manufacturing costs due to lack of production planning and the need to supply these products with a lower lead time than the initial order. Considering these assumptions, a mathematical model is presented for the proposed NP in order to maximize the expected total profit under stochastic storage space, supplier quantity discounts, and budget constraint. To suit a real-world NP problem, the model presented by Zhang (2010) has been used as the inspiration for a base model to formulate the problem. From the above discussion, we will address the following aspects.

- Determination of the purchased quantity.
- Choice of the discount segment for each product.
- Choice of either an emergency order or the lost sale condition for each product.

Before presenting the model, indices, parameters, and the decision variables are defined as follows.

Indices

- \(i = 1, \ldots, n\): Index of a product
- \(j = 1, \ldots, k\): Index of discount segment \(j\) for product \(i\) offered by a supplier
- \(k\): The number of quantity discounts for product \(i\) offered by a supplier

Parameters

- \(p_i\): Unit sales revenue of product \(i\)
- \(C_i''\): The budget limitation of the vendor
- \(c_{ij}^p\): The unit discounted price of product \(i\) on discount segment \(j\)
- \(C_i'\): The unit purchasing cost of product \(i\) where an emergency order is used
- \(q_{ij}^L\): Lower bound of order quantity of product \(i\) on discount segment \(j\)
- \(q_{ij}^U\): Upper bound of order quantity of product \(i\) on discount segment \(j\)
- \(D_i\): Demand rate of product \(i\)
- \(f_i(D_i)\): The probability density function followed by the demand of product \(i\)
- \(\pi_i\): Shortage cost of product \(i\) where the emergency purchase is not used
- \(h_i\): The holding cost of product \(i\) at the end of the period
- \(\tilde{\pi}_i\): Customer waiting cost for product \(i\) where the emergency purchase is used
- \(L_i\): Customer waiting time for product \(i\) where the emergency purchase is used
- \(A_i\): Fixed ordering cost of product \(i\)
- \(w_i\): Necessary storage space for product \(i\)
- \(\mu_w\): Mean of the available storage space
- \(\sigma_w\): Standard deviation of the available storage space

Decision variables

- \(Q_i\): Quantity of the purchased product \(i\)
- \(Q_{ij}\): Quantity of the purchased product \(i\) at price discount segment \(j\)
- \(y_{ij}\): 1 if product \(i\) is purchased at price discount segment \(j\), 0 otherwise
- \(y_i\): 0 if the emergency order is used for product \(i\), 1 otherwise
wwwThus, when an emergency order is chosen for product \( i \), i.e. \( y_i' = 0 \), the cost of an emergency order for each product includes the cost of purchasing the product without a discount:

\[
\sum_{i=1}^{I} (1 - y_i') C_i'(D_i - Q_i) f_i'(D_i) \, dD_i
\]

and the customer waiting cost is as follows:

\[
\sum_{i=1}^{I} (1 - y_i') \bar{x}_i L_i P\{D_i > Q_i\}
\]

Finally, the objective of maximizing the expected total profit is mathematically expressed as follows:

\[
\text{Max } Z = \sum_{i=1}^{I} \left[ p_i D_i - h_i (Q_i - D_i) \right] f_i(D_i) \, dD_i
\]

\[
+ \sum_{i=1}^{I} \left[ Q_i y_i' + (1 - y_i') D_i \right] p_i f_i(D_i) \, dD_i
\]

\[
- \sum_{i=1}^{I} y_i' \sigma_i (D_i - Q_i) f_i(D_i) \, dD_i
\]

\[
- \sum_{i=1}^{I} \left( 1 - y_i' \right) C_i'(D_i - Q_i) f_i(D_i) \, dD_i
\]

\[
- \sum_{i=1}^{I} \left( 1 - y_i' \right) \bar{x}_i L_i P\{D_i < Q_i\}
\]

\[
- \sum_{i=1}^{I} \sum_{j=1}^{J} c_{ij} Q_{ij} - \sum_{i=1}^{I} A_i
\]

subject to:

\[ Q_{ij} \leq q_{ij} y_{ij} \quad \forall i, j \]

\[ Q_{ij} \geq q_{ij} y_{ij} \quad \forall i, j \]

\[ Q_i = \sum_{j=1}^{J} Q_{ij} \quad \forall i \]

\[ \sum_{j=1}^{J} w_{ij} Q_{ij} + Z_{ij} \sigma_{ij} \leq \mu_{ij} \]

\[ \sum_{i=1}^{I} \sum_{j=1}^{J} c_{ij} Q_{ij} + \sum_{i=1}^{I} \int_{Q_i}^{\infty} (1 - y_i') C_i'(D_i - Q_i) f_i(D_i) \, dD_i \leq C'' \]

\[ \sum_{j=1}^{J} y_{ij} = 1 \quad \forall i \]

\[ Q_i, Q_{ij} \geq 0 \quad y_i', y_{ij} \in \{0, 1\}. \]

It is clear that the above model is a mixed integer nonlinear programming model (MINLP). Equation (1) is the objective function to maximize the expected profit including the total expected revenue presented in the first term, minus the total cost at the end of the period including the shortage costs in the case of using an emergency order, the cost of purchasing, the customer waiting costs, and the fixed ordering costs. It can be seen that the case of an emergency order has been applied in the proposed objective function using the binary variable \( y_i' \).

Constraints (2) and (3) guarantee quantity purchased limitations from the supplier. In contrast, Constraints (4) describes that the purchased quantity of the product \( i \) is the sum of all order quantities of product \( i \) in different price segments. Constraint (5) limits the stochastic storage space while Constraint (6) shows the budget restriction with respect to the cost of purchasing in the case of using an emergency order. Constraint (7) ensures that only one discount segment is eventually applied to the purchased quantity. Note that this constraint describes the proposed discount segment method. Finally, Constraint (8) ensures non-negative and integral constraints for the decision variables. It should be noted that \( c_{ij} < 1 \) for all products where \( c_{ij} \) is the initial purchasing cost. In addition, \( q_{i-1} < q_{i} < q_{i+1} \) should be considered.

3. Solving methodologies

As the proposed model is an MINLP, which is an NP-hard problem (D’Ambrosio, 2010), exact methods are complex and are not very effective in solving the problem. Therefore, two meta-heuristic algorithms are designed to find near optimal solutions – invasive weed optimization and a genetic algorithm. The framework of these algorithms is described in the following.

3.1. Invasive weed optimization

The invasive weed optimization (IWO) algorithm was first introduced by Mehrabian and Lucas (2006) and was motivated by a common phenomenon in agriculture — the colonization of invasive weeds. The name “weed” is reserved for those plants that are vigorous and a serious threat to desirable cultivated plants such as trees, vines, shrubs, and herbs. In addition, they have shown a very robust and adaptive nature. The algorithm process is simple and effective in converging to the optimal solution. The efficiency and the effectiveness of IWO are tested in detail through a set of benchmark multidimensional functions, including ‘Sphere,’ ‘Griewank,’ and ‘Rastrigin’ by Mehrabian and Lucas (2006). IWO has found successful utilization since then in different optimization problems such as the tuning of a robust controller (Mehrabian & Lucas, 2006), optimal positioning of piezoelectric actuators (Mehrabian & Yousefi-Koma, 2007), designing an E-shaped MIMO antenna (Zhang, Wang, Cui, Niu, & Xu, 2009), studying electricity market dynamics (Sahraei-Ardakani, Roshanaei, Rahimi-Kian, & Lucas, 2008), and solving various electromagnetic problems (Karimkashi & Kishk, 2010). In this regard, Karimkashi and Kishk (2010) presented linear array antenna synthesis – a standard antenna engineering problem – as an example for the application of IWO. Finally, IWO was applied to a U-slot patch antenna so as to achieve the desired dual-band characteristics. Ramu Naidu and Ojha (2014) solved six nonlinear
constrained optimization problems using IWO. Additionally, Ojha and Ramu Naidu (2014) presented a combination of particle swarm optimization (PSO) and invasive weed optimization (IWO), and incorporating the stochastic ranking method to handle the constraints, called PSO-IWO-SR.

The distinctive features of IWO in comparison with other evolutionary algorithms are the ways of reproduction, spatial dispersal, and competitive exclusion (Abu-Al-Nadi, Alsmadi, Abo-Hammour, Hawa, & Rahhal, 2013). The pseudo code for this algorithm is given in Figure 1.

The algorithm process continues until reaching a maximum number of iterations. In IWO, only the plants with better fitness can survive and produce seeds, while the others are eliminated. In this regard, the plant with the best fitness is closest to the optimal solution. These processes are addressed in detail in the following.

### 3.1.1. Initialization

The IWO algorithm process starts with initializing a population. This means that a population of initial solutions is randomly generated over the problem space. The fitness of the weeds initialized is evaluated depending upon the fitness function or the objective function chosen for the optimization problem.

### 3.1.2. Reproduction

Each member of the population is allowed to produce seeds within a specified region centered at its own position. The number of seeds depends on its relative fitness in the population with respect to the best and worst fitness as illustrated in Figure 2 (Pourjafari & Mojallali, 2012). The number of seeds for each member begins with the value $S_{\text{min}}$ for the worst member and increases linearly to $S_{\text{max}}$ for the best member as follows.

$$
\text{Number of seeds around weed } i = \frac{F_i - F_{\text{worst}}}{F_{\text{best}} - F_{\text{worst}}} (S_{\text{max}} - S_{\text{min}}) + S_{\text{min}} \quad (9)
$$

### 3.1.3. Spatial dispersal

After the reproduction step, the obtained seeds are distributed randomly over the search space such that they abide near to the parent plant using normally distributed random numbers with zero mean and variance $\sigma^2$. In this matter, the standard deviation $\sigma$ will be reduced from a predetermined initial value ($\sigma_{\text{init}}$) to a final value ($\sigma_{\text{final}}$) in each generation so that the algorithm gradually moves from exploration to exploitation with increasing generations. To do so, the standard deviation is decreased as follows:

$$
\sigma_{\text{cur}} = \left(\frac{\text{iter}_{\text{max}} - \text{iter}}{\text{iter}_{\text{max}}}\right)^n (\sigma_{\text{init}} - \sigma_{\text{final}}) + \sigma_{\text{final}} \quad (10)
$$

where $\text{iter}_{\text{max}}$ is the maximum number of iterations, $\sigma_{\text{cur}}$ is the standard deviation at the current generation, and $n$ is the nonlinear modulation index.

### 3.1.4. Competitive exclusion

After passing some iteration, the number of produced plants in a colony reaches its maximum ($P_{\text{max}}$). Thus, it is necessary to generate a competition between plants to limit the maximum number of plants in a population. Each weed is allowed to produce seeds and then is allowed to spread over the space. When all seeds have found their position, they are ranked together
with their parents (Mehrabian & Lucas, 2006). In this step, due to fast reproduction, a competitive mechanism is done to eliminate the undesirable plants with poor fitness and to allow fitter plants for reproducing more seeds. In this way, both weeds and seeds are ranked as a colony of weeds and the ones with high fitness survive and then are allowed to replicate.

Unfortunately, the original IWO and most of its improved variants are implemented in continuous space, and hence are not suitable for solving binary optimization problems. Since IWO employs a real-valued vector representation, the question arises of whether it can also be used for problem domains that need binary encoding. Therefore, this paper uses an approach to tackle the binary variables of the proposed problem. In the following, we have described the implementation approach in the IWO algorithm for the binary sections of the chromosome. Suppose \( k \) is a discrete variable in which \( 1 \leq k \leq N + 1 \). As this algorithm is applicable for only continuous variables, we define a function over a certain continuous space by using an integer output from the set \( \{1, \ldots, N\} \).

If \( 1 \leq x < N + 1 \), then the function of \( k = \lfloor x \rfloor \) would be appropriate for this purpose. By assuming \( 1 \leq x \leq N + 1 \), a discrete variable \( k \) can be obtained from the set \( \{1, \ldots, N\} \) using the mapping \( k = \min\{x, N\} \). To do so, a variable \( x \) is defined independently of the above set and then is converted to the variable \( x \); finally, the variable \( x \) would be converted to the variable \( k \). First, the variable \( r \) with \( 0 \leq r \leq 1 \) is selected randomly and independently of \( N \) and then is converted to \( 1 \leq r \leq N + 1 \) using the equation \( r = 1 + Nr \). In fact, this procedure can be described as the equation of a line between the two points \((0, 1)\) and \((1, N + 1)\). As a result, the variable \( x \) can be converted to the variable \( k \) using the mapping \( k = \min\{1 + Nr, N\} \).

Therefore, in the proposed IWO algorithm, we first define a continuous variable \( r \) between 0 and 1 (i.e. \( 0 \leq r \leq 1 \)) for the binary variable \( k \) and then create a new seed using normally distributed random numbers. At the end, the created seed can be converted to binary form using the mapping \( k = \min\{1 + Nr, N\} \).

### 3.2. Genetic algorithm

In recent years, genetic algorithms (GAs) have been developed to solve MINLP problems. Among the meta-heuristic algorithms, GA is a tool capable of solving MINLP models (Yokota, Gen, & Li, 1996). GAs have been applied successfully in various domains of artificial intelligence and optimization (Mousavi, Bahreininejad, Musa, & Yusof, 2014) and case-based reasoning systems (Passone, Chung, & Nassehi, 2006). Therefore, a GA is designed to solve the proposed model and also is used for comparison with the results obtained by IWO. The fundamental principle of GAs as introduced by Holland (1992) represents the features of problems by chromosomes. In GAs, the crossover and mutation operators are used with given probabilities. In general, a GA consists of the following steps (Ramezanian, Rahmani, & Barzinpour, 2012):

- **Step 1**: Create an initial population of chromosomes.
- **Step 2**: Evaluate the fitness of each chromosome.
- **Step 3**: Create new chromosomes by applying genetic operators such as crossover, mutation, and reproduction.
- **Step 4**: Evaluate the fitness of the new population of the obtained chromosomes.

#### 3.2.1. Chromosome representation

In GAs, a chromosome is a string or trail of genes that is represented as the coded figure of a solution (feasible or non-feasible). Designing a suitable chromosome is the most important stage in applying a GA to the solution process of the problem (Ramezanian et al., 2012). In this research, the chromosome is provided in four sections to satisfy all constraints all the time and hence to avoid generating infeasible solutions. The four sections of a chromosome are as follows.

- **Section 1**: An \( I \times J \) matrix \( Y_{ij} \).
- **Section 2**: An \( I \times J \) matrix \( Q_{ij} \).
- **Section 3**: An array \( Q_i \).
- **Section 4**: An array \( Y_{ij} \).

Each section of the chromosome is related to our decision variables. Figure 3 illustrates the general form of the proposed chromosome. It should be mentioned that the solution representation in IWO is similar to the chromosome representation in a GA.

#### 3.2.2. Evaluation and initial population

When a GA is employed for an optimization problem, a fitness value, which is the value of the objective function (defined in Section 2), needs to be assigned to a chromosome as soon as it is generated. An initial population (or a batch of chromosomes) is generated randomly. However, some of these chromosomes may not be feasible; so the generation of the chromosomes is controlled via a penalty method to generate feasible chromosomes.

#### 3.2.3. Selection

Selection creates an opportunity to deliver the gene of a good solution obtained to the next generation. In this paper, the roulette wheel of selection is utilized where chromosome selection in a mating pool is based on their probability of selection. The probability of selection of each chromosome is evaluated based on its fitness value.

#### 3.2.4. Crossover operator

Crossover exchanges some of the genes of chromosomes through the breakage and reunion of two selected chromosomes in order to generate a number of children.
predetermined parameter \( P_c \) is used to represent the probability of the crossover operator applied in a population. The arithmetic crossover is utilized for this purpose. Let us consider that \( X_i = (x_{i1}, x_{i2}, ..., x_{in}) \) and \( X_2 = (x_{21}, x_{22}, ..., x_{2n}) \) are selected as two parents. If \( a = (\alpha_1, \alpha_2, ..., \alpha_n) \), where \( 0 \leq \alpha_i \leq 1 \), the children are generated as follows:

\[
y_{1i} = \alpha_i x_{1i} + (1 - \alpha_i) x_{2i}, \quad i = 1, ..., n
\]
\[
y_{2i} = \alpha_i x_{2i} + (1 - \alpha_i) x_{1i}, \quad i = 1, ..., n
\]

It is worthwhile to note that the arithmetic crossover operator has a good performance in continuous space (Ramezanian et al., 2012). It is necessary to note that this operator is only applied for continuous-coded sections in the chromosome. For binary-coded sections of the chromosome, a two-point crossover is chosen to explore the solution space. In the two-point crossover, two positions are selected uniformly at random and the variables exchanged between the individuals between the points. Then two new offspring are produced.

### 3.2.5. Mutation and stopping criterion

The mutation operator makes an offspring solution by randomly modifying the parent’s features. This operator helps to generate a reasonable level of diversity in the population. It is worthwhile to note that the arithmetic crossover operator also serves the search by jumping out of local optimal solutions. According to our investigations in numerous experiments, \( S_{\text{max}} \) and \( S_{\text{min}} \) should be calibrated to 10 and 1, respectively. However, \( N_0, \text{iter}_{\text{max}}, S_{\text{min}}, \) and \( S_{\text{final}} \) should be tuned using RSM regarding the problem dimension.

RSM is a collection of statistical and mathematical techniques that are useful for modeling and analyzing problems in which a response of interest is influenced by several variables, and the objective is to optimize this response (Najafi, Niaki, & Shahsavard, 2009). Usually, the first step is to fit a first-order model and conduct a test of lack of fitness. Since the first-order model was inadequate and a small \( p \)-value (\( p = 0.0001 \)) was obtained for the lack of fitness test, a second-order model is utilized. The most popular second-order model in the literature is central composite design (CCD), which is as follows:

\[
E(Y) = \beta_0 + \sum_{i=1}^{k} \beta_i X_i + \sum_{i=1}^{k} \beta_{ii} X_i^2 + \sum_{i<j}^{k} \beta_{ij} X_i X_j
\]

where \( E(Y) \) is the expected value of the response variable, \( \beta_0, \beta_i, \beta_{ii}, \beta_{ij} \) are the model parameters, \( X_i \) and \( X_j \) are the input variables that affect the response \( Y \), and \( k \) is the number of factors. In this research, \( k \) factors that affect the response in the GA are population size (\( \text{PopS} \)), the maximum number of generations (\( \text{MaxG} \)), the crossover probability (\( P_c \)), the mutation probability (\( P_m \)), and the problem size (\( \text{dim} \)). In addition, as mentioned before, five factors affect the response in IWO. In this research, there are 2\( k+1 \) factorial points (fractional factorial), \( n \) central points (0,0,0,0,0), and 2\( k \) axial points (± 1, 0, 0, 0, 0), (0, ± 1, 0, 0, 0), (0, 0, ± 1, 0, 0), (0, 0, 0, ± 1, 0), (0, 0, 0, 0, ± 1). Since \( k \) is equal to 5 in this model, the total number of designed experiments is equal to 32 for both algorithms. Three levels of these factors are listed in Table 3.

In order to tune the algorithm parameters, three different problem sizes including 3, 5, and 7 products with three discount segments are considered. The values of the model parameters are generated from Table 4. The lower and upper bounds of some of these ranges are selected based on the case study presented by Taleizadeh et al. (2009) and Zhang (2010).

In Table 4, the term ‘\( U \)’ is related to the uniform distribution. The experiments are coded with MATLAB® 7.8 (R2009a) software. The experimental results are analyzed with Minitab® software.
16.2.2 software. The insignificant terms were eliminated with respect to their P-value. Therefore, second-order coefficients ($p < 0.05$) for IWO are listed in Table 5 and ANOVA is presented in Table 6.

The level of significance is 5%. Since the value of $F_o$ (30.11) is large, it is concluded that at least one variable has a non-zero effect and the regression is significant at a level of 5% ($p < 0.05$). On the other hand, the values of $R^2$ and $R^2_{adj}$ are 98.21 and 94.95% respectively; so the proposed regression model has been fitted well. In addition, the lack of fitness is not significant at the 5% level ($p > 0.05$), which confirms the good predictability of the model. Hence, the model is adequate. In Figures 4 and 5, one surface plot and an RSM contour plot for the obtained responses (profits) with regard to $\sigma_{\text{init}}$ (Zinit) and $\sigma_{\text{final}}$ (Zfin) are presented, respectively. The contour plots indicate that the highest profit (response) is obtained when both the $\sigma_{\text{init}}$ and $\sigma_{\text{final}}$ levels are high. In addition, the surface plot shows the shape of the response surface and gives a general idea of the profit for various settings of $\sigma_{\text{init}}$ and $\sigma_{\text{final}}$. Finally, the estimated regression function of the model fitness for IWO is given in Equation (13).

$$\text{fitness} = 25422540 - 6883 \ast (\text{dim}) + 7587 \ast (N_d) + 12100 \ast (\text{IterM}) + 6101 \ast (\text{Zinit}) + 5952 \ast (\text{Zfin}) + 29329 \ast (\text{dim}) \ast (\text{dim}) - 22362 \ast (N_d) \ast (N_d) - 14347 \ast (\text{IterM}) \ast (\text{IterM}) - 12474 \ast (\text{Zinit}) \ast (\text{Zinit}) - 11296 \ast (\text{Zfin}) \ast (\text{Zfin}) - 15545 \ast (\text{dim}) \ast (N_d) + 12510 \ast (\text{dim}) \ast (\text{IterM}) - 7001 \ast (\text{dim}) \ast (\text{Zinit}) - 7561 \ast (N_d) \ast (\text{IterM}) - 17227 \ast (N_d) \ast (\text{Zfin}) - 10538 \ast (\text{IterM}) \ast (\text{Zinit}) + 13178 \ast (\text{IterM}) \ast (\text{Zfin}) + 11867 \ast (\text{Zinit}) \ast (\text{Zfin}). \quad (13)$$

Thus, the objective function shown in Equation (13) is maximized using the GAMS® software to obtain the best values of the parameters in their corresponding ranges. A similar approach is taken not only to tune the parameters of IWO but also to calibrate the parameters of the GA. Finally, the optimal values of the IWO and GA parameters are presented in Table 7, where the problem size (dim) is five products.

4. Computational results

In this section, in order to assess the applicability of the proposed model, 30 problems are randomly designed with respect to the parameters listed in Table 4. To validate the results of the proposed algorithm, the problems are solved using GAMS/ BARON® 23.5 software on an Intel® core™ i7, 3.23 GHz laptop with 512 MB RAM. Note that the branch-and-reduce optimization navigator (BARON®) presented by Tawarmalani and Sahinidis (2005) is the algorithm for global solution of NLP and MINLP. They showed that this algorithm reduces root-node relaxation gaps by up to 100% and expedites the solution process, often by several orders of magnitude. BARON® implements the branch-and-bound approach enhanced with a variety of constraint propagation and duality techniques to reduce the ranges of variables in the course of the algorithm (Sahinidis, 2013). In this paper, the demand rate for each product follows a uniform distribution as follows:

$$f(D_i) = \frac{1}{b_i - a_i} \quad (14)$$

where $a_i$ and $b_i$ are the lower and upper bounds of the demand rate for product $i$, respectively. Both the GA and IWO are first implemented in three independent runs to solve each problem with the same input parameters. Then, the best fitness value of the problem and its corresponding CPU time (in seconds) are considered for comparison. The solutions of 30 different problems of various sizes obtained using the GA, IWO, and BARON® are presented in Table 8 with three categories. The second column in Table 8 represents the size of the problems. For example, 6*7 means there are six products and seven discount segments.

In order to validate the results obtained using the algorithms, they are compared with the results of BARON® by a quality measure called the relative percentage deviation of the objective function, which is defined for each solution as follows:

$$\text{Deviation}_{\text{obj}} = \frac{z_{\text{algorithm}} - z_{\text{BARON}}}{z_{\text{BARON}}} \times 100\% \quad (15)$$

A lower value of this measure implies a good performance of the proposed algorithm. It can be seen that both algorithms provide a near optimal solution. For instance, in Problem 27 the optimum result is $1,654,800$ while the results for the GA and IWO are $1,644,890$ and $1,653,437$, respectively. The values of the percentage deviation of the GA and IWO for this problem are 0.599 and 0.082, respectively. As shown in Table 8, the maximum percentage deviation in the GA is 24.748% while in IWO it is 5.433%. As a result, the results obtained by the GA and IWO are valid and near optimal. In terms of the percentage deviation, the results show that the IWO algorithm works better than the GA. On the other hand, the GA has better performance in terms of the computation time on some problems. Figure 6 displays the percentage deviation of the objective function for both algorithms with respect to the values listed in the Table 8.

Table 4. Data generation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>$\sim (55, 150)$</td>
</tr>
<tr>
<td>$A$</td>
<td>250</td>
</tr>
<tr>
<td>$c$</td>
<td>$\sim (5, 100)$</td>
</tr>
<tr>
<td>$x_i$</td>
<td>$\sim (5, 10)$</td>
</tr>
<tr>
<td>$\pi$</td>
<td>$\sim (2, 50)$</td>
</tr>
<tr>
<td>$d$</td>
<td>$\sim (15, 140)$</td>
</tr>
<tr>
<td>$d'$</td>
<td>$\sim (65, 190)$</td>
</tr>
<tr>
<td>$w$</td>
<td>$\sim (0, 10)$</td>
</tr>
<tr>
<td>$a$</td>
<td>$\sim (15, 140)$</td>
</tr>
<tr>
<td>$b$</td>
<td>$\sim (65, 190)$</td>
</tr>
<tr>
<td>$C$</td>
<td>70,000,000</td>
</tr>
<tr>
<td>$L$</td>
<td>$\sim (23, 120)$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$\sim (25,000, 300,000)$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>$\sim (30, 50)$</td>
</tr>
<tr>
<td>$h$</td>
<td>$\sim (1, 6)$</td>
</tr>
</tbody>
</table>
Moreover, it can be concluded that, in percentage deviation of the objective function is 5.433 (lower than that of the GA). Moreover, it can be concluded that, in percentage deviation of the objective function is 5.433 (lower than that of the GA). In this subsection, to assess the performance of IWO and compare it to the GA, the results of both algorithms are statistically analyzed in Table 8 by paired samples t-tests. It is worthwhile to mention that paired samples t-testing is a useful tool for testing the mean of the values of the expected total profit and computation times, and the relative percentage deviation obtained by IWO on all test problems is lower than that obtained by the GA. However, it can be seen from Table 9 that the GA uses less computation time than IWO on each problem. The comparisons in terms of the CPU times reveal that increasing the number of products and discount segments increases the computation times. Figure 7 shows the convergence curves of both algorithms on Problems 12 and 31. It can be seen that the proposed IWO searches out the better solutions, can provide better global searching ability, and can prevent trapping in local optima.

### Table 5. Estimated regression coefficients for $Y$

<table>
<thead>
<tr>
<th>Term</th>
<th>Coeff.</th>
<th>SE Coeff.</th>
<th>$T$</th>
<th>$P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>25,422,540</td>
<td>4737</td>
<td>5366.897</td>
<td>0.000</td>
</tr>
<tr>
<td>$dim$</td>
<td>-6,683</td>
<td>2424</td>
<td>-2.839</td>
<td>0.016</td>
</tr>
<tr>
<td>$N_0$</td>
<td>7,587</td>
<td>2424</td>
<td>3.129</td>
<td>0.010</td>
</tr>
<tr>
<td>$iter_{max}$</td>
<td>12,100</td>
<td>2424</td>
<td>2.517</td>
<td>0.029</td>
</tr>
<tr>
<td>$\sigma_{init}$</td>
<td>6,101</td>
<td>2424</td>
<td>2.455</td>
<td>0.032</td>
</tr>
<tr>
<td>$dim*dim$</td>
<td>29,329</td>
<td>2193</td>
<td>13,375</td>
<td>0.000</td>
</tr>
<tr>
<td>$N_0^*N_0$</td>
<td>-22,362</td>
<td>2193</td>
<td>-10,198</td>
<td>0.000</td>
</tr>
<tr>
<td>$\sigma_{iter}^*\sigma_{iter}$</td>
<td>-14,347</td>
<td>2193</td>
<td>-6,543</td>
<td>0.000</td>
</tr>
<tr>
<td>$\sigma_{init}^*\sigma_{init}$</td>
<td>-11,296</td>
<td>2193</td>
<td>-5,151</td>
<td>0.000</td>
</tr>
<tr>
<td>$dim*iter$</td>
<td>-15,545</td>
<td>2969</td>
<td>-5,236</td>
<td>0.001</td>
</tr>
<tr>
<td>$dim*\sigma_{iter}$</td>
<td>12,510</td>
<td>2969</td>
<td>4,213</td>
<td>0.001</td>
</tr>
<tr>
<td>$N_0^*\sigma_{iter}$</td>
<td>-7,561</td>
<td>2969</td>
<td>-2,359</td>
<td>0.038</td>
</tr>
<tr>
<td>$\sigma_{iter}^*\sigma_{iter}$</td>
<td>-17,227</td>
<td>2969</td>
<td>-5,802</td>
<td>0.000</td>
</tr>
<tr>
<td>$\sigma_{init}^*\sigma_{init}$</td>
<td>-10,538</td>
<td>2969</td>
<td>-3,549</td>
<td>0.005</td>
</tr>
<tr>
<td>$iter_{max}^*\sigma_{init}$</td>
<td>13,178</td>
<td>2969</td>
<td>4,438</td>
<td>0.001</td>
</tr>
<tr>
<td>$\sigma_{init}^*\sigma_{init}$</td>
<td>11,867</td>
<td>2969</td>
<td>3,997</td>
<td>0.002</td>
</tr>
</tbody>
</table>

$S = 11876.1; R^2 = 98.21%; R^2(adj) = 94.95%.$

### Table 6. Analysis of variance for fitness.

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Seq SS</th>
<th>Adj SS</th>
<th>Adj MS</th>
<th>$F$</th>
<th>$P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>20</td>
<td>84,946,853,646</td>
<td>84,946,853,646</td>
<td>4,247,342,682</td>
<td>30.11</td>
<td>0.000</td>
</tr>
<tr>
<td>Linear</td>
<td>5</td>
<td>7,775,487,289</td>
<td>7,775,487,289</td>
<td>1,555,097,458</td>
<td>11.03</td>
<td>0.001</td>
</tr>
<tr>
<td>Square</td>
<td>5</td>
<td>57,235,950,937</td>
<td>57,235,950,937</td>
<td>11,447,190,187</td>
<td>81.16</td>
<td>0.000</td>
</tr>
<tr>
<td>Interaction</td>
<td>10</td>
<td>19,935,415,420</td>
<td>19,935,415,420</td>
<td>1,993,541,542</td>
<td>14.13</td>
<td>0.000</td>
</tr>
<tr>
<td>Residual error</td>
<td>12</td>
<td>1,551,453,313</td>
<td>1,551,453,313</td>
<td>141,041,210</td>
<td>3.41</td>
<td>0.100</td>
</tr>
<tr>
<td>Lack-of-fit</td>
<td>6</td>
<td>1,246,918,910</td>
<td>1,246,918,910</td>
<td>207,819,818</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pure error</td>
<td>5</td>
<td>304,534,404</td>
<td>304,534,404</td>
<td>60,906,881</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>31</td>
<td>86,498,306,960</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$S = 11876.1; R^2 = 98.21%; R^2(adj) = 94.95%.$

**Figure 4.** Surface plot of $Y$ versus $\sigma_{init}$ and $\sigma_{iter}$ for IWO.

4.1. **Statistical comparison**

In this subsection, to assess the performance of IWO and compare it to the GA, the results of both algorithms are statistically analyzed in Table 8 by paired samples t-tests. It is worthwhile to mention that paired samples t-testing is a useful tool for testing the mean of the values of the expected total profit and computation times, and the relative percentage deviation obtained by IWO on all test problems is lower than that obtained by the GA. The summarized results of the tests are presented in Table 10, in which the null hypothesis of the test for each one of the metrics at a confidence level of 95% is as follows.

1. The mean of the expected total profit obtained by the GA is equal to that of IWO.
2. The mean of the computation times required by the GA is equal to that of IWO.
3. The mean of the relative percentage deviation obtained by the GA is equal to that of IWO.
It can be observed that, in terms of the expected total profit and relative percentage deviation metrics, IWO has the more desirable performance, whereas the GA has the better performance in solving the test problems in terms of computation time. This is mainly because the proposed IWO needs a relatively long time to implement the competitive mechanism in order to eliminate the undesirable plants. Therefore, IWO provides better total profit than that obtained by the GA. That is to say, IWO provides better quality solutions. In this way, Figure 8 shows this effectiveness better. In other words, IWO provides better total expected total profit for the vendor compared to the GA.

### 4.2. Sensitivity analysis

The problems designed in Table 8 are classified into three categories to survey the effect of the discount segments and the number of products on the expected total profit. The first category contains Problems 1–10, where the aim is to analyze the influence of discount segment variety on the objective function in which the number of products is fixed at three and seven products.

It can be seen that as the number of discount segments increases from five to six, the expected total profit using the GA and IWO increases from $3,997,550 to $4,232,967 and from $4,413,993 to $4,696,193, respectively. As a result, increasing the number of discount segments would result in a substantial increase in the expected total profit. Note that this ascending trend can be seen in the other problems of this category.

The second category contains Problems 11–20. Unlike the problems in the previous category, the aim of this category is to analyze the influence of the extent of product variety on the objective function and computation time in which the number of discount segments is fixed at three and six segments. For example, the expected total profit for the GA increases from $3,128,285 in Problem 17 to $4,642,373 in Problem 18, while for IWO increases from $3,251,235 to $5,344,069. That is, the results show that in some problems similar to the previous category, the objective function increases with an increasing number of products.

The second category contains Problems 21–30 in which the aim is to survey the effect of the discount segments and the number of products on the expected total profit simultaneously. Note that this category of problems is of high importance because these problems reflect the effect of the emergency order strategy. In other words, this category investigates how often the vendor will face either the lost sale condition or an emergency order. To do so, the results of IWO are used because, as mentioned before, its results are better than those obtained by the GA.

![Contour plot of Y versus σ_{init} and σ_{final} for IWO.](image1)

![Percentage deviations of the algorithms.](image2)

**Table 7. Optimal values of IWO and GA parameters.**

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>GA</td>
<td>PopS</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>MaxG</td>
<td>357</td>
</tr>
<tr>
<td></td>
<td>P_c</td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td>P_m</td>
<td>0.2</td>
</tr>
<tr>
<td>IWO</td>
<td>N_p</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>iter_max</td>
<td>200</td>
</tr>
<tr>
<td></td>
<td>σ_{init}</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>σ_{final}</td>
<td>0.003</td>
</tr>
</tbody>
</table>

Finally, the last category contains Problems 21–30 in which the aim is to survey the effect of the discount segments and the number of products on the expected total profit simultaneously. Note that this category of problems is of high importance because these problems reflect the effect of the emergency order strategy. In other words, this category investigates how often the vendor will face either the lost sale condition or an emergency order. To do so, the results of IWO are used because, as mentioned before, its results are better than those obtained by the GA.

It can be seen that as the number of discount segments increases from five to six, the expected total profit using the GA and IWO increases from $3,997,550 to $4,232,967 and from $4,413,993 to $4,696,193, respectively. As a result, increasing the number of discount segments would result in a substantial increase in the expected total profit. Note that this ascending trend can be seen in the other problems of this category.

The second category contains Problems 11–20. Unlike the problems in the previous category, the aim of this category is to analyze the influence of the extent of product variety on the objective function and computation time in which the number of discount segments is fixed at three and six segments. For example, the expected total profit for the GA increases from $3,128,285 in Problem 17 to $4,642,373 in Problem 18, while for IWO increases from $3,251,235 to $5,344,069. That is, the results show that in some problems similar to the previous category, the objective function increases with an increasing number of products.

Finally, the last category contains Problems 21–30 in which the aim is to survey the effect of the discount segments and the number of products on the expected total profit simultaneously. Note that this category of problems is of high importance because these problems reflect the effect of the emergency order strategy. In other words, this category investigates how often the vendor will face either the lost sale condition or an emergency order. To do so, the results of IWO are used because, as mentioned before, its results are better than those obtained by the GA.
the GA. In order to analyze better, these problems are divided into three parts. The first part includes Problems 21–23 under a constant number of discount segments. As mentioned before, $y_i^e$ denotes if the emergency order is used for product $i$. In Problem 21 $y_i^e = (0,0)$ while in Problem 22 $y_i^e = (0,1,0,1,1,0)$. In contrast, this variable is calculated as $(0,0,1,0,1,1,1,0,1,1,0)$ in Problem 23.

According to the computational results of Table 8, it can be seen that increasing the number of discount segments may result in a reduction in the vendor's tendency toward an emergency order. On the other hand, decreasing the number of discount segments has a limited effect on increasing the willingness of the vendor to choose an emergency order.

At the end, the third part includes Problems 27–30 in order to analyze the simultaneous effects of the discount segments and the number of products on the selection of either the lost sale condition or an emergency order. In these problems, $y_i^e$ is calculated as $(0,0)$, $(0,1,0,1,0)$, $(0,0,1,0,1,1,1,0,1,1,0)$, and $(1,0,0,1,0,1,1,1,0,1,1,0)$. The results show that the simultaneous increase of the number of products and discount segments has a substantial effect on the vendor's tendency to choose the lost sale condition. On the other hand, their simultaneous reduction may increase the willingness of the vendor to choose an emergency order because this reduction would result in saving the available budget for the vendor, and therefore the vendor can utilize the emergency order strategy. Therefore, the results of the sensitivity analysis show not only that the number of discount segments may affect the vendor's behavior.

Table 9. Optimization results for large-size problems.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Size</th>
<th>Best</th>
<th>Mean</th>
<th>Worst</th>
<th>CPU time (s)</th>
<th>Best</th>
<th>Mean</th>
<th>Worst</th>
<th>CPU time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>10^5</td>
<td>9,956,055</td>
<td>9,359,246</td>
<td>9,015,125</td>
<td>127</td>
<td>10,730,290</td>
<td>10,349,103</td>
<td>10,166,923</td>
<td>75</td>
</tr>
<tr>
<td>32</td>
<td>20^7</td>
<td>15,040,962</td>
<td>14,542,286</td>
<td>14,125,885</td>
<td>142</td>
<td>19,682,856</td>
<td>19,673,032</td>
<td>19,460,168</td>
<td>312</td>
</tr>
<tr>
<td>33</td>
<td>50^8</td>
<td>36,998,015</td>
<td>36,012,003</td>
<td>34,854,114</td>
<td>168</td>
<td>44,333,317</td>
<td>42,499,909</td>
<td>41,559,396</td>
<td>675</td>
</tr>
<tr>
<td>34</td>
<td>80^9</td>
<td>49,089,336</td>
<td>48,859,156</td>
<td>48,785,322</td>
<td>182</td>
<td>61,357,781</td>
<td>60,986,867</td>
<td>60,447,836</td>
<td>1,023</td>
</tr>
<tr>
<td>35</td>
<td>100^10</td>
<td>68,952,125</td>
<td>67,623,598</td>
<td>66,845,368</td>
<td>1100</td>
<td>86,952,140</td>
<td>86,215,015</td>
<td>85,542,148</td>
<td>1,278</td>
</tr>
<tr>
<td>36</td>
<td>120^11</td>
<td>72,657,304</td>
<td>71,123,005</td>
<td>68,952,110</td>
<td>1109</td>
<td>100,124,236</td>
<td>98,745,225</td>
<td>97,596,556</td>
<td>1,422</td>
</tr>
<tr>
<td>37</td>
<td>150^12</td>
<td>89,503,556</td>
<td>87,997,806</td>
<td>87,126,189</td>
<td>1120</td>
<td>112,003,548</td>
<td>111,906,223</td>
<td>111,769,020</td>
<td>1,795</td>
</tr>
<tr>
<td>38</td>
<td>200^13</td>
<td>109,902,629</td>
<td>109,096,426</td>
<td>107,548,982</td>
<td>1132</td>
<td>128,948,619</td>
<td>127,559,406</td>
<td>125,048,529</td>
<td>2,089</td>
</tr>
<tr>
<td>39</td>
<td>250^14</td>
<td>127,397,268</td>
<td>126,554,896</td>
<td>124,442,018</td>
<td>1158</td>
<td>142,878,905</td>
<td>146,991,826</td>
<td>146,720,006</td>
<td>2,745</td>
</tr>
<tr>
<td>40</td>
<td>300^20</td>
<td>154,230,948</td>
<td>153,012,805</td>
<td>150,559,285</td>
<td>1183</td>
<td>172,808,773</td>
<td>170,910,206</td>
<td>169,218,665</td>
<td>3,078</td>
</tr>
</tbody>
</table>
5. Conclusion and future research

This paper has presented a mixed integer nonlinear programming model for a newsboy problem taking into account total discounts, budget restrictions, and stochastic storage space. The purpose of this study was to maximize the expected total profit by assuming that, when a shortage of each product occurs, two strategies are available for the vendor to deal with the occurrence of the shortage, i.e. the lost sale condition and an emergency order. This paper focused on finding the optimal order quantity, as well as choosing either the lost sale condition or an emergency order. In this way, the proposed model considered the shortage costs and emergency order costs consisting of the cost of purchasing without discount and the customer waiting cost for each product. As the proposed problem was NP-hard, two meta-heuristic algorithms, namely a GA and IWO, were developed to solve it in which their parameters were tuned using RSM.

Several problems of both small and large dimensions were investigated in order to demonstrate the efficiency of the
proposed model and the solution methodologies. The computational results and statistical analysis showed that the results of both algorithms were valid and near optimal at a confidence level of 95%. Moreover, the proposed IWO had a more desirable performance in terms of the objective function and therefore outperformed the GA. The results of a sensitivity analysis showed that increasing the number of products and discount segments could result in a substantial increase in the expected total profit. In addition, they had a substantial impact on the vendor’s decision to choose either an emergency order or the lost sale condition.

The results showed that the simultaneous increase of the number of products and discount factor segments had a substantial effect in the vendor’s tendency to choose the lost sale condition. On the other hand, their simultaneous reduction may increase the willingness of the vendor to choose an emergency order because this reduction would result in saving the available budget for the vendor and therefore the vendor could utilize the emergency order strategy. Additionally, the results of the sensitivity analysis showed not only that the number of products and discount segments had a substantial impact on the vendor’s decision to choose either the emergency order or the lost sale condition, but also suggested that the decision maker should choose an appropriate discount segment in order to optimize the expected total profit.

Finally, the results confirmed the applicability of the proposed model and the methodologies adopted to solve the problem. For extension to future work, the following are recommended.

- Another objective such as service level can be considered.
- Another probability demand function such as the Poisson can be studied.
- The proposed model can be developed with fuzzy parameters.

Acknowledgements

The authors would like to acknowledge the efforts and the consideration of the editor and all of the anonymous reviewers for their valuable comments and suggestions to improve the quality of the paper.

Disclosure statement

No potential conflict of interest was reported by the authors.

ORCID

Amir Hossein Niknamfar http://orcid.org/0000-0002-2368-4324

References


