Non-Statistical Based Robust Identification of a Lightly Damped Flexible Beam Using Kautz Orthonormal Basis Functions

by

Hadi Esmaeilsabzali, Allahyar Montazeri, Javad Poshtan and Mohammadreza Jahedmotlagh

reprinted from

Journal of LOW FREQUENCY NOISE, VIBRATION AND ACTIVE CONTROL

VOLUME 27 NUMBER 3 2008

MULTI-SCIENCE PUBLISHING COMPANY LTD.
Non-Statistical Based Robust Identification of a Lightly Damped Flexible Beam Using Kautz Orthonormal Basis Functions

Hadi Esmaeilsabzali, Allahyar Montazeri, Javad Poshtan and Mohammadreza Jahedmotlagh
Electrical Engineering Department, Iran University of Science and Technology, Narmak 16846, Tehran, Iran
E-mail <hadisabzali@ee.iust.ir>

Received 21st October 2007

ABSTRACT
Robust identification of a lightly damped flexible beam with parametric and non-parametric uncertainties is presented; however, the main concern is about non-parametric uncertainties which are high-frequency high-amplitude unmodeled dynamics. Our approach is based on worst-case estimation theory in which uncertainties are assumed to be unknown but bounded and produces so-called hard bounds on model errors. Based on this theory, two methods named “Set Membership” and “Model Error Modeling” has been applied. We have also examined two outbounding algorithms (ellipsoidal and paralleloptopic) to solve the Set Membership identification problem. In order to properly deal with high-amplitude non-parametric uncertainties, the proposed methods are compared. It is shown that the combination of Set Membership approach with Model Error Modeling technique will result in superior identification in that it can better handle high-frequency high-amplitude non-parametric uncertainties. For each method the mode obtained and its associated uncertainty bound is mapped to the frequency plane so that it can be utilised by robust controller design methods such as $H_\infty$.

Key Words: Identification, Set Membership, Model Error Modeling, Vibration Control, Flexible Structures

I. INTRODUCTION
Robust control theory plays an important role in the application of control theory to practical problems. The main concept is to consider a physical system as an uncertain model which may be represented as a family of mathematical models. Using robust control techniques, all models in this family will be stabilised in an appropriate manner. This family is almost described by a nominal model and a bounded uncertainty. Thus, in order to design a robust control system, it is customary to have not only a nominal model, but also an uncertainty bound associated with this nominal model. One approach to determining this uncertainty bound is to impose a maximum allowable value of it by the controller designer. Once this uncertainty bound is not well-approximated, in the way that it does not exhibit the real system’s uncertainties, the designed controller may be poor in either performance or stability when it is utilised for the real system. An occasional alternative is to use “Robust Identification” methods in order to obtain not only a nominal model of a system but also a real description of its (parametric and non-parametric) uncertainties [1]. Because of the widespread use of robust control techniques in practical problems, and weakness of classical identification methods to produce suitable models for robust control theory, robust identification is an area...
which has received growing interest from researchers since the beginning of 1990’s [2, 5]. Robust identification algorithms use a priori information of a system and its input-output data (posteriori information) to produce a nominal model and its associated uncertainty. Two main philosophies for the description of model uncertainties have been used. The first one is based on statistical assumptions and produces the so-called “soft bound” on model uncertainty. The second approach is based on deterministic hypotheses and gives “hard bound” on uncertainties. Indeed in this approach, uncertainties are assumed to be “Unknown but Bounded” (UBB) [1]. Deterministic hypothesis on model uncertainties, leads to “Set Membership” or “Worst Case Estimation” methodologies [1, 2].

In all system identification problems, perturbations potentially arise form two main sources: a variance error due to the measurement noise and a bias term due to the effect of unmodeled or hard-to-model dynamics, i.e. dynamics that have not been included by nominal estimated model- also known as model error. The nature of these two error types is quite different. Variance error, which produces parametric uncertainties, is generally uncorrelated with the input signal (in an open loop data collection case), while bias error, to which non-parametric uncertainties are related, strongly depends on the nominal model structure and also the experiment input signal [1].

Three main approaches for robust identification have been addressed in the literature, namely:

1. Stochastic Embedding (SE)
2. Model Error Modeling (MEM)
3. Set Membership (SM)

SE is a frequency domain method based on statistical hypotheses about uncertainties. For instance, in [3] unmodeled dynamics relevant uncertainties are represented by a non-stationary stochastic process whose variance increases with frequency.

MEM is indeed a model validation tool but is also used for the purpose of robust identification [4, 5]. In comparison with other methods, MEM is more general in the sense that it is a time-domain method that can handle both statistical and deterministic assumptions on uncertainties. It can be shown that the advantage the MEM method is its ability in making separating noise and unmodeled dynamics. This is shown later in the next section where the MEM method is explained.

Finally SM is a time/frequency domain method, based on deterministic assumptions on system perturbations. In fact, uncertainties assumed to be unknown but bounded by a suitable norm, hence the term “Non-Statistical” may be associated with this method. In early works the idea was used for state estimation [6, 7]. Later, SM theory was employed for system identification [8, 9]. Both parametric and non-parametric uncertainties can easily considered in SM identification problems. In [8], [9], [10] and [11] just parametric uncertainties are considered while [1], [4], [11], [13], [14], [15] and [16] deal with both parametric and non-parametric uncertainties. This approach to robust identification is more popular than SE and other statistical based approaches, since:

1. It avoids tedious statistical theories.
2. Hard bounds on system uncertainties are greatly used by various robust controller design techniques.
3. Because of their deterministic nature, non-parametric uncertainties cannot be described well through statistical specifications.
4. The only information that is needed about parametric and non-parametric uncertainties to be utilised in the SM identification procedure is some measurement of their magnitude; In other words, using this framework, the nonparametric uncertainties can be easily handled like a bounded noise.
In this paper SM and MEM methods have been applied. It must be stated that although MEM approach can be used based on either statistical or deterministic assumptions of uncertainties specification, we have used it in a deterministic framework. In fact, the Model Error Modeling identification that has been performed in this paper is based on the Set Membership approach which is a deterministic (non-statistical) method. This will be illustrated further in the next section where SM and MEM methods are explained.

Fundamentally, lightly damped flexible structures are distributed-parameter systems, and thus have infinite dimensional analytic models. In order to design a controller, one has to have a finite dimensional model. Using truncated or reduced-order models, “spill over effect” is a possible phenomenon. Spill over effect is called to the degradation of controller performance due to excitation of unmodeled dynamics [17]. To tackle this problem, a robust controller is a beneficial tool. Therefore, robust identification of lightly damped flexible structures is an evident necessity.

In [18], the frequency domain data has been used for the robust identification of a flexible satellite boom, which can be considered as a lightly damped flexible beam, in the SM framework. Both parametric and non-parametric uncertainties have been included in the identification problem. The a priori information on the system consists of some measure of the measurement noise, a bound to system frequency response, and also the stability margin. The nominal model structure is the linear combination of “Kautz” basis functions, which is the best choice for the systems of oscillating nature. The SM problem has been solved using the LMI (Linear Matrix Inequality) method. The non-parametric uncertainty is the “hard-to-model dynamics”, and not “the high-frequency unmodeled dynamics”. On the contrary, in [19], the non-parametric uncertainty is the high frequency unmodeled dynamic, although its amplitude is too low. In this work the frequency domain data, which has been generated using the analytic model of a beam, has been used for SM robust identification of a flexible beam model. The a priori information on the system is an approximation of the system is natural frequencies, a bound on their damping ratios, and also an upper bound on the measurement noise. Because of the nominal model structure, which is the linear combination of second order transfer functions with unknown numerator and denominator coefficients, the solution to SM identification problem may not have a linear framework. So, a suitable transform has been introduced in order to convert the problem in to a linear framework and then the LMI method has been used to solve it. Finally, in [20], frequency domain data have been used to identify just the nominal model of a flexible beam. The model structure is the linear combination of second order transfer functions with unknown coefficients whose natural frequency and damping ratios have been tuned by iteration according to measured data and the estimated model. The unknown coefficients have been determined using LSE (Least Square Estimation) and RPE (Restricted Projection Estimation) methods. Moreover, a method for optimising the identification quality is proposed. However, in this work nothing special has been done on the specification of uncertainty bound associated with the identified model.

The aim of this paper is to use time domain input-output data and the deterministic (non-statistical) specification of system uncertainties for the robust identification of lightly damped flexible beams in two case studies. In order to evaluate the capabilities of identification algorithms, in the first case study, identification has been done using simulated data. The identification experiment simulation has been done using the finite element model of a simply-supported beam. Then, in the second case study, a experimental beam with clamped-clamped boundary conditions has been studied. While in [18] there are not any un-modeled dynamics and in [19] the unmodeled dynamic has low amplitude, in our work the main concern is about high frequency unmodeled dynamics, which have high amplitude. We have shown that due to the high amplitude of non-parametric uncertainties, using the SM approach independently cannot be suitable, and by combining it by MEM technique, the quality of identification may be increased.
Indeed, as is mentioned earlier, that the MEM technique has the ability to make separation between parametric and non-parametric uncertainties is the key point to the solution of this identification problem.

In the next section we introduce the main concepts of SM and MEM identification methods with respect to our problem. Section 3 presents the robust identification results for two lightly damped flexible beams. Finally, section 4 concludes the paper.

2. ROBUST IDENTIFICATION PROBLEM FORMULATION
2.1. The Set Membership Approach
Suppose that N samples of input-output data, generated by real system \( G(q) \), are:

\[
y_m(k) = G(q)u_m(k) + v(k), \quad k = 1, 2, \ldots, N \tag{1}
\]

where \( v(k) \) is the measurement noise and is bounded by a suitable norm:

\[
\|v(k)\|_\beta \leq \delta(k). \tag{2}
\]

It is possible to represent the real system as follow:

\[
G(q) = G(q, \theta) + \Delta G(q) \tag{3}
\]

where \( G(q, \theta) \) is the parameterized nominal model and \( \Delta G(q) \) stands for possible unmodeled dynamics and is also bounded by a suitable norm in the space of transfer functions. For our identification problem, we choose \( \infty \)-norm. Using this, the effect of the frequency response amplitude of unmodeled dynamics can be considered effectively.

Regarding (3), the input-output relationship (1) can be presented as:

\[
y_m(k) = [G(q, \theta) + \Delta G(q)]u_m(k) + v(k) \tag{4}
\]

\[
y_m(k) - G(q, \theta)u_m(k) = \Delta G(q)u_m(k) + v(k) \tag{5}
\]

Applying \( \infty \)-norm for both sides, we will get:

\[
\|y_m(k) - G(q, \theta)u_m(k)\|_\infty = \|\Delta G(q)u_m(k) + v(k)\|_\infty \tag{6}
\]

\[
\|y_m(k) - G(q, \theta)u_m(k)\|_\infty \leq \|\Delta G(q)u_m(k)\|_\infty + \|v(k)\|_\infty \tag{7}
\]

\[
\|y_m(k) - G(q, \theta)u_m(k)\|_\infty \leq \|\Delta G(q)\|_1 + u_k + v_k \tag{8}
\]

where \( u_k = \|u_m(k)\|_\infty \), and \( v_k = \|v(k)\|_\infty \) is the parametric perturbation (noise) bound, coming from a priori information on the system to be identified.

Now, we have to compute the non-parametric uncertainty bound. Our approach is affected by what has been addressed in [2], [13] and [14]. Regarding a priori information, let \( M = \|\Delta G(q)\|_\infty \), which is the maximum amplitude of high-frequency unmodeled dynamics frequency response, and \( \rho < 1 \) the radius of a disk in the complex plane which contains unmodeled dynamics poles. Moreover, let \( \Delta g(k) \) be the unmodeled dynamics impulse response. Then:

\[
\Delta G(e^{i\omega}) = \sum_{h=0}^{\infty} \Delta g(h)e^{-j\omega h} \tag{9}
\]

\[
\frac{1}{2\pi} \int_{-\pi}^{\pi} \Delta G(e^{i\omega})d\omega = \frac{1}{2\pi} \int_{-\pi}^{\pi} \sum_{h=0}^{\infty} \Delta g(h)d\omega \tag{10}
\]
Using *a priori information*, it is possible to define a bound for unmodeled dynamics impulse response as follow:

$$\Delta g(h) \leq M \rho^h \quad h = 0, 1, 2, \ldots$$  \hspace{1cm} (12)

Considering (11), it is possible to write:

$$\|\Delta G(q)\|_1 \leq M \sum_{h=0}^{\infty} \rho^h$$ \hspace{1cm} (13)

$$\|\Delta G(q)\|_1 \leq \frac{M}{1 - \rho}$$ \hspace{1cm} (14)

Therefore, the non-parametric uncertainty bound may be computed as follow:

$$\gamma = \frac{M}{1 - \rho}$$ \hspace{1cm} (15)

Thus (8) can be expressed as:

$$\|y_m(k) - G(q, \theta)u_m(k)\|_\infty \leq w_k$$ \hspace{1cm} (16)

where $w_k = y_{u \ell} + v_s$. Another way to determine the perturbation bound ($w_k$) for the Set Membership problem is to use a constant upper bound instead of a variable bound. In order to do this, we can choose the maximum value of the variable perturbation bound over all $N$ samples, and consider it in (16) for all data samples. This may increase the volume of uncertainty bound but seems to be more practical.

Next, in order to complete the Set Membership inequality in (16), we have to determine the structure of $G(q, \theta)$. Different model structures are available for the nominal model [21]. Among them, output error (OE) structure is a popular and well established model structure. To avoid high computational complexity due to nonlinear optimisation in the process of parameter estimation and to obtain a linear-in-model structure, instead of a transfer function with unknown numerator and denominator coefficients, we use the linear combination of orthonormal basis functions for the OE model structure. This choice has another advantage in the way that much more *a priori information* can be imported to the identification algorithm by proper choice of basis functions. In other words, by selecting basis functions whose dynamics are close to the dynamics of the real system, it will be conceivable to estimate the nominal model by a minimum number of parameters [22, 23]. Due to the resonant nature of our system, we use the so-called “Kautz” or two-parameter basis functions [24]:

$$G(q, \theta) = \sum_{j=0}^{N} \theta_j \psi_j(q)$$ \hspace{1cm} (17)

$$\psi_{2j-1}(q, b, c) = \frac{\sqrt{1 - c^2}(q - b)}{q^2 + b(c - 1)q - c} \frac{1}{q^{j-1}}$$

$$\psi_{2j}(q, b, c) = \frac{\sqrt{1 - c^2}(q - b^2)}{q^2 + b(c - 1)q - c} \frac{1}{q^{j-1}}$$ \hspace{1cm} (18)

$-1 < b < 1 \quad -1 < c < 1$
where \( n \) is the order of nominal model, and \( \psi_i(q) \) is a Kautz basis function. Now from (16) and (17) we will get:

\[
\left\| y_m(k) - \sum_{i=0}^{n} \theta_i \psi_i(q) u_m(k) \right\|_\infty \leq w_k
\]

(19)

Or equivalently:

\[
\left\| y_m(k) - \theta^T x_m(q,k) \right\|_\infty \leq w_k
\]

(20)

where \( x_m(q,k) \) is the regression (information) vector computed as:

\[
\theta = [\theta_1 \theta_2 \ldots \theta_n]^T
\]

and \( \theta = [\theta_1 \theta_2 \ldots \theta_n]^T \) is the vector of parameters. For each time stamp \( k=1, 2, \ldots, N \), (20) produces a so-called strip in the space of parameters. By intersecting these strips, a “Feasible Parameter Set” (FPS) will be obtained as follow:

\[
\Theta = \{ \theta: \bigcap_{k=1}^{N} \left\| y_m(k) - \theta^T x_m(q,k) \right\|_\infty \leq w_k \}
\]

(22)

In fact, \( \Theta \) is the set of all parameters compatible with input-output data, \textit{a priori} information on system, and the uncertainty bounds. For the case that inequalities are linear in parameters, as in (22), the FPS is a convex polytope in the space of nominal model parameters. The aim of the Set Membership robust identification problem is to compute the FPS and to determine an optimal point in FPS (in some sense) as the nominal model parameters. However, exact computation of FPS and nominal model parameters is a laborious task and requires a large amount of numerical computation, and hence is not usable in practical situations [25, 26, 27]. An alternative is to outbound the FPS by simple geometrical shapes such as “Ellipsoid” and “Parallelotope” (Fig.1) and consider their centre as the parameters of the nominal model [8, 9, 10, 14]. Then, these shapes are mapped to the frequency domain (e.g. Nyquist plane) using pre-mentioned model structure in (17). As a result, a closed uncertainty bound with specified upper and lower border will be generated around the estimated nominal model. It is worth mentioning that because of the greater DOF of the parallelotopes, they can outbound the FPS more tightly than ellipsoids. Although this can be observed explicitly in Fig. 1, it will be verified with respect to identification results in next section.

Figure 1. Ellipsoidal and Parallelotopic estimation of FPS

2.2. Model Error Modelling Technique

“Model Error Modelling” (MEM) is a time-domain technique with various applications in the area of system identification such as model validation and direct Model Error Modelling (i.e. combining simple models to obtain a suitable model of a system and its uncertainties), which will be used in this paper. More details about various aspects of MEM can be found in Ljung’s survey paper [4].

Consider (1) and let \( G(q, \theta^*) \) be the nominal estimated model of the system in (1). Although it is possible to obtain this model by several identification methods, in this paper \( G(q, \theta^*) \) is estimated using SM method as mentioned in the previous subsection.

Let “residual” sequence to be computed as:

\[
e(k) = y_m(k) - G(q, \theta^*) u_m(k)
\]

(23)
It is possible to consider the “Error System” whose input and output are $u_m$ and $\varepsilon$, respectively. Let $G_e$ be this system’s identified model. So:

$$\varepsilon(k) = G_e(q)u_m(k) + v(k)$$  \hspace{1cm} (24)

where $v(k)$ is the measurement noise as in (1). $G_e$ is also known as “Model Error Model” and is the estimate of unmodeled dynamics relevant error. As for the nominal model, $G_e$ can be identified by any identification method. Here, in this paper, $G_e$ is identified again by the SM technique. In other words, using a priori information the system, it is possible to define a suitable parametric model structure for $G_e$. Then by input-output data in (23), the model error model and its associated uncertainty will be determined. It is a simple fact that if the uncertainty region of $G_e$ contains zero element, the nominal estimated model $G(q, \theta^*)$ will be unfalsified. Having the system’s nominal model $G(q, \theta^*)$ and $G_e$ along with its uncertainty, it is possible to obtain a complete model of the system in (1). This task can be done by adding up the nominal model and the uncertainty bound of $G_e$. The obtained complete model then can demonstrate the system in (1) and its uncertainties in a suitable manner. This method may be known as “combination of SM and MEM”. Finally, it is easy to verify that MEM technique has the ability of separating bias and variance errors completely. Due to this fact, recently this approach to robust identification attracts some interests [1, 5].

3. CASE STUDIES
3.1 Simulation Results
This subsection presents the identification results for a lightly damped simply-supported flexible beam (Fig. 2).

![Figure 2. The simply-supported beam under study](image)

<table>
<thead>
<tr>
<th>Table I.</th>
<th>The beam properties under study</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>500 mm</td>
</tr>
<tr>
<td>Width</td>
<td>20 mm</td>
</tr>
<tr>
<td>Thickness</td>
<td>1 mm</td>
</tr>
<tr>
<td>Modulus of Young</td>
<td>2.07e+11</td>
</tr>
<tr>
<td>Density</td>
<td>7800 Kg/m$^3$</td>
</tr>
<tr>
<td>Damping</td>
<td>5e-3</td>
</tr>
</tbody>
</table>

The flexible beam which is considered here is assumed to be of steel, the exact specifications of which are presented in Table I. The identification experiment has been simulated using a “Finite Element” model of the beam. The input and output time domain signals for identification are force and displacement, respectively (Fig. 3). The input signal is the combination of 180 sinusoids with proper frequencies that have been picked according to the a priori information of system, which in this case is the rough estimate of beam FRF, obtained through impulse response averaging methods (Fig. 4). Distribution of these frequencies is a key point in the experimental design: around the beam’s natural frequencies, these are too close, whereas, their intervals become wider between two consecutive natural frequencies of the beam. This way, the excitation signal would be properly designed by stimulating the beam resonance frequencies, thus ensuring the identification quality enhancement. In order to simulate measurement noise, the output signal has been corrupted by a normally distributed Gaussian random signal with the variance of 1%.
Our aim is to identify the two first modes of the system and consider the two last modes as non-parametric uncertainty. Moreover, it is desirable to determine the uncertainty bound that is tight and has small volume in the control bandwidth (low frequency region) resulting in a good performance by the designed controller, and becomes wide in the uncertainty region (high frequency) to completely cover the unmodeled dynamics, resulting in the stability of the designed controller.

Firstly examine the SM algorithm introduced in the last section. For this purpose, we use Kautz basis functions where their parameters \((b, c)\) have been tuned with respect to the a priori information of the system (FRF). Two different outbounding algorithms for approximation of the FPS have been used: the ellipsoidal and the parallelotopic, shown in Fig. 5 and Fig. 6, respectively.

In comparison with Fig. 5, it can be seen that the tightness of the parallelotopic approximation is better. This can also be verified by comparing the volume of ellipsoidal and parallelotopic bounds which are \(2.3265 \times 10^{-17}\) and \(7.3212 \times 10^{-19}\), respectively. As stated earlier, this is because of the greater DOF of the parallelotopes. So, they can enclose the FPS better than ellipsoids. However, from the robust controller design and non-parametric uncertainties aspect these are not satisfactory because the uncertainty bound for the ellipsoidal algorithm is too large and thus the designed controller will not have a good performance. For the parallelotopic algorithm, the identification results are better, and the "tightness" of the uncertainty bound is adequate, but the real model is not covered by the
uncertainty bound at some frequencies. Although this is not an important deficiency in the control bandwidth region, for the high frequency region corresponding to unmodeled dynamics it may cause instability.

In order to increase the quality of estimation we have used the MEM method for better handling of high-frequency nonparametric uncertainties. This method was also introduced in section 2. As is stated earlier for identification of the nonparametric uncertainties (error model), we use the SM method and the Kautz basis functions again. The result of this approach is shown in Fig. 7.

Figure 5. Real model (dotted) and nominal identified model (solid) with its associated uncertainty bound (ellipsoidal approximation)

Figure 6. Real model (dotted) and nominal identified model (solid) with its associated uncertainty bound (parallelotopic approximation)
It is evident from this figure that the uncertainty bound has a good “tightness” and the real system is included by this bound at all frequencies. It must be stated that in the high frequency region, which corresponds to the uncertainty region, what is important is the upper border of the uncertainty bound that will be used in the robust controller design schemes (such as the Small Gain theorem). Therefore, the lower border of this bound has no importance in this frequency region and it is customary to restrict it with a predetermined amplitude. Using such a model for robust controller design will result in good performance/stability results.

3.2 Experimental Results
In this subsection the above method has been used in order to obtain the nominal model and uncertainty bounds of a laboratory Clamped-Clamped beam (Fig. 8). The identification experiment was done in the Modal Analysis Lab at Iran University of Science and Technology (IUST). Exact specifications of the beam are presented in Table II. The input signal is the sum of 1510 sinusoids with different frequencies and random phase. These frequencies have been selected according to the beam’s FRF (Fig. 9). The input and output time domain signals are force and acceleration, respectively (Fig. 10). A shaker has been used to apply the input signal and the output signal is sensed using an accelerometer. The input-output data are processed and amplified using 3560c pulse system and 2706 amplifier. The whole equipment is made by B&K Corporation.

Table II
The under study laboratorial beam properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>500 mm</td>
</tr>
<tr>
<td>Width</td>
<td>35 mm</td>
</tr>
<tr>
<td>Thickness</td>
<td>2 mm</td>
</tr>
<tr>
<td>Modulus of Young</td>
<td>2.07e+11</td>
</tr>
<tr>
<td>Density</td>
<td>7800 Kgm$^3$</td>
</tr>
<tr>
<td>Damping</td>
<td>5e-3</td>
</tr>
</tbody>
</table>
Figure 8. The clamped-clamped beam under study

Figure 9. FRF of the beam

Figure 10. Input-output identification data
As for the previous case, the aim of identification is to consider the first two modes as the control modes, and two last modes as unmodeled dynamics. The nominal model structure is the linear combination of Kautz basis functions. The Kautz parameters \((b, c)\) have been tuned according to the beam's FRF. Comparing the FRF of two beams, it is evident that in the present case the third and forth modes, to be considered as non-parametric uncertainty, have greater amplitude than the first and second modes, which are control modes. It must be stated that we wish to handle the worst case in the identification of a beam in the way that the amplitude of the control modes is lower than those of the modeled dynamics. In order to have this condition, we have adjusted the sensor and actuator so that the uncertainty
modes have the highest amplitude. Using the SM approach independently, it is predictable that the identification results do not have good quality. Fig. 11 shows the result of using an ellipsoid outbounding algorithm in order to approximate the FPS. It is evident that although the volume of the uncertainty bound is too huge ($V_{\text{ellipsoid}} = 0.0028$), the unmodeled dynamics are not included by this bound. The result of parallelotopic approximation of the FPS is shown in Fig. 12. It has not much more advantage over the last one except that its volume is less than the ellipsoidal one ($V_{\text{paralleloptope}} = 9.0643 \times 10^{-5}$).

To deal with these problems; i.e. determining an uncertainty bound which has small volume in the control bandwidth and become wide in the uncertainty region, we have examined the combination of SM and MEM methods again. The result of this technique is shown in Fig. 13. Using this model, the controller may have a good performance/stability result.

4. CONCLUSION

In order to design robust controllers one has to have a suitable model which consists of the nominal model and some measure of its uncertainties. Robust identification methods provide such models that are indicate the real uncertainties of the system. One of these techniques is the SM method that is based on deterministic (non-statistical) assumptions on uncertainties. This type of uncertainty representation is frequently by various robust control methods. In this paper, this method is used for the purpose of robust identification of lightly damped flexible beams. The main contribution is the combination of the SM and deterministic-based MEM techniques in order to properly handling of high-frequency high-amplitude non-parametric uncertainties of a lightly damped flexible beam and obtain a suitable uncertainty bound, while this task can not be done using the SM method independently.

REFERENCES


