Swarm-based structure-specified controller design for bilateral transparent teleoperation systems via $\mu$ synthesis

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This paper proposes a novel robust controller design for bilateral teleoperation systems using $\mu$ synthesis based on an inspired algorithm namely particle swarm optimization with time-varying acceleration coefficients. In the proposed structure, the $\mu$ synthesis problem is considered as a constraint optimization problem. Indeed, this technique proposes a novel alternative solution for solving the $\mu$ synthesis problem to design simple structure controllers satisfying robust stability and performance. The goal of the proposed structure is to achieve complete transparency and robust stability for bilateral teleoperation systems with uncertainty in time delay and task environment. To this reason, two local controllers are designed. One local controller is responsible for tracking the master commands and the other local controller is in charge of force tracking as well as guaranteeing the stability of a closed-loop system. The comparative results demonstrate the effectiveness of the proposed controller compared with the results obtained from D-K iteration controller.

Keywords: teleoperation; heuristic algorithm; model reduction; $\mu$ synthesis; D-K iteration method.

1. Introduction

Teleoperation systems are tools that indicate the capability of a human being to carry out operations in a remote environment. By means of these systems, a human operator can interact with remote and unknown environments, which can be hazardous or inaccessible, such as mining, sub-sea exploration and, more recently, in health care. In a teleoperation system, master and slave systems, communication channel, human operator and task environment are the key components (Fig. 1). Without requiring direct physical contact between human operator and task environment, a human operator is able to move a slave manipulator located in the remote environment by moving a master manipulator located in the local environment.

Stability and performance are two key issues of bilateral teleoperation systems. If the slave accurately reproduces the master’s commands and the master correctly feels the slave forces, then the human operator experiences the same interaction as the slave would. This is the major criterion for performance of teleoperation systems, called as transparency (Lawrence, 1993).
The time delay and task environment uncertainties make transparency and stability significantly compromised. According to these issues, different control approaches have been proposed in literature for teleoperation systems. Passivity theory (Hou & Luecke, 2005), non-linear control (Le et al., 2011), compliance control (Munir & Book, 2002), adaptive control (Hosseini-Suny et al., 2010) and optimal control (Ganjefar et al., 2011) are some of these approaches. Due to the uncertainty in the task environment and the time delay in communication channel, robust control theory is one of the significant approaches to control bilateral teleoperation systems (Sha Sadeghi et al., 2008). Among the robust control methods, \( \mu \) synthesis, one of the most powerful robust techniques, has been employed to design a robust controller for teleoperation systems (Leung et al., 1995). In spite of high efficiency, high-order \( \mu \) synthesis controllers may not be feasible for real-time implementation because of hardware and computational limitations.

To handle this problem, this paper proposes a structure-specified approach for designing a robust controller in bilateral teleoperation systems with uncertainties in the task environment and the communication time delay. This idea has been employed to design simple structure controllers in other robust control problems such as loop shaping (Nimpitiwan & Kaitwanidvilai, 2012), \( H_\infty \) problem (Zamini et al., 2009) and mixed \( H_2/H_\infty \) (Ho et al., 2005).

Structure singular value, denoted by \( \mu \), is utilized as a tool for the assessment of controller quality (Zhou & Doyle, 1998). In this point of view, \( \mu \) synthesis problem is considered as a constraint optimization problem. In the constraint optimization problem, robust stability measured via \( \mu \) stability analysis is assumed as a constraint whereas robust performance measured via \( \mu \) performance analysis is considered as an objective function which must be minimized. Particle swarm optimization (PSO) algorithm, a powerful inspired algorithm, is employed to solve this optimization problem. This algorithm tries to find the coefficients of structure-specified controllers, and satisfies the robust conditions. Recently, heuristic algorithms especially with stochastic search techniques seem to be a more hopeful approach and provide a powerful means to solve real-world optimization problems (Modares et al., 2010a,b; Alfi & Fateh, 2011a,b; Alfi & Modares, 2011; Wang et al., 2012). They can be a promising alternative to traditional techniques. The key features of heuristic algorithms such as genetic algorithm (GA) and PSO are: (i) the objective function’s gradient is not required; (ii) they are not sensitive to starting point and (iii) they usually do not get stuck into so-called local optima.

PSO as a population-based algorithm can solve a variety of difficulties associated with optimization problems. Compared with GA, PSO takes less time for each function evaluation as it does not use many GA operators such as mutation, crossover and selection operator (Alfi & Fateh, 2011b). Due to the simple concept, easy implementation and quick convergence, nowadays PSO has gained much attention and wide applications in different fields (Modares et al., 2010a,b; Alfi & Fateh, 2011b). According our knowledge, this is the first research to apply PSO for teleoperation systems via \( \mu \) synthesis.

Motivated by the aforementioned, in this paper, a novel PSO-based robust controller design is introduced for bilateral transparent teleoperation systems in the presence of uncertainties including time delay in communication channel and task environment. To this end, we use a modified PSO algorithm namely PSO with time-varying acceleration coefficients (PSO-TVAC). The key contribution of the
proposed controller structure is: (i) the closed-loop control system is robust stable and the \( \mu \) conditions including robust stability and performance are satisfied, (ii) the teleoperation system is complete transparent, (iii) it is simple and easy to implement with simple and low-order structure controllers. The preference of the proposed control method has been shown by comparison with those obtained the conventional technique namely D-K iteration for solving \( \mu \) synthesis problem.

This paper is organized as follows. Section 2 discusses the basic \( \mu \) synthesis problem. In Section 3, problem formulation is illustrated. Section 4 introduces the proposed approach. In this section, robust control design based on the proposed method for teleoperation systems is studied. Simulation result is presented in Section 5. Finally, conclusions are drawn in Section 6.

2. Preliminary

In this section, we describe a brief introduction to the \( \mu \) synthesis problem.

2.1 \( \mu \) synthesis problem

The structure singular value denoted by \( \mu \) depends on the underlying block structure of perturbations which is defined as follows: perturbation matrix \( \Delta \subset \mathbb{C}^{n \times n} \) is considered as

\[
\Delta = \{ \text{diag}[\delta_1 I_{r_1}, \ldots, \delta_S I_{r_S}, \Delta_1, \ldots, \Delta_F] : \delta_i \in \mathbb{C}, \Delta_j \in \mathbb{C}^{m_j \times m_j} \},
\]

where

\[
\sum_{i=1}^S r_i + \sum_{j=1}^F m_j = n.
\]

The block diagonal matrix \( \Delta \) includes two types of blocks: repeated scalar block (\( \delta_i \)) and full block (\( \Delta_j \)).

In Equation (1), \( n \) is the dimension of the block \( \Delta \) and the parameters \( \delta_i \) of the repeated scalar blocks can be real numbers only, if further information of the uncertainties is available. The Structure singular value \( \mu_\Delta(M) \) of a matrix \( M \in \mathbb{C}^{n \times n} \) with respect to a block structure \( \Delta \in \Delta \) is defined as

\[
\mu_\Delta(M) = \frac{1}{\min \{ \bar{\sigma}(\Delta) : \Delta \in \Delta, \det(I - M\Delta) = 0 \}}.
\]

If there is no \( \Delta \in \Delta \) such that \( \det(I - M\Delta) = 0 \), then \( \mu_\Delta(M) = 0 \).

Unfortunately, Equation (3) is not suitable for computing \( \mu \) since the optimization problem may have multiple local minima (Doyle & Packard, 1987). However, the upper and the lower bounds for \( \mu \) may be effectively computed as

\[
\max_{U \in \mathcal{U}} \rho(U) \leq \mu_\Delta(M) \leq \inf_{D \in \mathcal{D}} \bar{\sigma}(DMD^{-1}),
\]

where \( \mathcal{U} \) and \( \mathcal{D} \) are given in Equations (5) and (6), respectively.

\[
\mathcal{U} = \{ U \in \Delta : UU^* = I_n \},
\]

\[
\mathcal{D} = \left\{ \text{diag}[D_1, \ldots, D_S, d_1 I_{r_1}, \ldots, d_F I_{r_F}] : D_i \in \mathbb{C}^{r_i \times r_i}, D_i = D_i^* > 0, d_j \in \mathbb{R}, d_j > 0 \right\}.
\]
2.2 Robust stability and performance based on $\mu$ analysis

To fit the general framework, an interconnected system can be rearranged as shown in Fig. 2. In this figure, $K$ is the controller, $P$ the nominal plant and $\Delta$ is the uncertainty set (perturbation matrix). Moreover, $w$ denotes the exogenous input vector typically including command signals, disturbances, noises etc.; $z$ the error output usually consisting of regulator output, tracking errors, filtered actuator signals etc.; $v$ and $d$ are the input and output signals of dynamic uncertainties. Finally, $u$ and $y$ are the control input and the measurement signal, respectively. The general framework shown in Fig. 2 can be represented as

$$M(P, K) = Fl(P, K)$$

Let $M$ be partitioned accordingly as shown in Equation (7). In this partitioning, $M_{11}$ and $M$ are utilized for robust stability and robust performance analysis, respectively.

$$\begin{bmatrix} v \\ z \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} d \\ w \end{bmatrix}.$$  \hspace{1cm} (7)

The structure used for robust stability analysis based on $\mu$ is shown in Fig. 4. When $M_{11}$ is an interconnected transfer function formed with respect to the uncertainty set $\Delta$, the structure singular value of $M_{11}(s)$ indicates the robust stability of the perturbed system. Without loss of generality, assume that the uncertainties have been normalized. The standard configuration shown in Fig. 4 is robust stable if $M_{11}(s)$ is stable and $\mu_\Delta(M_{11}(s)) < 1$ (or $\|M_{11}\|_\mu < 1$). Figure 5 is also the standard configuration...
for robust performance analysis. The system performance specification can usually be interpreted as a reduction of $z$ with respect to $w$. The robust performance problem based on Fig. 5 is equivalent to robust stability problem shown in Fig. 4, if the uncertainty set $\Delta$ replace with $\tilde{\Delta}$ in Equation (8). With normalized uncertainties, the structure shown in Fig. 5 is robustly stable in the presence of uncertainty set $\tilde{\Delta}$ (robust performance in the presence of uncertainty set $\Delta$) if $\mu_{\tilde{\Delta}}(M(s)) < 1$ (or $\|M\|_{\mu} < 1$).

$$\tilde{\Delta} \in \tilde{\Delta} := \{\text{diag} [\Delta, \Delta_P] : \Delta \in B\Delta, \|\Delta_P\|_{\infty} \leq 1\},$$

$$B\Delta = \{\Delta \in \Delta : \tilde{\sigma}(\Delta) \leq 1\}. \tag{8}$$

Based on aforementioned conditions, designing of such a controller is so-called $\mu$ synthesis problem. D-K iteration is the most common technique to solve this problem. Upper limit of $\mu$ is considered instead of its value in this technique.

The D-K iteration method, for a fixed $K(s)$, is a convex optimization problem at each frequency $\omega$ as

$$\inf_{K(s)} \sup_{\omega \in \mathbb{R}} \inf_{D \in \mathbf{D}} \tilde{\sigma}[DF_i(P, K)D^{-1}(j\omega)], \tag{9}$$

where matrix $D$ is defined in Equation (6). In fact, Equation (9) is a different approach to solve main problem given in Equation (10).

$$\inf_{K(s)} \sup_{\omega \in \mathbb{R}} \mu_{\tilde{\Delta}}[F_i(P, K)(j\omega)]. \tag{10}$$
After the minimization over a range of frequency of interest, the resultant diagonal constant matrices $D$'s can be approximated, via curve fitting, by a stable, minimum phase, rational transfer function matrix $D(s)$, which will be used in the next iteration for $K(s)$. The D-K iterative $\mu$-synthesis algorithm can be summarized as follows:

**Step 1:** Start with an initial guess for $D$, usually set $D = I$.

**Step 2:** Fix $D$ and solve the $H_\infty$-optimization for $K$,

$$K = \arg \inf_K \|F_l(P, K)\|_\infty.$$ 

**Step 3:** Fix $K$ and solve the following convex optimization problem for $D$ at each frequency over a selected frequency range,

$$D(j\omega) = \arg \inf_{D \in \mathbb{D}} \tilde{\sigma}[DF_l(P, K)D^{-1}(j\omega)].$$

**Step 4:** Curve fit $D(j\omega)$ to get a stable, minimum-phase $D(s)$; go to Step 2 and repeat, until a stop criterion is achieved.

Although successful applications of the D-K iteration method have been reported, there exists some shortcoming. The main disadvantage of D-K iteration is that the algorithm may not converge in some cases, which lead to non-optimality of the resulting controller (Stein & Doyle, 1991). To overcome of this shortcoming, this paper introduces a different solution for solving $\mu$ synthesis problem.

### 3. Problem description

In this study, a control structure shown in Fig. 6 is used (Alfi & Farrokhi, 2008a,b). In this figure, $P$, $C$ and $y$ denote the transfer function of the local systems, the local controllers and the outputs, respectively, where the subscripts $m$ and $s$ designate the master and the slave, respectively. $T_{ms}$ and $T_{sm}$ denote the forward and the backward time delays, respectively. $F_e$ is the force exerted on the slave by its environment, $Z_e$ is the environment impedance, $F_h$ is the force applied to the master by the human
operator, $K_p$ and $K_f$ are the position and force scaling factors, respectively, $F_s$ is the input force applied to the slave and $F_r$ is the reflected force.

The performance system objectives are reflected by choosing appropriate weighting functions. The control scheme for the master and the slave sites are shown in Figs 7 and 8, respectively. In these figures, $W_1$ and $W_2$ are the weighting functions chosen to represent the desired tracking performance and control signal, respectively, and $W_3$ is used to reflect the frequency characteristics of the sensor noise. The following assumptions have to be stated first:

**Assumption 1** The slave system acts in a non-free task environment.

**Assumption 2** The position and force scaling factors $K_p$ and $K_f$ are equal to 1.

**Assumption 3** The task environment is passive.

The main goal of the proposed control scheme is to achieve stability and transparency. That is, the following conditions must be guaranteed:

(1) The closed loop of the overall system is stable.
(2) Position/velocity tracking is guaranteed. The position/velocity tracking means that the slave output $y_s$ has to follow the master output $y_m$ with an acceptable accuracy. Note that the master and the slave outputs can be considered position or velocity.

(3) Force tracking is guaranteed. It means that reflecting force $F_r$ has to follow the human operator force $F_h$.

These goals are achieved by designing two local controllers; one in the remote site $C_s$ guarantees the position/velocity tracking and the other one in the local site $C_m$ guarantees the force tracking as well as stability of the overall system. It is necessary to recall that, based on the designed slave controller in the first step, the master controller is designed in the next step.

4. The proposed approach

In this section, the proposed approach is described for solving the $\mu$ synthesis problem. To this end, first we briefly explain PSO algorithm.

4.1 PSO-TVAC algorithm

PSO is originally attributed to Kennedy & Eberhart (1995), based on the social behaviour of collection of animals such as birds flocking. In PSO algorithm, each individual of the swarm, called particle, remembers the best solution found by itself and by the whole swarm along the search trajectory. The particles move along the search space and exchange information with other particles according to the following equations:

$$V_{id} = wV_{id} + c_1 r_1 (P_{id} - X_{id}) + c_2 r_2 (P_{gd} - X_{id}),$$

$$X_{id} = X_{id} + V_{id}, \quad d = 1, 2, \ldots, N, \quad i = 1, 2, \ldots, S,$$

where $X_{id}$ represents the current position of particle, $P_{id}$ is the best individual particle position, $P_{gd}$ the best swarm position, the parameters $c_1$ and $c_2$ are cognitive and social parameters, respectively; the parameters $r_1$ and $r_2$ are random numbers between 0 and 1. Finally, $w$ is the inertia weight to balance the global and local search abilities. A large inertia weight facilitates global search while a small inertia weight facilitates local search. In an empirical study on PSO, Shi & Eberhart (1999) claimed that a linearly decreasing inertia weight could improve local search towards the end of a run, rather than using a constant value throughout. So the inertia weight is adapted non-linearly as follows:

$$w = (\text{iter}_{\text{max}} - \text{iter}_{\text{cur}}) \left( \frac{w_{\text{initial}} - w_{\text{final}}}{\text{iter}_{\text{max}}} \right) + w_{\text{final}},$$

where $w_{\text{initial}}$ and $w_{\text{final}}$ represent the initial and final inertia weights at the start of a given run, respectively; $\text{iter}_{\text{max}}$ is the maximum number of allowable iterations and $\text{iter}_{\text{cur}}$ denotes the current iteration number at the present iteration. However, due to the utilization of a linearly decreasing inertia weight, the global search ability at the end of the run may be inadequate. The PSO may fail to find the required optimal in cases when the problem is too complicated. But, to some extent, this can be overcome by employing a self-adapting strategy for adjusting the acceleration coefficients.

Suganthan (1999) observed that the fixed acceleration coefficients ($c_1$ and $c_2$) at 2 generate better solutions. However, through empirical studies, he suggested that the acceleration coefficients should not be equal to 2 all the time. Ratnaweera et al. (2004) improve the convergence of particles to the global
optima based on the way that make $c_1$ decrease and $c_2$ increase linearly with the increase of iteration numbers, namely PSO-TVAC. In PSO-TVAC algorithm, the parameters $c_1$ and $c_2$ are given by

$$
c_1 = c_{1i} + \frac{\text{iter}_{\text{cur}}(c_{1e} - c_{1i})}{\text{iter}_{\text{max}}},
$$

$$
c_2 = c_{2i} + \frac{\text{iter}_{\text{cur}}(c_{2e} - c_{2i})}{\text{iter}_{\text{max}}},
$$

where the subscripts $i$ and $e$ represent the initial and final values of $c_1$ and $c_2$, respectively.

In Ratnaweera et al. (2004), an improved optimum solution for most of the benchmarks was observed when changing $c_1$ from 2.5 to 0.5 and changing $c_2$ from 0.5 to 2.5, over the full range of the search. Moreover, Shi & Eberhart (1999) have observed that the optimal solution can be improved by varying the value of inertia weights from 0.9 at the beginning of the search to 0.4 at the end of the search for most problems. This modification to the original PSO concept has also been considered in Ratnaweera et al. (2004).

### 4.2 Methodology

In the proposed method, the $\mu$ synthesis problem is considered as a constraint optimization problem. Due to this reason, PSO-TVAC algorithm described above is employed to solve this constraint optimization problem. Before proceeding with the optimization operations, a performance criterion should be first defined. In general, the heuristic algorithm such as PSO needs to evaluate only the objective function to guide it. In this paper, the objective function is defined as follows:

$$
\min_{k(s)} \|M(s)\|_{\mu},
$$

such that: $\|M_{11}(s)\|_{\mu} < 1$,

where $M$ and $M_{11}$ are given in Equation (7). In the above optimization problem, the constraint is considered as a robust stability problem based on $\mu$ stability analysis whereas the objective function is calculated using robust performance based on $\mu$ analysis. Similar to D-K iteration method, the upper limit of $\mu$ is considered instead of the real value of $\mu$. In the proposed method, first a structure-specified controller (normally in the form of transfer function) should be determined. Then, the coefficients of transfer function are designated using PSO-TVAC algorithm such that the objective function given in Equation (15) is minimized without explicitly computing any gradients. Generally, the procedure of the proposed method is summarized as follows:

- **Step 1**: Determine a proper structure for the controller.
- **Step 2**: Set the parameters of PSO-TVAC algorithm.
- **Step 3**: Initialize a group of particles in a $D$-dimensional space as random points, where $D$ denotes the coefficients number of a controller. All of the particles must verify the robust stability constraint.
- **Step 4**: Calculate $P_{id}$, $P_{gd}$ and the objective function for each particle.
- **Step 5**: Update $w$, $c_1$ and $c_2$ according to iteration numbers given in Equations (13) and (14), respectively.
- **Step 6**: Update the velocities and positions of each particle according to Equations (11) and (12), respectively.
- **Step 7**: If each new position verifies the stability constraint, set it as the new position. Else, set the last position as the new position.
- **Step 8**: If the number of iterations is lower than a pre-defined value, go to Step 4. Else, go to Step 9.
Step 9: Consider $P_{gd}$ as the coefficient of desired controller. If the obtained controller is not efficient enough, the order of a structure-specified controller can be incremented.

5. Simulation results and discussion

In many literatures, a linear model of master–slave system has been utilized (Hokayayaem & Spong, 2006; Valdovinos et al., 2007). In order to evaluate the effectiveness of the proposed control structure, it is applied to the following simplified model, which has been utilized (Alfi & Farrokhi, 2008b).

\[
\begin{align*}
(J_m s^2 + B_m s + M_m g L_m) \theta_m &= u_m, \\
(J_s s^2 + B_s s + M_s g L_s) \theta_s &= u_s,
\end{align*}
\]  

(16)

where $J$, $M$ and $L$ are the moment of inertia, the mass and the length of the manipulators links, respectively; $B$ the viscous friction coefficient, $\theta$ the rotational angle, $g$ the gravity acceleration and $u$ the input; indices $m$ and $s$ are for the master and the slave, respectively. The considered parameters of the master and the slave are, respectively,

\[
\begin{align*}
J_m &= 2 \text{ kg m}^2, & B_m &= 3 \text{ Nm rad/s}, & L_m &= 0.2 \text{ m}, & M_m &= 0.6 \text{ kg}; \\
J_s &= 1 \text{ kg m}^2, & B_s &= 5 \text{ Nm rad/s}, & L_s &= 0.3 \text{ m}, & M_s &= 2 \text{ kg}.
\end{align*}
\]

By using the first-order Pade’ approximation, the modelling of time delay in the forward and in the backward path can be written as follows:

\[
e^{-Ts} \approx \frac{1 - (T/2)s}{1 + (T/2)s}.
\]

(17)

The nominal values of $T_{ms}$ and $T_{sm}$ are set to $T_{ms} = T_{sm} = 1.5$. The maximum values of time delay are equal to 3 s. Moreover, to consider the interaction between the slave and the remote environment, the following model for impedance of task environment is given by

\[
Z_e = B_e s + K_c, \quad 2 < B_e < 4 \quad \text{and} \quad 4 < K_c < 8,
\]

(18)

where the nominal values of viscous friction $B_e$ and the stiffness $K_c$ are chosen to be 3 and 6, respectively.

A structure-specified controller for master and slave is selected as a proportional integral derivative (PID) controller with first-order derivative filter. The controller structure is expressed in Equation (19). This controller is proposed in Kaitwanidvilai & Parnichkun (2008) for designing a robust controller.

\[
K(s) = K_p + \frac{K_i}{s} + \frac{K_d s}{\tau_d s + 1},
\]

(19)

where $\tau_d$ denotes the filter’s time constant and $K_p$, $K_i$ and $K_d$ represent the proportional gain, the integral time and the derivative time, respectively. All these parameters are supposed to be positive.

In the problem in hand, the parameters $K_p$, $K_i$, $K_d$ and $\tau_d$ must be obtained using PSO-TVAC. The parameters of PSO-TVAC algorithm as well as the bound of search space for each coefficient of master and slave controllers are listed in Tables 1 and 2, respectively. The weighting functions of the structures given in Figs 7 and 8 are also listed in Table 3.
Table 1  Parameters values in PSO-TVAC algorithm

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{1e}$</td>
<td>0.5</td>
<td>Number of particles</td>
<td>Master = 20, Slave = 25</td>
</tr>
<tr>
<td>$c_{1s}$</td>
<td>1.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c_{2e}$</td>
<td>1.5</td>
<td>$w_{\text{initial}}$</td>
<td>0.9</td>
</tr>
<tr>
<td>$c_{2s}$</td>
<td>0.5</td>
<td>$w_{\text{final}}$</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Table 2  Search space of the proposed controller parameters in PSO-TVAC algorithm

<table>
<thead>
<tr>
<th>Master controller</th>
<th>Range</th>
<th>Slave controller</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{pm}$</td>
<td>[0 1]</td>
<td>$K_{ps}$</td>
<td>[0 100]</td>
</tr>
<tr>
<td>$K_{im}$</td>
<td>[0 1]</td>
<td>$K_{is}$</td>
<td>[0 100]</td>
</tr>
<tr>
<td>$K_{dm}$</td>
<td>[0 1]</td>
<td>$K_{ds}$</td>
<td>[0 100]</td>
</tr>
<tr>
<td>$\tau_{dm}$</td>
<td>[0 5]</td>
<td>$\tau_{ds}$</td>
<td>[0 100]</td>
</tr>
</tbody>
</table>

Table 3  Weighting functions

<table>
<thead>
<tr>
<th>Master</th>
<th>Slave</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{m1} = \frac{s^2 + 15s + 2.4}{s^2 + 18s + 0.001}$</td>
<td>$W_{s1} = \frac{s^2 + 8s + 25}{s^2 + 10s + .01}$</td>
</tr>
<tr>
<td>$W_{m2} = 0.02$</td>
<td>$W_{s2} = 0.02$</td>
</tr>
<tr>
<td>$W_{m3} = 10^{-4} \frac{s}{0.001s + 1}$</td>
<td>$W_{s3} = 10^{-4} \frac{s}{0.001s + 1}$</td>
</tr>
</tbody>
</table>

The control performance is evaluated by applying two different inputs exerted by a human operator: step input and pulse input. In addition, simulation results are performed in two general parts. In the first part, the feasibility of the proposed control structure method against parameter uncertainties including the task environment and time delay in communication channel is represented. In the second part, the performance of the proposed method is compared with those obtained D-K iteration, which is the conventional method to solve the $\mu$ synthesis problem. In all the simulations, a Gaussian noise with mean zero and variance 0.005 is also considered as a measurement noise of sensor. Based on these, the proposed slave and master controllers are obtained as follows:

$$C_s(s) = 17.3 + 33.4 \frac{s}{s} + 29.4 \frac{s}{45.4s + 1}, \quad (20)$$

$$C_m(s) = 10^{-3} \left(40 + 22 \frac{s}{s} + 285 \frac{s}{3.43s + 1}\right). \quad (21)$$

5.1  Performance analysis in presence of uncertainties

5.1.1  Uncertainty in time delay. In this part, the performance of the proposed algorithm is investigated in different values of time delay including constant time delay and perturbed time delay,
whereas the nominal values for other parameters are used. First, two different values for constant time delay in communication channel are utilized: (i) Constant time delay equals 0.5 s. (ii) Constant time delay equals 3 s. The transparency response including force tracking and position tracking are demonstrated in Figs 9–12. Referring to these figures, a teleoperation system can preserve the transparency in the presence of time delay. Secondly, a perturbed time delay shown in Figs 13 and 16 is considered for step and pulse inputs, respectively. Figures 14, 15, 17 and 18 show the transparency response. From these figures, we can verify that the teleoperation system is also able to track the human command in spite of the perturbed time delay.

5.1.2 Uncertainty in the task environment. To cover wide operating conditions, three different cases shown in Table 4 are opted as the uncertainty of environment impedance. In these cases, a nominal time delay is considered. The results are illustrated in Figs 19–26 for step and pulse inputs, respectively. From these figures, the proposed control structure can cope up very well in the presence of environment impedance uncertainty.

5.2 Comparison

This section assigns the comprehensive comparison between the proposed controller structure and the D-K iteration controller. Using the D-K iteration method, the obtained order of slave and master controllers are 8 and 18, respectively. As the orders of these controllers are high for real-time control, to reduce order, we employ the optimal Hankel norm approximation. The reduced order controllers are as follows:

\[ C_{\text{s\_reduced}}(s) = \frac{1.808 \times 10^4 s^3 + 5.659 \times 10^5 s^2 + 4.155 \times 10^6 s + 5.706 \times 10^6}{s^4 + 1012 s^3 + 2.531 \times 10^4 s^2 + 1.741 \times 10^5 s + 174.1}, \]  
(22)

\[ C_{\text{m\_reduced}}(s) = \frac{-90.61 s^3 - 115.3 s^2 + 6.281 s + 25.05}{s^4 + 25.51 s^3 + 344.4 s^2 + 1797 s + 0.09982}, \]  
(23)
Fig. 10. Step response of position tracking: (a) time delay equal to 0.5 and (b) time delay equal to 3 s.

Fig. 11. Pulse response of force tracking for constant time delays.
Fig. 12. Pulse response of position tracking: (a) time delay equal to 0.5 and (b) time delay equal to 3 s.

Fig. 13. Perturbed time delay in the forward and the backward paths for step input.
These controllers are relatively complex in comparing with the simple structure-specified controllers given in Equations (20) and (21). It is noticeable that the main issue in controller order reduction is to preserve the stability and performance of the closed-loop system. The basic idea of order reduction is to minimize the difference between high-order and reduced-order closed-loop transfer function. To analyse the effect of reduction, the behaviour of the reduced-order controllers compared with the original controllers is acceptable (Figs 27 and 28). In these figures, because of high similarity in the features of robust stability, the robust performance features are compared. Figures 29–32 demonstrate the comparison of slave and master controllers obtained by the proposed structure-specified method and D-K iteration based on μ stability and performance analysis, respectively. Figure 29 shows that the slave controllers have the same condition in robust stability. The peak value of μ plot for the D-K iteration controller is 0.151, whereas for the structure-specified controller it is 0.162. It means that both of controllers have well enough stability margins. According to Fig. 30, the robust performance of slave
controllers is very similar to others in entire frequency ranges. In Fig. 30, peak values of $\mu$ plot equals 0.975 and 0.970 for the D-K iteration controller and the structure-specified controller, respectively. For the master controller, Fig. 31 illustrates that the peak values of $\mu$ plot for the D-K iteration and the structure-specified controller are 0.447 and 0.399, respectively. It concludes that both controllers have the appropriate robust stability conditions and the structure-specified controller has a slightly better stability margin. Referring to Fig. 32, the structure-specified controller has a much better stability performance condition with respect to the D-K iteration controller. In Fig. 32, the peak value of $\mu$ plot for the D-K iteration controller and the structure-specified controller equals to 0.990 and 0.714, respectively.

To evaluate the performance analysis of the closed-loop system, Figs 33–35 depict the time responses of a system by considering the nominal model. As these figures show, due to the existence of undershoot in the step response related to the D-K iteration controller, the proposed structure-specified controller exhibits the better performance in transparency response. Indeed, because of existence the
right zero in the D-K iteration controller given in Equation (23), the transient response is slower. This fact can be seen in Figs 33, 35, 36 and 38. In addition, the objective functions of both slave and master for the structure-specified controllers are depicted in Figs 39 and 40, respectively.
Fig. 20. Step response of position tracking for environment impedance uncertainty (Case 1).

Fig. 21. Step response of position tracking for environment impedance uncertainty (Case 2).

Fig. 22. Step response of position tracking for environment impedance uncertainty (Case 3).
Fig. 23. Pulse response of force tracking for environment impedance uncertainty.

Fig. 24. Pulse response of position tracking for environment impedance uncertainty (Case 1).

Fig. 25. Pulse response of position tracking for environment impedance uncertainty (Case 2).
Fig. 26. Pulse response of position tracking for environment impedance uncertainty (Case 3).

Fig. 27. Robust performance of D-K iteration controller and its reduced order designed for the slave site.

Fig. 28. Robust performance of D-K iteration controller and its reduced order designed for the master site.
Fig. 29. Comparison between the slave controllers based on $\mu$ stability analysis.

Fig. 30. Comparison between the slave controllers based on $\mu$ performance analysis.

Fig. 31. Comparison between the master controllers based on $\mu$ stability analysis.
Fig. 32. Comparison between the master controllers based on $\mu$ performance analysis.

Fig. 33. Step response of force tracking of D-K iteration and structure-specified controllers.

Fig. 34. Step response of position tracking of the structure-specified controllers.
Fig. 35. Step response of position tracking for D-K iteration controllers.

Fig. 36. Pulse response of force tracking for D-K iteration and structure-specified controllers.

Fig. 37. Pulse response of position tracking for the structure-specified controllers.
Fig. 38. Pulse response of position tracking for D-K iteration controllers.

Fig. 39. The cost function of slave controller given in Equation (14).

Fig. 40. The cost function of master controller given in Equation (14).
6. Conclusions

This paper introduced a novel robust control design for the bilateral teleoperation system in presence of uncertainties including time delay and task environment. The robust controllers are designed based on \( \mu \) stability and performance analysis via an inspired algorithm namely PSO-TVAC. This approach can be considered as an alternative solution to the \( \mu \) synthesis problem. The performance of teleoperation system was investigated in terms of transparency and robust stability. To this end, two local controllers were designed. The first controller is responsible for tracking the master commands, whereas the second controller is in charge of force tracking as well as guaranteeing the stability of the overall closed-loop system. Simulation results confirmed the efficiency of the proposed control structure in comparison with the D-K iteration method.

References


