7^{th} International Congress on Advanced Electromagnetic Materials in Microwaves and Optics – Metamaterials 2013
Bordeaux, France, 16-21 September 2013

Spatially Isotropic SRR-based Unit Cell for Conformable Metamaterials

A. Ajami [1], A. Al-Najjar[2], A. Bülow[1], H. Shakhtour[1], D. Heberling[1]

[1] RWTH University of Aachen, Institute of High Frequency Technology, Melatener straße 25
52074, Aachen, Germany
[2] German University in Cairo (GUC), Cairo, Egypt
ajami@ihf.rwth-aachen.de

Abstract – The patterned metamaterials (MTMs) working in microwave region contain basically split ring resonators (SRRs). Both metallic and complementary SRRs (CSRRs) are mostly designed on a planar surface oriented perpendicularly or parallel to the incoming wave, i.e. the functionality of these unit cells depends on the incident angle of the incoming wave and the designed MTM is mostly spatially anisotropic. Using a spherically formed substrate coated with some bent SRRs on its surfaces can solve the spatial anisotropy. In this work we present a spatially isotropic unit cell (SIUC) using the bent SRRs mounted on a sphere. Based on the geometrical isotropy of the sphere and the magnetic and electric resonances of the bent SRRs the optimal design is achieved. The spatial isotropy is compared between the proposed SIUC and the conventional planar unit cell by considering the stability of the retrieved electromagnetic parameters for different angular orientations of the unit cells related to the incoming wave. The SIUC leads to a three-dimensional metamaterial with macroscopic spatial isotropy, which initiates new applications for MTM in the microwave region, where deforming and conforming of metamaterials are required.

I. INTRODUCTION

Since the first realization of artificial MTMs by Smith et al. [1] in the year 2000, which consisted of a combination of split ring resonators (SRRs) and thin wires (TWs), it became evident that the normal formation of MTMs (especially in the microwave region) would take generally the shape of rigid and inflexible slabs, since MTMs are mostly based on conductors etched on conventional rigid dielectric media. The design idea emerged from the famous monolithic fabrication technology used in printed RF devices. The inflexible form of MTM structures restricts their applications. Moreover the planar formation limits the functionality of the MTM to only one angular direction of emission (spatial anisotropy).

Different and various trials of spatial isotropization of MTMs were attempted [2, 3], however, mostly in THz frequencies and without achieving an accepted level of spatial isotropy. Therefore, a need for an ultimately flexible and spatially isotropic structure was evident.

A spatially isotropic unit cell (SIUC) can lead to a macroscopic spatial isotropy of the whole bulk MTM containing these UCs. This makes MTMs convenient for many applications in microwave region. In this work a SIUC for this purpose is achieved. Section II shows the way from planar unit cell (UC) to a spherical one and in section III a method for the mathematical evaluation of spatial isotropy is suggested. The optimal design of a SIUC is shown in section IV. Spatial isotropy is investigated and compared by illuminating two unit cells i.e. SIUC and conventional (planar) unit cell with a plane wave coming from different angular directions. It will be evident that the proposed SIUC can provide spatial isotropy.

II. THE WAY FROM PLANAR TO SPHERICAL UC

The development of a SIUC started from a SRR-based planar UC shown in Fig. 1a. This UC consists of a SRR etched on a substrate (FR-4 in this case) and resonates at 16 GHz with perpendicular orientation to the incoming wave (the surface vector of the UC is perpendicular to the poynting vector and $\vec{E}$ is parallel to the gap of SRR). During studying the effects of bending of such UCs for the sake of conformal applications, an enhancement in the spatial isotropy of the structure was observed. The cell was then bent in a cylindrical shape such that it could fit some applications with cylindrical geometry (Fig. 1b). Due to this bent shape, rotating the structure around its bending axis caused a smaller difference in its transmission factor compared to that of the rotation of the planar UC (Fig. 2a & b). This is due to the fact that by the bent SRR the area exposed to the incident electromagnetic wave does not change much for different angles of incidence. Therefore the emerged inductance does not change with the rotation of this UC. One can imagine a bent UC around other axes. The final result is a spherical

UC shown in Fig. 1c. In consideration of geometrical aspect a sphere shows the perfect spatial isotropy and thereby it is possible to have even two SRRs on the top and bottom side of the sphere as shown in Fig. 1c.

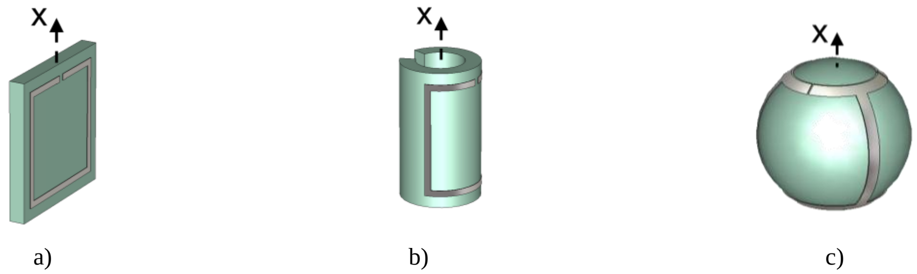


Fig. 1: a) A planar SRR b) A bent SRR around a cylinder c) spherical unit cell including bent SRRs

Fig. 2 shows the transmission factors of the UCs shown in Fig. 1 when rotating them around their x-axes (see Fig. 1). The transmission factors ($|S_{21}|$) of every UC were calculated using the CST-MWS simulation program. The "unit cell" boundary conditions were used, which means that the UC is repeated in x- and y-directions periodically, whereas the first linear floquet mode (TE_{00}) was used for the excitation in z-direction. The UCs were rotated around their x-axis with an increment of 10^0 from 0^o to 360^0. It should be noted, that the mutual couplings between the three connected SRRs of the spherical UC increase the effective inductance of the cell compared to the planar and the bent UCs, hence having a lower resonance frequency in this case. To establish a better comparison between these results the spatial isotropy shall be explained mathematically. Section III shows how the boxplots can be used to determine the spatial isotropy of a unit cell.

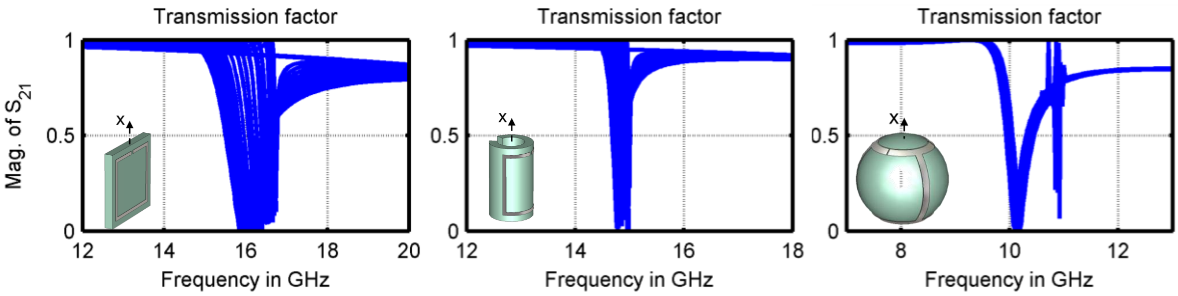


Fig. 2. Transmission factors of the UCs ,shown in Fig. 1, for rotation of the UCs around their x-axis (see Fig. 1)

III. MATHEMATICAL EXPLANATION OF THE SPATIAL ISOTROPY

As shown in the previous section to study the spatial isotropy, the parameter of a UC should be considered at different angular orientations of the UC related to the incoming wave for every observation frequency. Considering only one frequency, 36 values are achieved in our experiment for a whole rotation of a UC (rotating the UC from 0^o to 360^0 with an increment of 10^0). The question is now how the spatial isotropy can be explained mathematically from this group of values. Obviously the variation of these values can be considered as the factor to present the spatial isotropy (i.e. the stability against rotation) of the UC at this frequency. Instead of calculating the variance we used a more convenient way to depict the variation of the values namely accumulation box plot. In descriptive statistics the accumulation of some values can be explained by using a box plot. The bottom and the top boundaries of this box show usually the first (25%) and third (75%) quartiles of the values and the band inside the box shows usually the second quartile (the median). The minimum and the maximum values are shown out of the box too (Fig. 3 right). Fig. 3 left compares the variations of the $|S_{21}|$ depicted in Fig. 2 at the resonance frequency of the UCs. The variation of $|S_{21}|$ of the spherical UC is minimum, i.e. the spherical UC shows more spatial isotropy than the others.

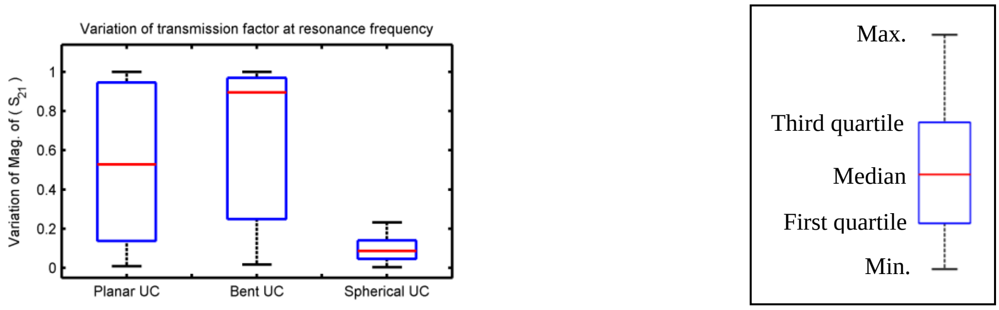


Fig. 3: Comparison between the variations of the transmission factors (left) of the UCs shown in Fig. 1 at their resonance frequencies. The box plot properties used for these comparisons (right)

IV. OPTIMIZATION OF THE SIUC

Considering the prototype spherical UC (shown in Fig. 1c) one could ask, how many SRRs around a sphere are needed to get the best spatial isotropy. It is clear, that the SRR-areas cannot be arbitrary small. Too small SRRs cause too high resonance frequency, since the dimensions of the UC will be comparable to the resonance wavelength and the homogeneity condition [4] for such MTM will not be valid any more.
Another challenge is the positioning of the gaps of SRRs. Having equal gaps above and below the SRRs requires even number of oblique SRRs. Using 2 oblique SRRs makes the UC again planar. But 4 oblique SRRs would be a good compromise. Since the SRR-areas would not become too small according to the homogeneity condition (compared to 6 oblique SRRs). With 4 oblique SRRs we can have 2 gaps above and 2 gaps below and by every oblique SRR we can have only one gap. Having further geometrical symmetry, 2 gaps on the top side and 2 gaps on the bottom side can be placed with 90 deg. rotation around the x-axis (Fig. 4a).
Finally the areas of SRRs should be analytically determined, so that the appropriate inductances are achieved to have the same resonance frequency for both kind of SRRs (SRR including one gap and SRR including two gaps). For this the area B shown in Fig. 4b should be equal to twice that of the projection of A (A = B/2), since the SRRs including the area B have 2 capacitive gaps (see Fig. 4b). It should be noted, that the areas A & B are the projected areas onto the orthogonal plane to the incoming $\vec{H}$ field. According to the coordinate system shown in Fig. 4b the projected area A will be calculated as follows:

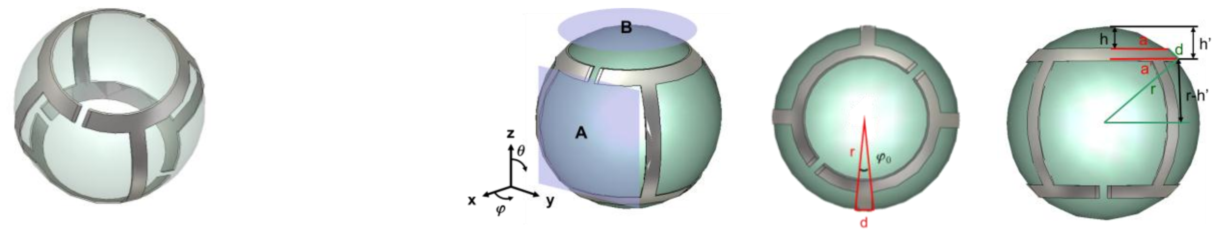
$$A = \int_{\varphi_1}^{\varphi_2}\int_{\theta_1}^{\theta_2} r^2 \sin^2(\theta)\cos(\varphi)\, d\theta\, d\varphi \overset{!}{=} \frac{B}{2} = \frac{\pi a^2}{2} \tag{1}$$

Where $\theta_2 = \pi - \theta_1$, $\varphi = \varphi_2 - \varphi_1 = \frac{2\pi}{N-2} - \varphi_0$, N is the number of SRRs (here 6) and $\varphi_0 = 2\arcsin(\frac{d/2}{r})$ (see Fig. 4b). From this follows:

$$r^2 \sin(\frac{\varphi}{2})(\sin(2\theta_1) + \pi - 2\theta_1) \overset{!}{=} \frac{\pi a^2}{2} \tag{2}$$

Where $\theta_1 = \arcsin(\frac{a'}{r})$. On the other hand $h = r - \sqrt{r^2 - a^2}$, $h' = r - \sqrt{r^2 - a'^2}$ and $a' = a + \Delta a$ with $\Delta a = \sqrt{d^2 - (h' - h)^2}$. Using equation (2), a' can be calculated as follows:

$$a' = a + \frac{1}{2a}\left(\frac{\sqrt{(r^2 - a^2)(4r^2 - d^2)}}{r^2}\, da - (\frac{da}{r})^2\right) \tag{3}$$



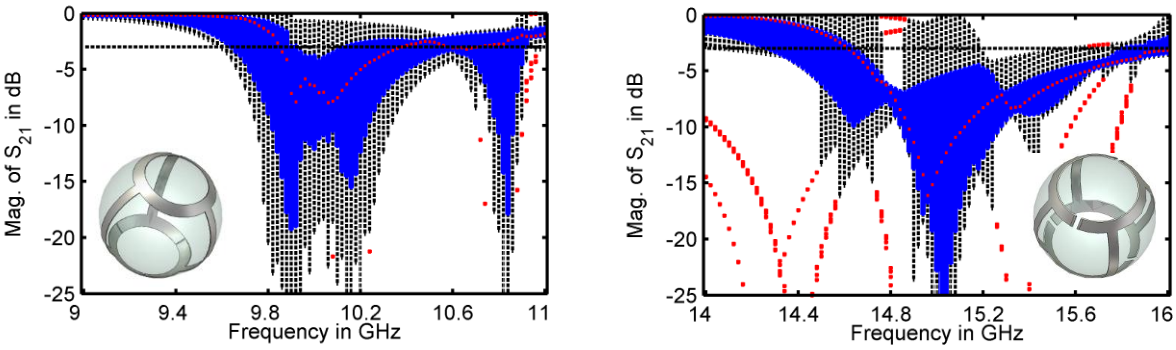
a) b)

Fig. 4: Design of a spatially isotropic UC: a) Symmetry and determination of the SRRs b) calculation of areas, gaps and resonance frequencies of SRRs

For known N, r and d, a and a' can be calculated. As a prototype design a spherical UC with diameter = 2.5 mm was designed (Total length of the unit cell including the distances to the excitation surfaces was 2.9 mm). According to the effective homogeneity condition [4] this UC can be observed as a MTM-UC below 25.86 GHz (Length of the UC along the propagation direction $= L = 2.9\,mm \le \lambda_0 / 4 \Rightarrow f_{operating} \le c_0 / (4L)$, where c_0 is the light velocity). Considering this diameter ($r = 1.25\,mm$) and for $N = 6$, and $d = 0.2\,mm$, a and a' were calculated and optimized using simulations: $a = 1.03\,mm$ and $a' = 1.13\,mm$.

To verify our optimization based on given mathematical calculations we studied the total spatial isotropy of this UC. It was rotated around all three axes (x, y and z) from 0^o to 360^o with an increment of 30^o and the variations of its transmission factors were compared to those from the initial design (Fig. 1c). Fig. 5 shows the boxplots of the optimized SIUC compared to the one in Fig. 1c over the whole frequency range of observation. According to this figure, considering e.g. the -3dB threshold, the $SIUC_{opt.}$ exhibits a wider bandwidth (BW_{-3dB}) of stability. The bandwidth (BW_{-3dB}) of stability amounts 1.8% and 7.36%, respectively.

It should be noted that as shown in Fig. 5 the resonance frequency of the $SIUC_{opt.}$ is different for some excitation angles from the expected resonance frequency. The reason is that using the upper and the lower SRRs causes more couplings in this UC compared to the $SIUC_{ini.}$ shown in Fig. 1c.



a) b)

Fig. 5: Comparison of spatial isotropy between a) the initial and b) the optimized UCs: Magnitude of S_{21} for the incident planar wave from different angular directions

But the $|S_{21}|$s and the bandwidth of boxplots cannot represent sufficient information about the electromagnetic properties, which we are actually looking for. Therefore the $|S_{21}|$s are used to retrieve the effective electromagnetic parameter of these UCs for different cases (incoming wave from different directions). The effective electromagnetic parameters can be retrieved from the simulated results (complex S-Parameter) [5]. For instance, the variations (boxplots) of the retrieved $Re\{\underline{\mu}_{eff.}\}$ are shown in Fig. 6. The $Re\{\underline{\mu}_{eff.}\}$ is here especially interesting, since we expect from our UCs (due to their SRR-based constructions) negative $Re\{\underline{\mu}_{eff.}\}$, which should be achieved for different angular illuminations to be sure that the UCs are spatially isotropic. Although the $SIUC_{ini.}$ shows negative Re{$\underline{\mu}_{eff.}$}, this is not seen for all angular illuminations.

On the other hand the $SIUC_{opt.}$ shows very good response concerning spatial isotropy effect. For the frequency range of 250 MHz (from 15.15 to 15.4 GHz) the $SIUC_{opt.}$ exhibits for all arbitrary orientations a negative Re{$\underline{\mu}_{eff.}$} around -1 (see Fig. 6 right). In other words at this frequency range we can build a bulk MTM consisting of this UC, which represents indeed negative Re{$\underline{\mu}_{eff.}$} independent of the UC orientations. This spatially isotropic MTM can be used to realize many applications of MTMs, where a conformal and/or flexible bulk MTM is required.

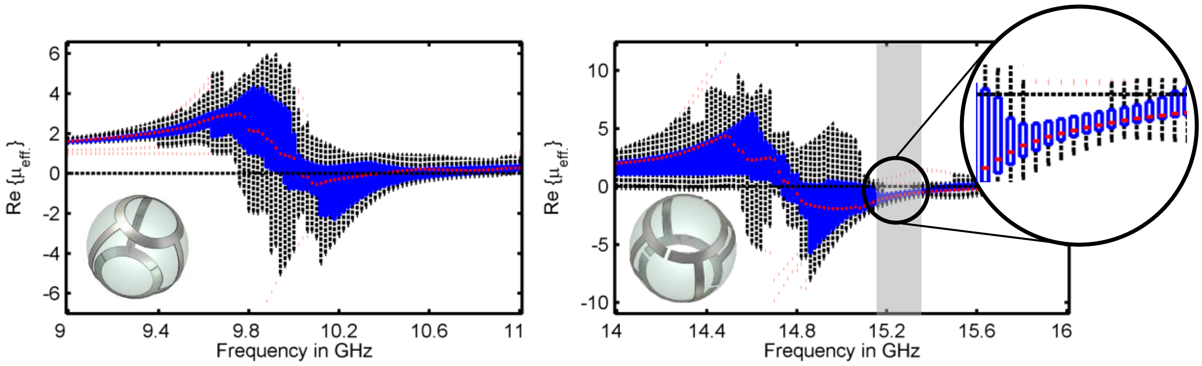


Fig. 6: Retrieved Re{$\underline{\mu}_{eff.}$} of the initial (left) and optimized (right) SIUCs for rotation of the UCs around their x-,y- and z-axis

V. Conclusion

A 3D spatially isotropic metamaterial unit cell is proposed. The unit cell is based on several bent SRRs, which are cleverly located on a spherically shaped substrate. A sphere provides the highest geometrical isotropy. This fact along with clever positioning of the SRRs a spatially isotropic response is achieved. To prove the spatial isotropy the proposed SIUC was rotated around all three axes and its effective electromagnetic parameters were calculated for every rotational angle. These results were compared with those obtained from a planar UC and it is shown, that the spherical UC shows much better spatial isotropy. The optimized SIUC can be used to build a spatially isotropic bulk MTM, which would in turn make MTMs conformable and more flexible. Consequently many applications of MTMs can be realized.

References

[1] D. R. Smith, W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz, *Physical Review Letters 84, 4184*, 2000

[2] J. D. Baena, L. Jelinek, R. Marqusa, J. Zehentner, *Applied Physics Letters 88, 134108*, 2006

[3] M. Hudlicka, J. Machac, "Triple wire medium for use in isotropic metamaterials", *978-2-87487-001-9, EuMA* 2007

[4] C. Caloz and T. Itoh, "Electromagnetic metamaterials: Transmission line theory and microwave applications", *JOHN WILEY & SONS, INC., Publication*, 2006

[5] A. Ajami, H. Shakhtour and D. Heberling, "A length-independent method to retrieve the effective parameters of materials and determine the interactions in metamaterial-structures", The Fifth International Metamaterial Congress, Barcelona, 2011