Practical multi-objective controller for preventing noise and vibration in an automobile wiper system

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Abstract

This paper presents an approach using a multi-objective controller to prevent noise and vibration generated by the wiper blade during its wiping operation. Firstly, this paper focuses on the experimental approach to collect noise and vibration data from a car wiper system during its operation and secondly, to develop black box model of the wiper system using nonparametric approach of system identification known as nonlinear auto regressive exogenous Elman neural network (NARXENN). Finally, a novel closed loop iterative input shaping controller whose parameters are tuned simultaneously by a Pareto based multi objective genetic algorithm (MOGA) are proposed and simulated in such a way that it can prevent unwanted noise and vibration in the wiper system. The main contribution of this work rather the previous studies of automobile wiper system is to develop a novel multi-objective control strategy whereby an automobile wiper blade is moved within its sweep workspace in the least amount of time with minimum noise and vibration.

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1. Introduction

A good wiping performance of an automobile wiper system is not only to provide a clear vision through the windscreen but also to operate silently. In order to meet these two important aspects it is typically relied on the wiper physical attributes such as blade rubber, windscreen glass material, friction between blade tip and windscreen, windscreen curve shape and attack angle of the blade. A computational modeling validated by experimental test which considered attack angle and friction coefficient influence carried out by Chevennement et al. [1]. The effects of wiping speed and windscreen wetness on noise generated of wiper blade was investigated by Zhang [2] and was shown that wiping noise is strongly affected by windshield wetness, while less affected by wiping speed. It is frequently happened that the wiper system generates low frequency noise and vibration known as chatter [3]. In the open literature it is evidenced that chattering noise in the wiper system is due to deformation of wiper blade during the wiper turnover. This chatter noise is not only annoying to the occupants but also causes vision disturbance to the driver. To date, there are a number of published articles concerning on noise and vibration in the wiper system. Abu Bakar et al. [4] conducted model testing to determine the natural frequencies of a wiper and conducted experiments to investigate vibration and noise of a wiper. They found that the wiper produces chatter noise below 100 Hz. The windscreen also experiences streaking visual deterioration effect during the experiment. They inferred that non-uniform water films on the windscreen may disturb contact between the rubber blade and the windscreen interfaces that lead to vibration. Goto et al. [5], Grenouillat et al. [6] and Okura and Oya [7] investigated squeal noise reduction using 2D and 3D mathematical models. Stallart et al. [8] and Chevenement et al. [9] developed a finite element model to support the optimization of the control configuration, study the dynamic instability of a flexible wiper system and suppressing wiper squeal noise. Fujii and Yamaguchi [10] suggested a new method for dynamic behavior of wiper blade using an optical approach. Chang et al. [11] worked on control of the chaotic motion of an automotive wiper system by using bifurcation diagram and Lyapunov exponent. Various Control methods have also been adopted for other purposes in the wiper system. An experimental study has been carried out in order to compensate the delay caused by various environmental conditions as well as other intelligent operations of a smart wiper. Three different sensory devices which measure the rain intense according to auditory, tactile and vision parameters are determined. A fuzzy logic controller is employed for position control of wiper and eliminates uncertainties [12]. Park et al. [13] developed an intelligent method comprising of image processing technique and fuzzy logic...
decision maker to assign the interval of wiper movement and its speed based on rain intensity without driver's intervention. An accurate control of wiper blade for an independent wiper system in opposition sides which are actuated via two separate motors are investigated by Lévine [14]. A feedback PID controller is employed in collaboration with a sort of feed-forward controller called flatness in order to avoid collision and obtain more precise trajectory tracking of each wiper.

The various techniques of system identifications are broadly used as a fundamental requirement in engineering and scientific applications such as time series prediction, pattern recognition, symbolic regression or prediction of dynamic model of a system. Due to hyper nonlinear characteristic of a wiper system accurate nonlinear system identification is required for extracting black box of system. Hence, A nonlinear auto regressive exogenous (NARX) in cascade with Elman neural network (ENN) is utilized for the purpose of system identification of nonlinear wiper system.

NARX is one the well-known system identification models which proved to be greatly efficient and accurate for nonlinear system identification [15–19]. There have been a number of popular techniques which developed and applied to identify NARX models that best estimate output behavior of a nonlinear system based on the input state. These techniques are classified into two major sorts of parametric and non-parametric approaches of system identifications. Although parametric identification approach such as least square (LS) or recursive least squares (RLS) algorithms have got broad applications for parameter estimation in modeling slowly varying dynamic systems [20], the technique does not work efficiently for a system whose characteristics change abruptly with time [21]. Parameter estimation constitutes a procedure that makes it possible to adjust a model with a specific structure. For this purpose, it is necessary to determine the appropriate order and parameters for the model that best fits input–output data obtained during the experiment. This task can be very time consuming indeed and it is very important to choose a correct order for the model, since a lower order may imply that the model could not adequately describe the real dynamics of the process, while a higher order could increase model uncertainties [22]. In order to control vibration and noise of wiper system which is considered as a flexible manipulator with several modes of frequencies it was proven that nonparametric approaches and specifically NN performed better at higher resonant modes than conventional RLS even in problems associated with non-minimum phase characteristics of the system [20]. Once the system is not sufficiently excited, the problems associated with non-minimum phase characteristics of the amplitudes and instances of convolved impulses are identified accurately [33–35]. On the other hand, inaccurate application of Impulse instances can cause significant degradation in system performance. One way to overcome this shortcoming of open-loop input shaping is using closed-loop input shaping (CLIS) [36].

Since control of wiper system’s noise and vibration involves several conflict objectives which should be in command and optimized simultaneously a need for an effective multi objective optimization is necessary in this study. The main advantage of MOGA is its versatility for including a variety of objectives and constraints while designing the controller. Generally, it can be said that it is not an analytical approach which deals with Pareto set problem. So, evolutionary computations are employed commonly in order to find the Pareto fronts of different objectives of problem. Pareto approaches for first time has been proposed by Goldberg in order to figure out Schaffer’s VEGA problem [37]. Fitness sharing is introduced to keep diversity of solutions over the Pareto line by Fonseca and Fleming, 1993 [38]. Up to present several multi objective problems have been studied by researchers in control field [39–41].

A novel combination of CLIS and IL controllers is developed while a Pareto multi objective genetic algorithm (MOGA) is utilized to estimate the most appropriate values of controller gains to design a robust controller for vibration and noise reduction of automobile’s wiper blade without sacrificing its speed of response. In fact the proposed controller applies a hybrid of an open loop controller which overcomes the unwanted vibration of wiper due to its high elasticity characteristic in one hand and closed loop part of controller is tasked to deal with the external disturbance which are likely to influence the operation of wiper system in both time and frequency domain simultaneously.

2. Experimental model preparation

The wiper model used in this study is the uni-blade type wiper which is typically found in the Proton Iswara. A pipe hose with running water on the top of windscreen facilitated that simulates a rainy or wet condition for operating wiper at speed of Bang–Bang input. In the experiment, a 16 input channels PAK MK II Muller BBM signal analyzer, a Kistler Type 8794A500 tri-axial accelerometer and a shaft encoder were used to measure the acceleration of end point and displacement of hub angle of the wiper. The signals are converted into FRF through PAK analyzer. The analyzer was connected to a laptop to display the results. The experiment equipments were shown in Fig. 1.
3. Methodology

3.1. NARX model

The NARX model structure is defined by

\[ y(k) = F(y(k-1), \ldots, y(k-n_y), u(k-1), \ldots, u(k-n_u)) + e(k) \]  

in which the effect of noise is assumed additive at output of the model. \( F(\cdot) \) is a nonlinear function, \( y_k, u_k \) and \( e_k \) are output, input, and noise respectively where \( n_y, n_u \) and \( n_e \) are maximum lags on observations and exogenous inputs [42]. In order to identify the NARX model, the corresponding \( F(\cdot) \) function should be approximated first; so that in this study the nonlinear function \( F(\cdot) \) is estimated by ENN.

3.2. Elman neural network algorithm

Elman neural network was proposed in 1990 by Elman in which a layer was integrated to conventional network in order to undertake feedback network of the hidden layer, as a step delay operator to act for memory purpose, so that the system has an ability to adapt to time-varying characteristics [43]. Hence, in the structure of an ENN there is an additional undertake layer called context layer besides the three conventional namely input, hidden and output layers that making the identification of dynamic characteristics.

Suppose an ENN like it is shown in Fig. 4 in which the vectors of input, middle and output layers’ nodes are labeled with \( u, x \) and \( y \) respectively. Also, \( W_1, W_2 \) and \( W_3 \) represent the respective connection weights of input, middle and output layers. The nodes of input layer play the role of signal transmission while nonlinear functions of \( M(\cdot) \) and \( O(\cdot) \) are introduced as transfer functions of middle and output layers which in this study the tan sigmoid function is used. Furthermore, the previous moment output values of hidden layer were stored in memory and return to the input, so it can be considered a step delay operator. The mathematical modeling of Elman Neural Network can be expressed as following equations:

\[ X(k) = M(W_2, u(k) + W_1, u(k-1)) \]  
\[ y(k) = O(W_3, x(k)) \]

A back propagation (BP) algorithm as it broadly used and discussed in literature was adopted for training process of neural network [44]. BP uses the error sum of squares function between output of network and target values:

\[ E(W) = \sum_{n=1}^{k} [y_n(W) - T_n(W)]^2, \quad n = 1, 2, 3, \ldots, k \]

where \( T_n (W) \) is the target vector of output.

Schematic model of proposed system identification named Nonlinear Auto Regressive Elman Neural Network (NARXENN) is illustrated in Fig. 5.

3.3. Closed loop iterative input shaping controller using MOGA

A command shaper is utilized to convolve the instantaneous impulses at suitable time locations in the reference input. Then a
Fig. 3. Time domain response of actual wiper lip: (a) end-point acceleration of wiper lip and (b) hub displacement.

Fig. 4. Frequency domain response of actual wiper lip: (a) PSD of end-point acceleration and (b) Yule–Walker spectral density of end-point acceleration.

Fig. 5. Developed NARXENN for system identification of wiper system.
collocated PD controller set in the path of hub-displacement error between measured hub displacement of wiper and desired shaped reference input of wiper system in corporation of PID type iterative learning controller. In order to reduce the end-point acceleration of wiper lip as much as possible and meanwhile sensitive to the time response of system a MOGA optimizer is used to estimate the most appropriate values of PD gains as well as IL scale factors. The schematic diagram of the proposed controller can be seen in Fig. 6.

3.4. Iterative learning

In IL algorithm, at each iteration of the learning process, input signals \((u_k(t))\) are recorded in memory in order to use them in next iteration. The errors obtained from the current iteration are saved in memory as \(e_k(t)\) and the output signals as \(y_k(t)\). Note, that \(k\) denotes the current value while the time variable \(t\) may be continuous or discrete. The learning algorithm then evaluates the system’s performance error which can be obtained as

\[
e_k(t) = y_d(t) - y_k(t),
\]

where \(y_d(t)\) is supposed to be the desired parameter and \(y_k(t)\) is the actual one of the system in each iteration. These data are used to compute the new input signal \((u_{k+1}(t))\) in the next iteration to modify the control inputs and reduce the system performance error gradually after some iterations. In other words, it can be shown that as the number of iteration \(k(t)\) increases, i.e., \(k \to \infty\) for \(k \in [0, \text{stopping criterion}]\), the track error \(TE\) converges to zero \((TE \to 0)\). The basic principle of IL method can be seen in Fig. 7.

Most of the algorithms proposed by Arimoto et al. in the literature show that the \((k+1)\)th input to the system can be obtained by \(k\)th input plus an error coefficient that may consist of a coefficient of track error \((TE = q_{\text{desired}} - q_{\text{actual}})\), derivative and integral of track error. These mathematical expressions are similar to the description of classic PID controller; therefore the IL algorithms can be described as the Proportional–Derivative (PD), Proportional–Integral (PI) and Proportional–Integral–Derivative (PID) type learning algorithms [30]. Learning control laws of IL utilized in this study is mathematically expressed as follows:

\[
l_{k+1} = l_k + \left[\varphi + \psi \int dt + \Gamma \frac{d}{dt}\right] e_k
\]

where

- \(l_{k+1}\) is IL of next step
- \(l_k\) is the current IL gain
- \(e_k\) is the output error \((TE = q_{\text{desired}} - q_{\text{actual}})\)
- \(\varphi, \psi, \Gamma\) are learning parameters of iterative learning algorithm.

3.5. Input shaping

The residual vibration resulted from a series of impulses utilized in the system can be derived from second order system transfer function [45]. In order to achieve more robustness of system and zero vibration result due to errors resulted of imprecise natural frequencies and damping ratio values, the performance of command shaper can be enhanced via equaling the derivative of vibration residual to natural frequency to zero. This type of input shaping is named zero vibration derivatives input shaping (ZVDIS) and the values of its amplitude and time location of three impulses have been calculated and stated as [46].

\[
A_1 = \frac{1}{(G+1)^2}, \quad A_2 = \frac{2G}{(G+1)^2}, \quad A_3 = \frac{G^2}{(G+1)^2}
\]

\[
t_1 = 0, \quad t_2 = \frac{T_d}{2}, \quad t_3 = T_d.
\]

where \(A_1, A_2\) and \(A_3\) are amplitudes and \(t_1, t_2,\) and \(t_3\) are time locations of impulses while \(G = \exp (-\zeta \pi / \sqrt{1 - \zeta^2})\), \(T_d = 2\pi / \omega_d\) is damped period of system and \(\omega_d\) is damped natural frequency of system.

3.6. MOGA

The key difference between GA and MOGA is laid on fitness assignment approach. By contrast the single objective optimization problems in which the fitness of cost function can be directly corresponded as final measure to find the global optimum of objective function, in multi-objective optimization there is no straight approach to find the global optimal of multiple conflicting cost functions.

Main goal in a multi-objective optimization is to obtain the minimum of

\[
\min Y(x) = \begin{bmatrix} y_1(x) \\ y_2(x) \\ \vdots \\ y_n(x) \end{bmatrix}, \quad x \in \mathbb{R}.
\]

![Fig. 6. Schematic representation of closed loop iterative input shaping controller using MOGA.](image-url)
The Pareto sets are contained the solutions which are not dominated by any other solutions. A variable \( x \in R \) is called a Pareto optimal if there is not another \( x' \in R \) such that \( y_j(x) < y_j(x') \) for all \( i = 1,2,\ldots,n \) and exists at least one \( j (j = 1,2,\ldots,n) \), such that \( y_j(x) < y_j(x') \).

In Fig. 8 the intersection points of frontier \( \partial Y \) and normal lines originated from points on the line \( AC \) and \( EB \) which are known as Utopia line toward \( O \) are Global Pareto optimal points, the points laid on line \( CD \) are Local Pareto and points are set on line \( ED \) are neither Local nor Global Pareto points.

Fitness sharing type of MOGA is implemented in this study [47]. Fitness sharing models individual competition for niche resources in a closed environment. Individuals similar to one another (according to some measure of similarity) mutually decrease each other’s fitness by competing for the same resources. Even if originally considered; less isolated individuals are thus given a greater chance of reproducing, favouring diversification. Finding a good trade-off description means achieving a (diverse) not uniform, sampling of the trade-off surface in objective function space. In the sharing scheme proposed here, share counts are computed based on individual distance in the objective domain, but only between individuals with the same rank. Sharing works by providing an additional selective pressure to that imposed by ranking, in order to counter the effects of genetic drift. Genetic drift becomes more important as more individuals in the population are assigned the same rank. The niche count of each individual was initially set to zero and then incremented by a certain amount for every individual in the population with the same multi-objective rank, including itself. The lethality of each generation is maintained using mating restriction technique [48].

There are several common operations in GA and MOGA optimizations. In this study binary encoding is used in order to convert real value objectives into binary. After ranking the individuals based on the multiplication of their fitness values and exponential pressure, the new individuals for next generation are chosen by stochastic universal sampling method. Also, a multi-point crossover technique is used in this study [49]. The mutation process executed after every 200–300 bits transfers from the crossover process. Two random positions of the string are chosen and the bits corresponding to those positions are interchanged. [50].

### 4. System modeling

In order to validate the capability of applied system identification model for acceptable extent of uncertainty and nonlinearity a uniformly distributed white noise signal in the same frequency range of system was generated in the path of the output of system. The mentioned noise input in the time and frequency domains are illustrated in Fig. 9.

For the modeling process, input–output data were collected for a wiper system. Then, performing the one value at the moment the best maximum lag of the data in NARX model was found to be 7. Subsequently, ENN with two hidden layers, each with 10 tansigmoid neurons and two linear output layers was trained. The process is adjusted until the prediction output satisfied a model validation test and model mean squared errors (MSE) level reached to 0.000048. The fitting accuracy of predicted system for one step ahead prediction the corresponding end-point acceleration and hub-angle responses of the actual system compared to NARXENN are shown in Fig. 10.

In Fig. 11 the MSE of NARXENN versus epochs of learning as well as error between actual and predicted end-point acceleration are illustrated.

The illustrated results of actual and predicted power spectral density (PSD) and Yule Walker power/frequency of end-point acceleration in frequency domain in Fig. 12 prove that, there is an acceptable similarity between system identification results and actual results in frequency.

![Image](https://via.placeholder.com/150)

**Fig. 8.** Illustration of Pareto optimal set.

![Image](https://via.placeholder.com/150)

**Fig. 9.** Output uncertainty as white noise in: (a) time domain and (b) frequency domain.
Fig. 10. Time domain modeling of wiper lip response: (a) end-point acceleration of wiper lip and (b) hub displacement.

Fig. 11. (a) Error between actual and predicted end-point acceleration and (b) MSE of NARXENN.

Fig. 12. Frequency domain modeling of wiper lip response: (a) PSD of end-point acceleration and (b) Yule–Walker spectral density of end-point acceleration.
5. Results and discussions

5.1. Establish the cost functions

Three cost functions of wiper system’s dynamic characteristics are defined to be considered in this study. Integral of absolute end-point acceleration (IAEA), maximum overshoot of hub displacement and rise time of hub displacement response are objectives that aimed to be minimized and defined as

- Integral absolute value of end-point acceleration (IAEA):
  \[ IAEA = \int_0^T |yEA(t)| \, dt, \tag{10} \]
  where \( |yEA(t)| \) signifies the acquired signal of end-point acceleration of wiper lip. IAEA in the merely states the integral of area of acceleration response of wiper lip respect to time such that it is measure of power spectral density in frequency domain. IAEA is a representative of noise level of wiper while two other objectives assess the performance of controller in trajectory tracking in time domain.
- Rise time: The time required for system hub displacement response to rise from 5% to 95% of the final steady state value of the desired response.
- Maximum overshoot: The maximum peak value of the hub displacement response curve measured from the desired response of the system.

In the engineering pursuit, designers frequently come across with trade-off problems. In the design of proposed closed loop iterative input shaping controller such trade-off is emerged in relationship between rise time and vibration amplitude. Typically, in flexible manipulator and structure dynamics control often the low levels of residual vibration cannot be obtained with a command that produces the fastest rise time [32–35]. In most cases, to achieve the low levels of vibration and highest robustness, the rise time must be increased which is not desirable. IAEA and maximum overshoot are objectives in accord; the rise time index of wiper lip is in obvious conflict with two aforementioned objectives.

5.2. Applying the single objective controller

Regarding the frequency domain representation of the experimental wiper blade in Fig. 4 the first mode natural frequency of the system which causes the major reason of vibration and noise of wiper can readily be estimated as 11.5 Hz. In order to utilize the ZVDIS controller for wiper blade the accurate value of wiper damping ratio is required besides the associated natural frequency. To avoid complex and time consuming calculation for obtaining the damping ratio of system the nearby value of wiper damping ratio is estimated using single objective GA for open loop end-point acceleration of wiper system.

A GA optimization with population size of 60 in which each chromosome is comprised of single individual of 12 bits, maximum generation of 50 as stopping criteria; double point crossover with rate of 0.8 and mutation rate of 0.005 aiming to minimize single objective IAEA can for obtaining the damping ratio of wiper system and its corresponding time location and amplitude of the impulses.

In Fig. 13 the history of GA convergence to obtain the fittest cost value that represents the accurate estimation of damping ratio of wiper system and its corresponding individual are demonstrated. With the estimated damping ratio of system as 0.15684 and knowing the first natural frequency of system as 11.5 Hz, the amplitude and time location of impulses in Eqs. (7) and (8) are easily calculated. This shaped signal outcome from shaper is used as reference to the wiper system instead of the desired reference, aiming to reduce vibration at the end point without the influence of feedback controller with optimized scale factors.

A GA optimized IS controller is implemented in the path of Bang–Bang input of wiper system in the absence of external disturbance. The effectiveness of designed controller for accurate input tracking and chatter noise reduction are shown in Figs. 14 and 15 respectively. Comparisons of the responses of the wiper system without controller and using ISC signifies that IS scheme is capable of reducing the end-point vibration and chatter noise of system while resulting in better input tracking performance.

In other stage, for developing a robust controller even in the presence of external disturbance the feedback part of controller comprising of a colocated PD controller in alliance with proportional, derivative and integrative iterative learning controller PIDILC are introduced. In addition, since ISC scheme always causes delay in the system response, which in turn results in longer rise time a fitness sharing MOGA optimization is augmented.

In order to assess the effectiveness of the proposed controller in the presence of external disturbance a harmonic disturbance \( \tau_d \) is imposed to wiper over the time as

\[ \tau_d = 2\sin(6t) \tag{11} \]

5.3. Applying the multi-objective controller

5.3.1. Deciding on MOGA

In the field of flexible structure control in the presence of known disturbance using iterative learning (IL) incorporation
of input shaping (IS), adjusting the corresponding scale factors are implemented passively in the controller chip (microcontroller or microprocessor); so, the main concern of such problems confronting to use multi-objective optimizers is to find the most accurate solutions rather than time efficiency or elapse time of searching algorithm. MOGA with its present features in this paper was adequately verified in the previous well-known papers in the same field to be robust in both theory and experiment for flexible link control\[26,32–35,41\]. Hence, due to close inherent similarity of wiper blade and flexible link and knowing that in the current study only data acquisition part is carried out experimentally while controller results are obtained through the simulation, the industrial engineers by default trust to the algorithm that is tested and thrived in real world rather the new and untested one in experiments even if the latter is more efficient. On the other hand, main contribution of this survey is to propose a pioneer and practical controller for the industry purpose (particularly Automobiles companies) while it is a rule of thumb in industrial automation control that as long as a fresh algorithm has not been tested experimentally in real life it would not be reliable despite its theoretical superiorities.

Moreover, flexible structures dynamics unlike the solid ones are highly nonlinear so that the investigation of their control is to a great extent experimental oriented. Damping ratios (inherent dynamic characteristics) and natural frequencies of a real world wiper blade in an experimental rig are high sensitive factor of such structures and not uniformly influenced by the solutions that are obtained from Pareto-fronts. This makes too complex and fragile in a real world task to estimate the best aggregation vector coefficient of a scalar trend MOEA like Multi-Objective Evolutionary Algorithm Based on Decomposition (MOEA/D). In such a problem, algorithms with crowding distance operation in objective space only e.g. Non-dominated Sort Genetic Algorithm-II (NSGA-II) would be the main shortfall. A NSGA-II is more sensitive to sharing factor than MOGA so that makes more difficult to rely on in real world problem without having previous experimental experience of solution space. MOGA because of contribution of Kernel estimation of population density based on the Epanechnikov kernel to keep diversity of solutions as well as its good performance in similar flexible manipulator control test [35] has been proven to be the most reliable algorithm that profoundly obtains Pareto solutions which satisfy controller results in experiment as it was operated in simulation.

All the reasons are contemplated and according to previous experience of authors [26,32,33] and taking into account the fact that the proposed design are improvised to introduce a novel application of MOEA to a narrative robust controller for vibration and noise reduction of a practical flexible structure in real world

Fig. 14. Time domain response of wiper lip without disturbance: (a) end-point acceleration of wiper lip and (b) hub displacement.

Fig. 15. Frequency domain response of wiper lip without disturbance: (a) PSD of end-point acceleration and (b) Yule–Walker spectral density of end-point acceleration.
and industry, the less computational burden and meanwhile the most reliable and practical algorithm that fulfills all the requirements in experimental set up would be MOGA. However, it is believed that once the functionality of the proposed technique in industry is verified, more elaboration and modification like considering more advance and efficient MOEA, faster processors etc. can be included in next versions. Therefore, the abilities and competitiveness of other efficient MOEA in other fields are not downplayed.

5.3.2. Performance measurement of Pareto based MOGA

In a multi objective Pareto based optimization problem it shall be assumed that the true Pareto front is unknown; therefore the only means of evaluation available is to compare the MOGA solutions with each other. Two reliable metrics for evaluating the convergence performance of MOGA namely Hypervolume indicator and ratio of non-dominated solutions \( \text{RNDS} \) are adopted in this study \([51,52]\). Hypervolume indicator assesses the convergence of algorithm toward Pareto front as well as preserving the distribution of Pareto front throughout objectives space. In other words, this metric explores the extent of the objective space covered by a set of solutions which is restricted by setting a suitable reference point. In case of minimization problem, like the case of current paper the reference point is set in such a way that exceeds the constraint of each objective. Hence, once applying this metric to compare the performance of multiple algorithms or efficiency of a single algorithm in successive iterations as the amount of covered hypervolume increases, the one is supposed to perform well.

- **Hypervolume indicator of set \( S \)**

Let the reference point be denoted as \( R=(r_1,r_2,...,r_k) \), the hypervolume indicator of \( S \) (denoted as \( \text{Hv}(S) \)) is defined as the volume of the hypercube restricted by all points in \( S \) and \( R \). For a two-dimensional objective space, let the reference point be set in such a way that \( r_1=0 \) and \( r_2=0 \). And the size of the union of all such rectangles covered by the solutions is used as the measure. This concept can be extended to any number of dimensions to give the general hypervolume metric \([51]\).

\[
\text{Hv}(S) = \text{Leb}(\cup_{x \in S} [f_1(x, r_1) \times f_2(x, r_2) \times \ldots \times f_k(x, r_k)])
\]  

(12)

In Fig. 16 the schematic operation of Pareto front in indicating the effective performance of MOGA algorithm in terms of distributed non-dominated solutions as well as their numbers in count for two-dimensional objective space are illustrated.

It can be seen that as the number of non-dominated solutions and their distribution throughout the objective space increased the Hypervolume indicator’s value represents the greater value according to (12) calculation.

- **Ratio of non-dominated solutions**

This measure simply counts the number of solutions which are members of the latest Pareto-optimal set \( S^* \). The ratio of non-dominated solutions of \( j \)th generation \( \text{RNDS}(S_j) \) can be written as follows:

\[
\text{RNDS}(S_j) = \frac{|S_j - x \in S_j : r < x|}{|S_j|}
\]

(13)

where \( r < x \) means that the solution \( x \) is dominated by the solution \( r \). An \( \text{RNDS}(S_j) = 1 \) means all solutions are members of the current Pareto-optimal set \( S^* \), and an \( \text{RNDS}(S_j) = 0 \) means no solution is a member of the \( S^* \). It is an important measure that, although the number of obtained solutions \( S_j \) is large, if the ratio of non-dominated solutions \( \text{RNDS}(S_j) \) is 0, it may be a worse result that denotes the deficiency of algorithm performance. The illustrative representation of \( \text{RNDS}(S_j) \) is shown in Fig. 17.

MOGA initialized with a random population consists of 60 individuals and maximum generation of 100 as termination criterion. The population is represented by binary strings each of 30 bits, called chromosomes. Each chromosome consists of five separate strings constituting two terms for proportional and derivative scale factors of PD controller and the rest three are specified to proportional, derivative and integrative of II. gains. Hence, in the applied MOGA there will be five decision makers in which the first two values of the first rows are used to calculate the values of proportional and derivative gains of PD controller and three separate scale factors if ILC are thus formed with the

![Fig. 16. Improvement of Hypervolume indicator in two successive generations: (a) jth generation and (b) j+1th generation.](image-url)
values of the first row of the randomly generated initial population. Using educated guess a reasonable range of these parameters that ensure stability of system is defined. Then random binary strings are converted into real values. The crossover rate and mutation rate for this optimization process were set at 85% and 0.009%, respectively. Moreover, Epanechnikov fitness sharing genetic technique was used to ensure that the best solution of each generation is selected for the next generation so that the next generation’s best will never degenerate and hence guarantee convergence of the GA optimization process.

The performance of applied MOGA are assessed and illustrated in Figs. 18 and 19 by means of Hypervolume indicator as well as $R_{NDS}(S)$. From Fig. 18 it can be deduced that the overall number of Pareto front members found in each generation and their diversity throughout the objective space are increased as the number of generations goes on so that the maximum value of Hypervolume is obtained at last generation. Furthermore, in Fig. 19 superb performance of applied MOGA is shown in terms of $R_{NDS}$ such that this ratio has got to the closest value to unity in a quite stable state in the last 18 generations.

Pareto optimal sets of pair objectives for wiper dynamic characteristics including IAEA, maximum overshoot and rise time are shown in Fig. 20.

From Figs. 20a and b it can be deduced that maximum overshoot and IAEA are in obvious conflict with rise time so that makes the decision tough for designer to choose the best trade-off. However, the non-dominated Pareto fronts depicted in Fig. 20c proves that IAEA and maximum overshoot are highly non-competing, and it is important for the decision maker, as it conceptually reduces the complexity of the problem.

The explicit objective values for various optimized values of collocated PD and IL scale factors are shown in Table 1.

From the instances of trade off between the vibration reduction and different time domain performance measures of wiper blade in Table 1, was observed that the fastest response of system is obtained in sol. 2 as 0.1340 s while the IAEA is an undesired value of 61342.96 and 76.382 7% maximum overshoot which are not appropriate for practical case. On the other hand, the least amount of vibration objectives are found in sol. 3 for which the IAEA and maximum overshoot are 37.7812 and 0.0730% respectively for the longest rise time of 0.2160 s. From Table 1, can be seen that even though sol.3 has a greater vibration reduction and wiper hub trajectory tracking than sol. 4 but this is achieved at the expense of longer system delay or rise time. Also, diverse compromise of objectives can be seen in other solutions of Table so that each of them can be obtained by adjusting the corresponding gains of collocated PD as well as ILC.

For more investigation on Pareto front solutions for IAEA, rise time and maximum overshoot of wiper blade is shown in Fig. 21. The $x$-axis shows the design objectives and the $y$-axis signifies normalized values formed of each objective. The existence of trade off between a pair of objectives is evident from crossing lines between adjacent objectives and parallel lines prove harmony between the objectives solutions.

To emphasize the overshoot and delay or rise time of the system’s response for the three nominated Pareto solutions of closed loop iterative input shaping controller by MOGA, the leading edge of the hub displacement is shown in Fig. 22a. It can be seen that, sol. 3 has the minimum extent of overshoot with acceptable rise time compared to two other solutions shown in Fig. 22a. Also, comparisons of IAEA measures of these three trade-off solutions are illustrated in Fig. 22b. Performance of designed controller using three nominated trade-off solutions in the vibration and noise reduction of wiper end point are illustrated in Fig. 23. It has been shown that sol. 3 performed well in vibration and noise reduction with satisfactory level of rise time in relative terms.

Normalized performance measurements of some nominated Pareto solutions for comparing the dynamic characteristics, vibration and noise reduction of wiper blade system are demonstrated in Fig. 24. The main aim of the designed closed loop iterative input shaping controller is to achieve good tracking control of hub displacement response and vibration reduction at the end point of the wiper blade system simultaneously without sacrificing its speed of response. So, regarding this aim and earlier results sol. 3 is preferred to other non-dominated Pareto front solutions that the estimated values of IAEA, maximum overshoot and rise time of wiper blade will be 48.6007s, 0.4676 and 0.1551 respectively.

From Fig. 25 it is evident that vibration reduction at the end point and Bang–Bang input tracking of IS controller in the presence of disturbance have not been satisfactory. However, the optimized controller succeeded to reduce end-point vibration and reject the distortion greatly at the cost of a very small amount of delay in the system response. The PSD and Yule–Walker plots of the wiper end-point for developed controller are shown in Fig. 26. From the performance measurements of various
controllers illustrated in Fig. 27 it can readily be observed that substantial attenuation is achieved with applying optimized proposed closed loop iterative input shaping controller compared to Bang–Bang input and even solitary IS controller in the presence of disturbance.

6. Conclusion

This study centered a practical multi disciplinary control of a flexible structure particularly for an automotive wiper blade system. A model characterizing a flexible wiper structure has been devised using identification system. A dynamic model based experiment is developed at initial stage of the work. Then, the inputs and outputs of the wiper system are used to model the dynamic characteristics of the structure using a modified nonparametric approach called NARXENN. With attaining the estimated transfer function of system with high accuracy, an effective controller was required in order to attenuate unwanted chatter noise generated from system and reduce the oscillation of system as much as possible. A novel closed loop iterative input shaping controller was developed in which the command shaper is designed to suppress the vibration of flexible part of system while iterative part is devised to deal with the probable uncertainty during operation of wiper. In order to achieve significant amount of noise and vibration deduction within reasonable response time duration caused by IS that are in conflict objectives, a fitness sharing MOGA is used as well. After tuning the corresponding gains of controller, It has been observed that the proposed controller succeed to eliminate unwanted chatter noise reached out into the automotive as well as improve the oscillation of

![Graphs and charts illustrating the Pareto front for a problem with two objectives.](image)

**Table 1**

<table>
<thead>
<tr>
<th>Solution no.</th>
<th>Controller parameters</th>
<th>Objectives</th>
</tr>
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<tr>
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<td>$K_d$</td>
</tr>
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<tr>
<td>5</td>
<td>0.6032</td>
<td>0.5863</td>
</tr>
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</table>

Fig. 20. Pareto front illustrations of pair objectives.
flexible wiper lip to a desirable extent during an acceptable rise or delay time of system. It was observed that the developed controller could excellently overcome the deficiency of IS alone where the environmental or external disturbance intervened into the wiper system.

Fig. 21. Instances of three objectives’ Pareto set.

Fig. 22. Time domain response of wiper lip for trade off solutions: (a) end-point acceleration of wiper lip and (b) hub displacement.

Fig. 23. Frequency domain response of wiper lip for trade off solutions: (a) PSD of end-point acceleration and (b) Yule–Walker spectral density of end-point acceleration.

Fig. 24. Normalized performance measurements of nominated solutions of Pareto front.

Fig. 24. Normalized performance measurements of nominated solutions of Pareto front.
Fig. 25. Time domain response of wiper lip in presence of disturbance: (a) end-point acceleration of wiper lip and (b) hub displacement.

Fig. 26. Frequency domain response of wiper lip in presence of disturbance: (a) PSD of end-point acceleration and (b) Yule–Walker spectral density of end-point acceleration.

Fig. 27. Normalized performance measurements of various controllers.

References


