ON CLASSICAL, BAYESIAN AND FUZZY HYPOTHESES TESTING

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Let $X_1, \cdots, X_n$ be independent and identically distributed with density function $f(x|\theta)$, where $\theta$ is a one dimensional parameter. Consider testing simple versus simple hypotheses

$$
\begin{align*}
H_0 &: \theta = \theta_0 \\
H_1 &: \theta = \theta_1,
\end{align*}
$$

where $\theta_0$ and $\theta_1$ are fixed numbers, based on a random sample. One can find the best solution for this problem in the different frameworks as follows:

<table>
<thead>
<tr>
<th>Test</th>
<th>$\theta$</th>
<th>Critical region (by)</th>
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<tr>
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<td>fixed and unknown parameter</td>
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<tr>
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<tr>
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<td>random variable with unknown prior</td>
<td>Likelihood Ratio</td>
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</table>

Now consider fuzzy hypotheses

$$
\begin{align*}
H_0 &: \theta \simeq \theta_0 \quad (\theta \text{ is approximately } \theta_0) \\
H_1 &: \theta \simeq \theta_1 \quad (\theta \text{ is approximately } \theta_1),
\end{align*}
$$

where $\theta \simeq \theta_i$, $i = 0, 1$ are expressed by two membership functions $m_0(\theta), m_1(\theta)$ in fuzzy community and by two prior probability laws $\pi_0(\theta)$ and $\pi_1(\theta)$ in Bayesian community. A few authors had tried to find the best test for testing fuzzy hypotheses, [1,2,4]. In this paper we show that the best test for fuzzy hypotheses in the Bayesian framework is simply equivalent to Neyman-Pearson Lemma in the classical statistics.

Key Words: Classic, Bayes and empirical Bayes test, fuzzy hypotheses, Neyman-Pearson lemma, likelihood ratio test.

References: