Abstract — This paper addresses the problem of energy detection of an unknown deterministic signal over a general fading channel model. More particularly, a closed-form mathematical expression is derived for the energy detector's probability of detection over $\alpha-\mu$ generalized fading channel with selection combining diversity reception. The derived expression can be used to study the performance of energy detector in many known fading channel models with and without selection combining; this can be achieved by choosing some specific values for both $\alpha$ and $\mu$ parameters in the proposed general expression. Nakagami-$m$, Weibull, Gamma, Rayleigh and Exponential fading distributions are special cases of the derived general expression.

Keywords—energy detector; selection combining diversity; fading channels; $\alpha-\mu$ generalized fading distribution.

I. INTRODUCTION

Electromagnetic spectrum, as a natural resource, is limited. It has been divided into specific bands that are assigned to suitable applications. The licensed subscriber has the full permission to use the band they bought whenever and wherever they need to, as long as their licenses are valid. However, vast amounts of the spectrum are not used efficiently, indeed, they are under-utilized. Cognitive radio (CR), which is a clever telecommunication system that can sense and adapt its parameters to avoid interference on licensed users [1], is one solution to this underutilization problem. The CR user is considered as a rental or secondary user of the spectrum and it has to decide when it can access the spectrum and what band to use in order not to cause any kind of interference to any licensed user. This leads to the fact that cognitive radio network must accurately sense the spectrum and adapts its transmission in accordance with the results of its sensing operation and the situation of the channel to be used. There are several spectrum sensing techniques used to enhance the spectrum utilization [2]. The ultimate goal of these techniques is to enable rental (secondary) users benefitting from the white spaces in the spectrum that are spatially/temporally free of primary users. The energy detector proposed in [3], is one of the main and simplest techniques frequently used in cognitive radio networks to enable opportunistic spectrum access.

Several fading distribution models have been suggested to describe the statistics of the received mobile signal envelope [4]. Indeed, the short-term signal envelope variation is properly depicted by several main distributions such as Rice, Nakagami-$m$, Weibull, Rayleigh, Hoyt and others. Each of these fading distributions is suitable for certain channel conditions. In some situations, no distributions satisfactorily match experimental observations, although one of them may produce moderate fitting. This motivates the need for a general distribution that can give up better fitting to real measurements and can include several fading distributions as special cases. One of these general fading distributions is the $\alpha-\mu$ distribution recently proposed in [5][6]. It is an umbrella distribution and involves as special cases several main distributions such as Nakagami-$m$, Rayleigh, Gamma, Weibull, exponential, and one sided Gaussian. In addition, its probability density function, cumulative distribution function, and moments come-out in uncomplicated closed-form formulas. Furthermore, it can describe the non-linearity of the wireless propagation environment. These features make the $\alpha-\mu$ distribution very attractive.

Fading channels can extremely affect the transmitted signals and decreasing the overall signal to noise power ratio (SNR) at the reception end. In this case, antenna diversity reception techniques, that combine the outputs of multiple fading branches together, can be used to boost the SNR at the receiver. Selection diversity combining (SC), equal gain combining (EGC), switch and stay combining (SSC), and maximum ratio combining (MRC) are examples of combining methods used in antenna diversity reception [7].

During the last decade, a lot of interest has been paid to the issue of detecting unknown deterministic signals over a variety of fading channel models with or without diversity reception at the receiver [8][9][10]. Indeed, in [8] the average detection probability of energy detector is derived for Rayleigh, Rician and Nakagami-$m$ faded signals. An alternative analytical approach have been proposed by Digham et al. in [9], where closed-form expressions are obtained for the average detection probability undergoing Rayleigh and Nakagami-$m$ fading with square law combining and square law selection diversity methods. In [10], the moment generating function (MGF) technique and the probability density function (PDF) technique are employed to evaluate the performance of energy detector undergoing Rician and Nakagami-$m$ fading with several diversity combining techniques. However, this yields a wide collection of performance expressions that are applicable only for certain fading models with specific model parameters. To avoid this drawback, Fathi and Tawfik have recently proposed
a versatile performance expression for energy detector over the \( \alpha - \mu \) generalized fading channels [11]. Nevertheless, no diversity combining techniques are considered.

In this contribution, we suggest to extend the results in [11] by considering selection combining diversity reception at the receiver. A new closed-form formula is derived for the average detection probability of the energy detector over \( \alpha - \mu \) generalized fading channels with selection combining diversity reception.

The remainder of the paper is structured as follows. In section 2, the system model is described. Section 3 exposes the derived mathematical expression for the probability of detection of the energy detector with SC. Section 4 presents some special cases of the derived general expression. Numerical examples are presented and discussed in Section 5. Conclusions are reported in Section 6.

II. SYSTEM MODEL AND PERFORMANCE ANALYSIS

A. The Energy Detector (ED)

The ED is simply a threshold-based binary decision device; its output is one of two hypotheses \( H_0 \): white space, i.e. only Additive White Gaussian Noise (AWGN), or \( H_1 \): occupied, i.e. existence of primary user. The decision is made by comparing the aggregated energy of a Band-Pass-Filtered (BPF) received signal, in an observation period of time against a predetermined detection threshold \( \lambda \). Fig. 1 exhibit a block diagram of the well known ED.

Fig. 1. Block diagram of the ED.

The received signal \( r(t) \) can be either AWGN denoted by \( n(t) \), or unknown transmitted signal \( s(t) \) faded by a channel gain \( h \) and added to a noise \( n(t) \) as follows:

\[
r(t) = \begin{cases} 
  n(t) & H_0 \\
  h s(t) + n(t) & H_1 
\end{cases} 
\]

(1)

If the fading channel is characterized to have small-scale variations with nonlinear propagation medium, then the envelope \( h \) of the fading signal obeys the \( \alpha - \mu \) general fading distribution, where \( \alpha \) denotes a positive parameter, and \( \mu > 0 \) denotes the inverse of normalized variance of \( h^\alpha \). The PDF of the signal envelope \( h \) is expressed as

\[
f_h(h) = \frac{\alpha \mu \gamma^{\frac{\alpha - 1}{2}}}{\Gamma(\mu) \gamma^{\frac{\alpha}{2}}} e^{-\frac{\mu}{\gamma}}
\]

(2)

where \( \gamma = h^2 E_s/N_0 \) is the instantaneous signal to noise power ratio (SNR) of the signal envelope, \( E_s \) is the energy of the signal accumulated over the observation period, \( N_0 \) is the power spectral density of the noise, and \( \gamma = h^2 E_s/N_0 \) is the average SNR.

B. Conditional probabilities of detection and false alarm

There are two important probabilities to discuss when distinguishing between two hypotheses related to statistically random variates. They are the false alarm probability \( P_f \) and the detection probability \( P_d \). Fig. 3 shows the miss detection probability \( P_m = 1 - P_d = Pr(Y < \lambda | H_1) \) and the false alarm probability \( P_f = Pr(Y > \lambda | H_0) \).

Fig. 2. The PDF \( f_h(h) \) of the \( \alpha - \mu \) general fading distribution for several values of \( \alpha \) and \( \mu \).

Since the decision statistics \( Y \) is the sum of square values of the received signal amplitudes, its PDF has to be derived from \( f_h(h) \) by simple change of variables [11], as follows:

\[
f_Y(y) = \frac{\alpha \mu \gamma^{\frac{\alpha - 1}{2}}}{\Gamma(\mu) \gamma^{\frac{\alpha}{2}}} e^{-\frac{\mu}{\gamma}}
\]

(3)

where \( \gamma = |h|^2 E_s/N_0 \) is the instantaneous signal to noise power ratio (SNR) of the signal envelope, \( E_s \) is the energy of the signal accumulated over the observation period, \( N_0 \) is the power spectral density of the noise, and \( \gamma = h^2 E_s/N_0 \) is the average SNR.

Fig. 3. False alarm and miss detection probabilities.
The false alarm $P_f$ and detection $P_d$ probabilities can be computed, respectively, in AWGN as follows [9]:

$$P_f = \frac{r(u^2)}{r(u)}$$

$$P_d = Q_u(\sqrt{2\gamma \sqrt{\lambda}})$$

where $\Gamma(.,.)$ denotes the incomplete gamma function, $Q_u(.,.)$ denotes the generalized Marcum Q-function, and $u$ denotes the time-bandwidth product which is equal to half number of symbols in the observation time period.

III. ENERGY DETECTOR PERFORMANCE ANALYSIS OVER GENERALIZED FADING CHANNELS

In this section, the analysis of the ED over the $\alpha - \mu$ generalized fading channel is revisited to derive a mathematical formula for the average detection probability when SC diversity technique is employed at the receiver.

To obtain the Receiver Operating Characteristics (ROC) when considering AWGN and $\alpha - \mu$ fading channel, equation (5) should be averaged over the PDF of the fading channel. Note that the false alarm probability given by (4) has no terms relating to fading channel parameters, and so, doesn’t change.

In [11], the average detection probability of the ED is obtained over the $\alpha - \mu$ general fading distribution, but with no diversity reception. When diversity is used, multiple branches are combined and the combiner output is then compared to a threshold value to distinguish between the two hypotheses $H_0$ and $H_1$. The combining technique used by the combiner determines the shape of the output decision statistics variable. In this paper, we discuss the selection combining diversity technique, where the diversity branch with highest SNR is chosen by the selection combiner. The PDF of the SNR for any single branch in the $\alpha - \mu$ fading channel is given by (3). For $L$ diversity branches, the SNR of the combiner output is equal to the maximum of $\{Y_1, Y_2, ..., Y_L\}$, where $Y_i$ is the $i$-th branch instantaneous SNR. Assuming that the average SNRs for all branches are equal, let’s denote it by $\hat{Y}$, then for any single branch, the probability that its SNR $Y_i$ is less than some value $\gamma$ is given by:

$$Pr(Y_i \leq \gamma) = \int_0^\gamma f_{Y_i}(y)dy = \int_0^\gamma \frac{a_{\mu} \gamma \frac{a_{\mu} + 1}{2} e^{-\mu(\frac{\gamma}{\hat{Y})^{a_{\mu}/2}}}}{2r(\mu) \hat{Y}^{a_{\mu}/2}} dy_i = 1 - \frac{r(\mu, \mu(\frac{\gamma}{\hat{Y})^{a_{\mu}/2}})}{r(\mu)}$$

(6)

The cumulative distribution function (CDF) of the output SNR of $L$ i.i.d selection combiner branches is derived as follows:

$$CDF(\gamma) = Pr(Y_1 \leq \gamma, Y_2 \leq \gamma, ..., Y_L \leq \gamma)$$

$$= \prod_{i=1}^L \left(1 - \frac{r(\mu, \mu(\frac{\gamma}{\hat{Y})^{a_{\mu}/2}})}{r(\mu)}\right)$$

(7)

Now, the PDF of the SNR at the output of the combiner $f_{SC}(\gamma)$ is the derivative of $CDF(\gamma)$, and it is calculated as:

$$f_{SC}(\gamma) = \frac{L a_{\mu} \frac{a_{\mu} + 1}{2} e^{-\mu(\frac{\gamma}{\hat{Y})^{a_{\mu}/2}}}}{2r(\mu) \hat{Y}^{a_{\mu}/2}}
\times Q_u(\sqrt{2\gamma \sqrt{\lambda}})$$

(9)

The Q-function $Q_u(\sqrt{2\gamma \sqrt{\lambda}})$ can be rewritten into series representation [9][12] as follows:

$$Q_u(\sqrt{2\gamma \sqrt{\lambda}}) = \sum_{n=0}^\infty \frac{f(n+u^2)}{r(n+u)} e^{-\gamma}$$

Substituting (10) into (9) and using binomial and then multinomial expansion for the term $\left(1 - \frac{r(\mu, \mu(\frac{\gamma}{\hat{Y})^{a_{\mu}/2}})}{r(\mu)}\right)$, it follows after solving the integral in (9) based on the general Laplace transform [13, 2.2.1-22] that:

$$\tilde{P}_{d,a,\mu,sc} = C \sum_{n=0}^\infty \sum_{l=0}^{L-1} \left(\frac{(\mu-1)}{i} \sum_{m=0}^{\mu-l} \frac{m! (\frac{a_{\mu} + m}{2} + n)}{a_{\mu} - \frac{2n}{\hat{Y}}} \right)$$

$$\times \beta_{mu}(\mu) \alpha_{l,k} \Delta(l-w)$$

(11)

where

$$C = \frac{-a_{\mu} \mu \sqrt{2(\mu - a_{\mu} - 1)}}{2r(\mu) \hat{Y}^{a_{\mu}/2}}$$

$$z = \frac{l(l+1)\mu}{k^{a_{\mu}/2}}$$

and $w = \frac{1}{2} \alpha(\mu + m) + n - 1$. In addition, $\alpha_{l,k}$ is the Meijer-G function, $\Delta(k,a) = \frac{a}{k^{a_{\mu}/2}} \beta_{mu}(\mu)$ denotes the multinomial expansion coefficient which can be computed recursively as illustrated in [4, 9.124].

IV. SOME SPECIAL CASES

A. No diversity reception with $\alpha - \mu$ general fading distribution

In the case of no diversity at the receiver, i.e. single branch $L = 1$, the derived formula for the detection probability of the ED with SC diversity reception in (11) reduces to a previously known result found in [11, Eq. (8)], as follows:
B. Rayleigh distribution with and without diversity reception

The probability of detection for the ED with SC diversity reception when considering Rayleigh faded signals can be found from (11) by setting \( \alpha = 2 \) and \( \mu = 1 \):

\[
\hat{P}_d = A \sum_{n=0}^{\infty} \frac{\alpha^n \mu^k}{(n\mu)^k} \left( \frac{\nu}{\nu + 1} \right)^n \frac{1}{n!} \left( \frac{\nu + 1}{\nu} \right)^{-\nu - 1} \left( \frac{\nu + 1}{\nu} \right)^{-\nu - 1} \left( \frac{\nu + 1}{\nu} \right)^{-\nu - 1}
\]

For the no diversity case \( L = 1 \), equation (13) reduces to:

\[
\hat{P}_d,\text{Ray,SC}= \sum_{j=0}^{\infty} \frac{r^{j+u}}{L^{j+u}} \left( \frac{1}{L^j} \right) \left( \frac{1}{1+\nu} \right)^{L^j+u} \left( \frac{1+\nu+1}{\nu} \right)^{-\nu - 1}
\]

V. NUMERICAL RESULTS AND DISCUSSION

The performance of the ED is quantified by depicting the complementary Receiver Operating Characteristics (ROCs) \( P_m = 1 - \hat{P}_d \) with the effect of the various parameters \( \alpha, \mu, L, \) and \( \nu \). From the \( \alpha - \mu \) general fading distribution, other fading distributions can be derived based on specific values for both \( \alpha \) and \( \mu \). The following \( \{ \alpha, \mu \} \) pairs were taken as test cases for the subsequent complementary ROCs [11]. \{2.5\} Nakagami-m \( (m=5) \); \{1.5\} Gamma (Chi-Square \( \alpha=5 \)); \{1.5,1\} Weibull \( (K=1.5) \); \{1,1\} Exponential; \{2,1\} Rayleigh.

Fig. 4 shows the complementary ROCs of the ED over \( \alpha - \mu \) fading channel without SC; it is the special case when setting \( L=1 \) in (11) yielding (12). By selecting different values of \( \alpha \) and \( \mu \), the results for several well known distributions are shown. One can notice from Fig. 4 that the performance of energy detector is degraded when going from the non-fading case (only AWGN channel) to the fading case with several distributions. Exponential faded channel results in the worst performance while Nakagami-m faded channel with severity parameter \( m=5 \) yields the best performance. Note that Rayleigh faded channel (special case when \( m=1 \)) yields better performance than Nakagami-m faded channel with \( m=5 \). Thus, the higher is the \( m \), the higher is the detection probability.

Fig. 5 shows the complementary ROCs of the ED over \( \alpha - \mu \) fading with SC diversity \( (L=2) \). When comparing Fig. 4 and Fig. 5, one can notice that the SC antenna diversity technique greatly enhances the performance of the ED. For example, when \( P_f=0.2 \), all curves in Fig. 4 have \( P_m \) less than 0.4, while in Fig. 5 they are less than 0.2. This means that the more is the diversity branches, the less is the miss detection probability.

The effect of increasing the average SNR (\( \nu \)) on the complementary ROCs of the ED is depicted in Fig. 6. One can notice that, the miss detection probability improves greatly when increasing \( \nu \) from 10dB to 25dB. Fig. 7, Fig. 8 and Fig. 9 illustrate the effect of increasing the number of SC diversity branches (from \( L=1 \) to \( L=5 \)) on the complementary ROCs of the ED for Rayleigh, Nakagami-m and weibull faded signals, respectively. It is noticed that the miss detection probability is greatly reduced when increasing the number of diversity branches.
Fig. 7. Complementary ROCs of the ED for Rayleigh fading channel \((\alpha = 2, \mu = 1)\) with different values of SC diversity branches \(L\), \(u=5\), and \(\gamma = 20\) dB.

Fig. 8. Complementary ROCs of the ED for Nakagami-\(m\) fading channel \((\alpha = 2, \mu = m = 5)\) with different values of SC diversity branches \(L\), \(u=5\), and \(\gamma = 20\) dB.

Fig. 9. Complementary ROCs of the ED for Weibull fading channel \((\alpha = K = 1.5, \mu = 1)\) with different values of SC diversity branches \(L\), \(u=5\), and \(\gamma = 20\) dB.

VI. CONCLUSION

A new expression for the detection probability of the ED over \(\alpha - \mu\) generalized fading model with SC antenna diversity is derived in this paper. The derived expression covers several known fading distribution models as special cases as well as it can be used with and without SC diversity reception. Complementary ROCs were drawn for the ED where enhancement of the probability of detection was achieved by using SC antenna diversity reception. Currently, we are analyzing the detection probability of the energy detector undergoing \(\alpha - \mu\) fading with other diversity combining techniques such as EGC, MRC, SLC, etc.

REFERENCES