A Comparative Study for 3D Surface Modeling of Coal Deposit by Spatial Interpolation Approaches

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Abstract

The planning stages of mining require comprehensive and detailed analyses. The proper determination of the orebody boundary is one of the most important points to provide optimum model structure and projections. The limits can be determined by different methods based on the site geology. Although some three dimensional (3D) models were proposed for providing detailed information concerning a mine deposit, developing a solid model via a 3D approach is novelty. In other words, surface modeling should be performed for creating a solid model and determining limits of the deposit. In this way, sensible generation of the surface model can be achieved. This study investigated the estimation capability of the polynomial approach, which is a novel spatial interpolation method, for modeling a coal deposit surface. The performance of the proposed model was compared with some conventional methods in the literature. The results showed that the polynomial interpolation method is an effective method to employ for surface modeling of a mine deposit.

Keywords: coal deposit, polynomial interpolation, spatial interpolation, surface modeling.

1. Introduction

Location-based assessment of quality values of an ore is important for preparing optimum mine plans. These appraisals can be conducted for both interpolation and visualization purposes. The interpolation/visualization process in mining aims to estimate the value of the blocks that are stated by locations and the solid models developed by location information. A solid model obtains some necessary information and additional useful tools to investigate the position and shape of an orebody.

The solid modeling was originally developed for defining 3D objects in computer aided design (Mantyla & Tamminen, 1983; Liu et al., 2001; Wang et al., 2001; Calcagno et al., 2008). The solid modeling approach completely and unambiguously defines the volume of a three dimensional object. Solid models can be manipulated via set operations. For example, a new solid model can be created by computing the volumetric union, difference or intersection of two solids. Some researchers have carried out studies on solid modeling in the earth sciences (Fisher & Wales, 1990; Lemon & Jones, 2003; Zhu et al., 2012).

In recent years, 3D solid modeling of a mine deposit has been provided by some software packages. If surfaces are determined, a solid model can be obtained easily among these surfaces. The importance of surface modeling has emerged recently since faces obtained by the surface modeling are defined by limits of both the solid model and the block model.

From a general perspective, interpolation is the process of predicting the values of attributes at unsampled sites. Contrary to classic modeling approaches, spatial interpolation methods incorporate information about the geographic position of sample...
points (Schloeder et al., 2001; Apaydin et al., 2004; Calcagno et al., 2008; Dag & Mert, 2008; Xie et al., 2011). The methods in the literature generally focus on modeling of the surface and they gain importance in generating more sensitive surfaces. Some of these methods, such as inverse distance weighting, nearest neighbor, splines, triangulation, kriging and trend surface methods, are used for surface estimation (Nayak et al., 2002; Falivene et al., 2010). In the present study, the polynomial regression approach, an alternative methodology, is used for surface modeling. This interpolation method is simpler than others in controlling only polynomial order. The availability of the method for mine deposit is examined based on real borehole data information.

The rest of this article is structured as follows: Section 2 describes the problem and methodology. In this section, a general theory is given about the polynomial and conventional interpolation methods. Section 3 covers the experimental studies, while Section 4 presents the results and discussions and Section 5 concludes the paper.

2. Methodology
2.1 Description of problem
There are three types of 3D model: wireframe, surface and solid models (Fig. 1). Solid models of geologic structures can provide necessary data and clues for analyzing the systems by researchers and practitioners. These models completely and unambiguously define the stratigraphy for the site being modeled, including complex boundaries and embedded seams (Lemon & Jones, 2003).

As a concrete site definition, a solid model can be created using different methods, one of which is solid modeling between two surfaces. In this method, the solid model is created by extending the top surface down to bottom surface of geologic units. Surface modeling primarily requires building the solid model. As is known, the surface modeling is carried out by estimation techniques. These techniques in mine planning software include the inverse distance weighting, nearest neighbor, splines, triangulation and kriging methods. Surface modeling, the first step of the process, is crucial as it affects the sensibility of other modeling and planning stages. For this reason, the utility of different methods over conventional methods should be investigated.

2.2 Conventional interpolation methods
2.2.1 Triangulation interpolation
Triangulation is a significant topic especially in geographic information systems where a meshwork of triangles cuts through a surface. Triangular networks are referred to as Delaunay triangulations. The process of triangulation consists of determining the natural neighbors of successive points on a map. Figure 2 shows the steps of determining triangles. In Figure 2b, point 2 is the first neighbor of point 1 if the circle contains no other points. The circle is expanded as shown in Figure 2c so that points 1 and 2 lie on its perimeter. The interior of the circle is checked to see if any points are enclosed. If one point is found, it is the second neighbor. Thus, point 3 is the second neighbor of point 1. The process repeats until all of the three neighbors of the triangles have been identified (Fig. 2d).

In the triangulation interpolation (TI), the equation of a simple planar surface passing through the vertices of the triangle is determined, an elevation (z) having

Fig. 1 3D models: (a) wireframe; (b) surface; and (c) solid.
specified locations \((\varphi, \lambda)\) is then found by this equation. The equation of a simple planar surface is:

\[
zb = b_0 + b_1 \varphi + b_2 \lambda
\]

where \(b_0, b_1, \text{ and } b_2\) are unknown coefficients that can be found by solving three normal equations of the known coordinates \((\varphi_i, \lambda_i, z_i)\) of three points of triangle \((i = 1, 2, 3)\).

\[
zb = b_0 + b_1 \varphi_i + b_2 \lambda_i
\]

\[
zb = b_0 + b_1 \varphi_2 + b_2 \lambda_2
\]

\[
zb = b_0 + b_1 \varphi_3 + b_2 \lambda_3
\]

2.2.2 Ordinary kriging

Geostatistics is the application of the regionalized variable theory to the estimation of mineral deposits. A geostatistical analysis can be divided into four main steps as follows:

1. Determination of semivariogram models,
2. Testing semivariogram models,
3. Estimations with kriging,
4. Determination of estimation errors,

All of these steps should be systematically performed in a geostatistical study. Depending on the stochastic properties of random fields, different types of kriging apply. Ordinary kriging (OK) is the most commonly applied method. In geostatistics, the spatial variability of a regionalized variable is characterized by the semivariogram function \(\gamma(h)\):

\[
\gamma(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} [Z(x_i) - Z(x_i + h)]^2
\]

where \(h\) is the separation vector, \(\gamma(h)\) is the semivariance, \(N(h)\) is the number of data pairs and \(Z(x)\) is a random variable defined at point \(x\) (Goovaerts, 1997).

The ordinary kriging estimator of an unknown average value at a point \(x_0\) by \(Z^*(x_0)\) is
\[ Z^*(x_0) = \sum_{i=1}^{N} \lambda_i Z(x_i) \]  

(4)

where \( \lambda \) is the weight and \( Z(x_i) \) is the observed value. To ensure that the estimate is unbiased the weights are made to sum to 1 (Webster & Oliver, 2007).

### 2.3 Polynomial interpolation

Polynomials are often used when a simple empirical model is required. The polynomial model can be used for interpolation or extrapolation, or to characterize data using a global fit. Polynomial interpolation (PI) is a process of finding a theoretical model whose graph will pass through a given set of points. For the polynomial equation, the measured data \((x, y, z)\) are converted to the normalized values \((\phi, \lambda)\)

\[ \phi_k = \frac{x_k - \bar{x}}{\text{std}_x}, \quad k = 1, 2, \ldots, n_k \]  

(5)

\[ \lambda_k = \frac{y_k - \bar{y}}{\text{std}_y}, \quad k = 1, 2, \ldots, n_k \]  

(6)

where \( \bar{x} \) and \( \bar{y} \) are the mean values of \( x \) and \( y \), while \( \text{std}_x \) and \( \text{std}_y \) are the standard deviation of \( x \) and \( y \), and \( n_k \) is the number of points.

Polynomial interpolation fits a polynomial to the surface. The equation of the polynomial for surface modeling is:

\[ z^*(\phi, \lambda) = \sum_{i=0}^{n_\phi} \sum_{j=0}^{n_\lambda} b_{ij} \phi^i \lambda^j \]  

(7)

where \( z^* \) is the predicted value of \( z \) at specified values of \( \phi \) and \( \lambda \), \( n \) is the degree of \( \phi \), \( m \) is the degree of \( \lambda \), and \( b_{ij} \) is the coefficient of the term \( \phi^i \lambda^j \), which is calculated using the least squares method. The deviations, \( z^*_k - z_k \), are considered and sum of the square of error must be as follows (Davis, 2002):

\[ \sum_{k=1}^{n_k} (z^*_k - z_k)^2 = \text{minimum} \]  

(8)

The least square fitting method is used to determine coefficients \( (b_{ij}) \) of the polynomial function for minimum deviations. The degree of the polynomial is the maximum of \( \phi \) and \( \lambda \) degrees and it is very important in practice. In surfacing with polynomial regression methods, the degree of polynomial depends on the number of points and degree of freedom. As much as possible, it must start with the highest degree and the most suitable coefficient must be determined with statistical tests. In the polynomial surface model, the degrees of \( \phi \) and \( \lambda \) terms can be up to 5. The name of the model type ‘poly’ followed by the degree of \( \phi \), or \( \phi \) and \( \lambda \) is specified, for example, if a \( \phi \) degree of 3 and a \( \lambda \) degree of 2 are specified, the model name is poly32.

### 3. Case studies

#### 3.1 Field description and data set

The study area, Çöllolar field, is located in Afsin-Elbistan lignite deposit about 15 km away from Kahramanmaras city, in southeast Turkey. The Afsin-Elbistan lignite deposit covers an area of 900 km², contains 3.4 billion metric tons of reserves and is the biggest lignite basin and one of the most important resources for electrical energy production in Turkey (Dag, 1997). There are a total of six sectors: A (Kısıkoy), B (Çöllolar), C (Afsin), D (Kuşkayası), E (Çobanbey) and F.

Many researchers have studied on the geology of the Afsin-Elbistan lignite basin in the past (Mert, 2010). It is known that in the east part of the basin outcrops of Neogene formations were observed and these formations are covered by Quaternary deposits. The total thickness of these materials is about 300–400 m. The Neogene lithologies are listed from top to bottom as:

1. Red and brown coarse-grained clastics;
2. Reddish brown and sandy marl sediments;
3. Greenish and bluish plastic clay and marl units;
4. Lignite;
5. Gyttja; and
6. Greenish and bluish plastic clay and marl units with lignite streaks (Yorukoglu, 1991; Tutluoglu et al., 2011).

Figure 3 gives both geology and location maps of the study area. The data used in this study were obtained from 212 exploration boreholes performed by the General Directorate of Mineral Research and Exploration of Turkey (MTA) in 2005. The data were identified from hole id and coordinate values of east \((x)\), north \((y)\) and top elevation of coal seam \((z)\). The data were randomly divided into two subsets: the training set (150 samples) and the testing set.
(62 samples). The locations of boreholes are shown in Figure 4.

3.2 Triangulation Interpolation (TI) model
The triangulation networks are used to establish the surface of the solids. The surfaces defining the volume enclosed by a solid model are composed of triangles. Figure 5 shows the set of the Delaunay triangles formed across map with training data points. The elevation values of the testing data locations are estimated with equation of planar surface and belong to triangle covered locations. Estimated values and parameters used in estimation are given in Table 1 as an illustration.

3.3 Ordinary Kriging (OK) model
Kriging estimation largely depends on the variogram model. Variogram is a measure used to describe the
spatial relationship for a single variable. The data variability in the spatial system was evaluated by experimental variogram and theoretical models were fitted to the experimental variograms. Figure 6a, b shows both experimental and theoretical variogram models. The parameter values calculated for spherical and Gaussian models are given in Eqs (9) and (10).

\[
\gamma(h) = \begin{cases} 
50 + 94 \left[ \frac{3}{2} \left( \frac{h}{498} \right) - \frac{1}{2} \left( \frac{h}{498} \right)^3 \right] & h \leq 498 \\
50 + 94 & h > 498
\end{cases}
\] (9)

\[
\gamma(h) = 55 + 90 \left[ 1 - e^{-\frac{h^2}{498^2}} \right]
\] (10)

3.4 Polynomial Interpolation (PI) model

The polynomial surface interpolation model fits a polynomial to the dataset. Different surface fit options and methods are tested and the best polynomial surface model is determined by the data according to the goodness of fit statistics. The equation of the determined polynomial model, poly55, is as follows:

\[
Z(x, y) = b_0 + b_1 x + b_2 + b_3 x^2 + b_4 xy + b_5 y^2 + b_6 x^3 + b_7 x^2 y \\
+ b_8 xy^2 + b_9 y^3 + b_{10} x^4 + b_{11} x^3 y + b_{12} x^2 y^2 \\
+ b_{13} xy^3 + b_{14} y^4 + b_{15} x^5 + b_{16} x^4 y + b_{17} x^3 y^2 \\
+ b_{18} x^2 y^3 + b_{19} xy^4 + b_{20} y^5
\] (11)

The coefficients of the polynomial model are given in Table 2. Figure 7 shows the surface of the fitted model and data location.

4. Results and discussion

For informing the validation of the developed models, the relationship between the estimated and measured values belonging to the top elevation of the coal seam should be considered. Performances of the models have been tested using the testing data. Figure 8 shows the measured and estimated values of the models.

There are many indicators to define performance of estimation. Relative error (RE), root square mean error (RMSE) and the coefficient correlation (r) are well-known performance indicators. The RE and RMSE values are calculated as follows:

\[
RE = 100 \times \frac{|y - y^*|}{y}
\] (12)

\[
RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (y_i - y_i^*)^2}
\] (13)

where y and y* denote measured and estimated values, and N is the number of data, respectively. Because the
Table 1 Estimated elevation values and parameters used in the triangulation model

<table>
<thead>
<tr>
<th>Id</th>
<th>Sample locations East-x</th>
<th>North-y</th>
<th>Locations of triangle vertices East-x</th>
<th>North-y</th>
<th>Elevation-z</th>
<th>Eq. coefficients†</th>
<th>Sample elevation</th>
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<td></td>
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<td>b₁</td>
<td>b₂</td>
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<td>1,066.95</td>
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</table>

†Where locations (x, y) are normalized by mean and standard deviation (std) of locations of triangle vertices (x, y).

Fig. 6 Variogram models for the top elevation of the coal seam: (a) spherical model; and (b) Gaussian model.
average RE values are smaller than 10%, it can be expressed that these methods can be accepted as successful (Bardossy & Fodor, 2004; Tutmez, 2012). The RMSE and r indicators are used to characterize the performance of the model—the higher r and the lower RMSE, the better the model. Data variability is important in spatial data analysis. For this reason, the standard deviation (std) of the measured and estimated elevation values were compared in addition to r, average RE and RMSE.

The results are given in Table 3. As seen in this table, the PI model has better model performance owing to the nearer std (18.530), the higher r (0.851), the lower average RE (0.648) and RMSE (10.215).

5. Conclusions
Sensitively generated surface models have a crucial importance for the optimum planning and three dimensional modeling in mining applications such as geological, hydrological, geotechnical and solid modeling. In this study, three different interpolation methods were used for modeling a surface of coal seam and results were compared.

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As a result of the experimental studies, some merits of the polynomial interpolation method have been recorded. First, the polynomial surface modeling method has provided more accurate results. Second, it is computationally simpler and also a practical method. Hence, the polynomial interpolation approach could be applied for modeling surfaces of coal seams as well as orebodies. Finally, a future work on the same topic is projected. In the projected study, it is aimed to combine the polynomial interpolation method and soft computing-based interpolation methods for providing new resource estimation tools.

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References


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