Simultaneous Topology, Shape and Size Optimization of Truss Structures by Fully Stressed Design Based on Evolution Strategy

This is the pre-print version of the published paper in “Engineering Optimization” [1]. Some minor modifications were performed in the subsequent stages.


Corrigendum:

The ground structure depicted for the 68-bar test problem (Figure 2f) has 73 bar. The following 5 bars should not exist in the ground structure of this problem:

\( A_{3,5}, A_{6,8}, A_{9,11}, A_{12,14}, A_{15,17} \)

where the subscript denotes the connecting nodes of the bar.
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Abstract The most effective scheme of truss optimization considers the combined effect of topology, shape and size (TSS); however, most available studies on truss optimization by meta-heuristics concentrated on one or two of the above aspects. The presence of diverse design variables and constraints in TSS optimization may account for such limited applicability of meta-heuristics to this field. In this article, a recently proposed algorithm for simultaneous shape and size optimization, Fully Stressed Design based on Evolution Strategy (FSD-ES), is enhanced to handle TSS optimization problems. FSD-ES combines advantages of the well-known deterministic approach of Fully Stressed Design with potential global search of the state-of-the-art ES. A comparison of results demonstrates that the proposed optimizer reaches the same or similar solutions faster and/or is able to find lighter designs than those previously reported in the literature. Moreover, the proposed variant of FSD-ES requires no user-based tuning effort which is desired in a practical application. The proposed methodology is well tested on a number of problems and is now ready to be applied to more complex TSS problems.

Keywords: fully stressed design; adaptive penalty; truss optimization benchmarks; evolution strategies, resizing.
1 Introduction

Truss optimization can be considered from three distinct perspectives. Topology optimization deals with selection of nodes and their connectivity. On another level, shape optimization seeks to find the optimal coordinates of existing nodes. Finally, cross-sections of truss members can be optimized, which is known as size optimization. In each case, the optimization problem is subjected to some constraints on nodal displacements, member stresses, critical buckling loads, natural frequencies, etc. The objective function is usually set to minimize the structure weight which usually correlates with the overall cost.

While methods for shape and size optimization are quite similar in available truss optimizers, a great variety of ideas for topology optimization have been developed in recent decades. Shape annealing (Reddy and Cagan 1993) forms new topologies by employing some predefined rules, called shape grammar, on an existing design. Shape and size variables are also modified one-at-a-time using simulated annealing to direct the search process. A revised version was applied to more intricate structures such as a transmission tower (Shea and Smith 2006) and a roof truss (Shea and Cagan 1999) by modifying the topology, shape and size. However, flexibility of the topology optimization phase was quite limited.

In evolutionary structural optimization (ESO) (Xie and Steven 1993), the design domain is discretized into 2D or 3D elements and inefficient element are gradually eliminated. The improved version, called BESO (Huang and Xie 2007), allows for reactivation of eliminated elements and smoothing of the topology. However, the whole topology optimization strategy resembles a local search where the algorithm usually converges to a close local optimum. Application of stochastic optimizers, which may provide potent global search in cost of more evaluations, is not practical since this approach employs a rather costly 2D or 3D finite element analysis. Additionally, the intricacy of the final topology relies on the fineness of the mesh, and the BESO algorithm has four control parameters to be tuned by the user.
The most common strategy to handle truss topology optimization is the ground structure method (Topping 1983), where a subset of an excessively connected structure is determined by the optimizer. Engineering intuition can be utilized to form a reasonable ground structure for the problem at hand and evaluation is quite fast since FE analysis is carried out using 1D elements, making it more desirable for population based optimizers. An extension of this idea with a customized genetic algorithm as an optimizer has also been tried (Deb and Gulati, 2001).

This study aims at reinforcing a recently introduced ES-based truss optimizer, called Fully Stressed Design based on Evolution Strategy (FSD-ES), such that it can handle simultaneous TSS problems as well. A variant of this algorithm, which handles only size and shape optimization, outperforms existing shape and size optimizers (Ahrari and Atai 2013). In particular, FSD-ES combines the reliable global search of meta-heuristics with engineering knowledge on truss analysis to make the most of each evaluation. This enables it to handle optimization of more complex structures with a large number of design variables. Moreover, unlike most available TSS truss optimizers in literature which require an ad hoc tuning of several control parameters for a new problem, the proposed FSD-ES is quasi parameter-free from the user’s perspective.

The literature on TSS optimization employing the ground structure concept is surveyed and a brief review of the available FSD-ES and other ES-based optimizers is provided in section 2. The enhanced algorithm is elaborated in details in section 3. In section 4, a systematic experimentation on a selection of test problems is conducted and obtained results from the proposed algorithm are compared to the best available results in the literature.

1.1 Related work

The advent of deterministic truss optimizers dates back several decades (Topping 1983). A simple yet efficient technique to optimize truss structures is the Fully Stressed Design approach (Topping 1983), in which members are resized after each analysis so that the stress ratio
approaches unity. Later, this resizing technique was extended to problems with deflection and buckling constraints as well as inelastic analyses (Saka 2003). The rationale underlying this method is that in an optimally sized structure all members should reach their stress limit. Although logical, this is not necessarily valid for indeterminate truss structures or those subjected to multiple load cases (Topping 1983). In such problems the maximum stress ratio at some members could be smaller than unity (Topping 1983). It does not consider any objective function and hence, such a methodology can be potentially outperformed by using a mathematical programming method Topping (1983). A combination of both methods where layout is derived by mathematical programming and subsequently resized by Fully Stressed Design approach was demonstrated to be useful; however, both methods were originally designed to handle continuous values for member sections and their application to discrete truss problems is much more complicated (Saka 2003). Both methods gradually eliminate excessive members, but neither can reactivate a previously eliminated member, which could be beneficial during the optimization process.

Due to the shortcoming of robustness when using deterministic approaches, applications of meta-heuristic algorithms in structural optimization have gained much interest in the recent decade (Hare et al. 2013). Although meta-heuristic methods, in comparison to deterministic approaches, may require a significantly larger number of solution evaluations, this issue has become less important due to rapid development of computers and parallel processing. On the other hand, meta-heuristics perform a guided random sampling, thereby allowing the algorithm to explore a larger fraction of the search space (Lamberti and Pappalettere 2011). This enables handling of nonlinear, multimodal and discontinuous truss problems, where deterministic approaches are prone to be trapped in undesirable local minima. In contrast to mathematical programming approaches that can only eliminate a member (Topping 1983), meta-heuristics may reactivate a removed member or node during the
optimization process. Additionally, structural optimization is seldom a continuous variable problem, which highly limits the applicability of gradient-based methods.

Most previous studies on truss optimization by meta-heuristics, even those published recently, deal with truss optimization on one or two levels. For example, size optimization by Harmony Search algorithm (Lee et al. 2005, Degertekin 2011), Artificial Bee Colony (Sonmez 2011), Big-Bang-Big Crunch algorithm (Kaveh and Talatahari 2009b), Reduced Space and Sequential Quadratic Programming (Chen and Huang 2012), Charged System Search (Kaveh and Talatahari 2010), Chaotic Imperialist Competitive Algorithm (Talatahari et al. 2012), evolution strategies (Papadrakakis et al. 1998), Particle Swarm Optimization (Li et al. 2007, Lu et al. 2012), Genetic Algorithm (Toğan and Daloğlu 2008), Ant Colony Optimization (Kaveh et al. 2008), and also some hybridized algorithms (Ayvaz et al. 2009, Kaveh and Talatahari 2009a, Rahami et al. 2011) are worth mentioning. Hasançebi et al. (2009) compared performance of seven different stochastic optimization techniques for size optimization of truss structures and concluded that evolution strategies and Simulated Annealing are the most reliable approaches.

More sophisticated schemes consider the joint effect of shape and size (Hasançebi 2008, Farajpour 2011, Kaveh and Talatahari 2011, Lee et al. 2011, Miguel and Miguel 2012, Kaveh and Khayatzad 2013), or topology and size (Ruiyi et al. 2009, Su et al. 2011). Nevertheless, studies on simultaneous topology, shape and size (TSS) optimization are comparatively scarce, although this is the most rigorous, complete, and effective scheme (Luh and Lin 2008). The diversity in the types of design variables may account for this trend. Boolean, continuous and discrete variables should be reasonably employed for topology, shape and size optimization, respectively (Rajan 1995). Additionally, the number of design variables grows excessively for more intricate structures, which quite often demands a large population size (Deb and Gulati 2001, Hultman 2010). Accordingly, some techniques aimed at alleviating
problem complexity of TSS optimization of truss structures have emerged during the past decade, a number of which are discussed.

Genetic Algorithms (GAs) have been widely utilized in TSS optimization of truss structures (Rajan 1995, Deb and Gulati 2001, Rahami et al. 2008, Hultman 2010). When using Binary-coded GAs (BGAs), continuous variables are discretized (Rahami et al. 2008), for which the discretization step, which determines precision of the optimized results, should be tuned. Deb and Gulati (2001) proposed a real-valued GA in which the search range of member areas is assumed symmetric, for example, \([-A, A]\), and members with areas less than a predefined threshold, \(+\varepsilon\), are considered passive, i.e. eliminated from in the structure. This strategy was also employed in some recent studies (Luh and Lin 2008, Wu and Tseng 2010), resulting in continuous treatment of all variables.

Another strategy to moderate problem complexity of TSS optimization is to use a two-stage approach. First, the structure topology is optimized while the cross-sectional area of members and shape of the truss remain fixed. After an optimized topology is found, size as well as shape of the obtained topology is optimized. Such a strategy greatly alleviates the problem complexity as it reduces the number of object variables at each stage. Luh and Lin (2008, 2011) exploited this strategy for TSS optimizing using Ant Colony or Particle Swarm Optimization methods. Although for the investigated problems this two-stage strategy appeared beneficial, it cannot always provide the global optimum since TSS optimization is not a separable problem (Deb and Gulati 2001, Rahami et al. 2008). The obtained results were outperformed by another method based on Differential Evolution (Wu and Tseng 2010), which considers the joint effects of topology, shape and size. Nonetheless, in the latter research, the drawbacks of continuous values for member areas and specifying the critical area, \(\varepsilon\), remained unsolved.
A remarkably efficient strategy is to activate or deactivate a non-basic node or member, a node or member that can be eliminated, with similar probabilities (Hasançebi and Erbatur 2002a, Hasançebi 2007). This strategy leads to an inherent bias towards topologies with small number of nodes and members, since the number of acceptable topologies in which a non-basic node is active is much more than those where this node is passive.

Noilublao and Bureerat (2011) applied multi-objective EAs on TSS optimization of a slender truss tower, where a second objective is introduced using either the natural frequencies, frequency response function (FRF), or force transmissibility (FT). Sequential cellular particle swarm optimization (SCPSO), suggested by Gholizadeh (2013), provided good solutions for some TSS test problems. However, the algorithm needed tuning of five control parameters and selection of a grid for each problem. The large amount of tuning required substantially diminishes the practicality and robustness of the algorithm. The firefly algorithm (FA) (Miguel et al. 2013), in contrast, has a few control parameters and thus more practical, but the best solutions found by this method were slightly heavier.

2 Evolution Strategies and ES-based Truss Optimizers

One of the main streams of evolutionary algorithms, the evolution strategies (ES), follows the principles of natural selection, including recombination, mutation and selection. In the canonical form, a number of $\lambda$ offspring are produced by recombination and mutation of $\mu$ (usually less than $\lambda$) parents. Selection is performed over the recently generated offspring (“Comma” scheme), or the combination of parents and offspring (“Plus” scheme). These two schemes are denoted by $(\mu|\rho,\lambda)$-ES and $(\mu|\rho+\lambda)$-ES respectively. The extra parameter, $\rho$, specifies the number of parents that recombine to generate each offspring. More details on ES and their variants can be found in a comprehensive review by Beyer and Schwefel (2002).

Among ES variants, Covariance Matrix Adaptation Evolution Strategy (CMA-ES), is considered as the state-of-the-art ES for unconstrained continuous optimization (Beyer and
Sendhoff 2008, Kramer 2010). This optimizer ranked the best among the optimization algorithms that participated in the CEC-05 (García et al. 2009) and BBOB-2009 (Hansen et al. 2010b) workshops. CMA-ES employs normal distribution for sampling, mutates all variables simultaneously, performs non-elite (“Comma”) selection, recombines design variables using Global Weighted Recombination (GWR), and adapts the covariance matrix of sampling distribution. However, TSS optimization problems are neither unconstrained nor continuous and some of the peculiar features of ES, and CMA-ES in particular, are inevitably modified while tailoring them for truss optimization. For example, in many previous studies, a discrete distribution replaced the normal distribution (Hasançebi 2008, 2009, Thierauf and Cai 1998, Lagaros et al. 2002), the discrete recombination replaced the intermediate recombination for size variables (Papadrakakis et al. 1998, Thierauf and Cai 1998, Lagaros et al. 2002, Ebenau et al. 2005, Hasançebi 2008, Hasançebi et al. 2009), only a fraction of variables were mutated (Lagaros et al. 2002, Papadrakakis et al. 1998, Ebenau et al. 2005, Hasançebi 2007, 2008, Hasançebi et al. 2009), or the elite individuals were preserved (Hasançebi 2008, Ebenau et al. 2005).

Unlike previous efforts for truss optimization by ES, FSD-ES tries to abide by the principles of the state-of-the-art ES (Ahrari and Atai 2013). The next section elaborates the steps of the reinforced FSD-ES, which can handle topology optimization as well.

3 FSD-ES for simultaneous TSS optimization

In this section the problem representation, algorithm steps, and rationale underlying each strategy are explained in detail so that the role of each part is clarified and that the results are reproducible. Some concepts and relations are quite similar to those utilized in the earlier version (Ahrari and Atai 2013), however, the new variant of this algorithm can encompass TSS optimization problems as well.
3.1 Problem Representation

In FSD-ES, each candidate design is configured by three vectors:

- $M_{1\times e}$ is a vector of Boolean variables that determines connection of members in a candidate design.
- $X_{1\times 3n}$ is a vector of continuous variables that determines nodal coordinates.
- $A_{1\times e}$ is a vector of continuous/discrete variables that determines member areas.

Parameters $e$ and $n$ denote the number of members and nodes in the ground structure, respectively. Such representation resembles the work of Rajan (1995), where Boolean, continuous, and discrete variables were allocated for topology, shape, and size respectively.

Matrices $M=[m_{ji}]_{2\lambda\times e}$, $X=[m_{ji}]_{2\lambda\times 3n}$, $A=[a_{ji}]_{2\lambda\times e}$ store the values of topology, shape and size of all individuals in the current iteration, where $2\lambda$ is the population size.

3.2 Initial Values

For the first iteration, recombinant design can be randomly selected within the bounds. As this design is not evaluated, it does not necessarily belong to the given discrete set. The recombinant point, denoted by the subscript R, consists of vectors of design variables ($M_R$, $X_R$, $A_R$) and their corresponding vectors of strategy variables ($\sigma_{MR}$, $\sigma_{XR}$, $\sigma_{AR}$). An independent mutation step is allotted for each design variable. The values of the strategy variables are set to a fraction of the corresponding search range: 1/2, 1/3, and 1/4 for Boolean, discrete, and continuous variables, respectively.

3.3 Mutating topology variables

First, the strategy parameters of topology variables are mutated:

$$\sigma_{Mj} = \text{exp}(\tau_0 N_j)(\sigma_{MR} \otimes [\text{exp}(\tau N(0,1)) \ldots \text{exp}(\tau N(0,1))])$$

where the sign $\otimes$ refers to element-wise multiplication, the index $j$ refers to the $j$-th individual, $\sigma_{Mj}$ is the vector of step sizes for topology variables of this individual, $N_j$ is a random number sampled from the standard normal distribution, $\tau_0$ and $\tau$ are learning rates which are set to...
0.5(N_{VAR})^{0.5} and 0.5(N_{VAR})^{0.25} respectively. \( N_{VAR} \) is the total number of (independent) design variables, equal to the sum of the number of topology (\( N_{\text{top}} \)), shape (\( N_{\text{shape}} \)) and size (\( N_{\text{size}} \)) variables. \( N_{\text{shape}} \) and \( N_{\text{size}} \) are set to the number of independent coordinates and member section variables respectively. The value of \( N_{\text{top}} \), however, is the binary logarithm of the number of topologically distinct acceptable designs.\(^1\)

Once the value of \( \sigma_{Mj} \) is determined, it is used to mutate topology variables. The truncated normal distributions are used to sample \( M_j \). The centre of mutation is \( M_R \), the standard deviation is \( \sigma_{Mj} \), and [0, 1] is the truncated range. Since \( M_j \) consists of Boolean variables, a rounding strategy is subsequently performed. Unlike conventional rounding strategies which replace the continuous variable by the closest (scaled) discrete value (Chen and Chen 2006), a stochastic rounding technique is used in which each component of \( M_j \) becomes 1 with the probability of its value and 0 otherwise. This kind of rounding does not change the expected value of that component.

After rounding, the generated topology, \( M_j \), is accepted if it is stable and includes basic nodes of the structure, otherwise discarded and this step starts again from the beginning. To check stability, the condition number of the stiffness matrix of the generated topology is calculated while the shape and size of the design are equal to their corresponding values in the recombinant design. If the condition number is larger than a predefined value, e.g. \( 10^{10} \), the design is considered unstable.

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\(^1\) This can be easily estimated by uniform sampling of a reasonable number of topologies (for example 1,000) subset of the ground structure and calculating the ratio of the acceptable topologies to all sampled topologies. As the overall number of topologies is known, the number of acceptable topologies can easily be estimated.
3.4 Mutating Shape Variables

Having determined the topology, shape of the design is determined in this step. Step sizes are mutated first:

$$\sigma_{Xj} = \exp(\tau_0 N_j)(\sigma_{XR} \otimes [\exp(c_1 \times \tau N(0,1)), \ldots, \exp(c_{3n} \times \tau N(0,1))])$$  (2)

where $$\sigma_{Xj}$$ is the vector of step sizes for shape variables of the $$j$$-th individual, $$N_j$$ is identical to its value in section 3.3, $$c_i=1$$ if the corresponding node is active and $$c_i=0$$ otherwise. This denotes that step sizes of coordinates of passive nodes are not mutated. Similarly, coordinates of passive nodes remain equal to the corresponding values of $$X_R$$, since their variation have no effect on fitness and just perturbs current values which are retained from previous iterations. Coordinates of active nodes are modified by mutation of the corresponding variables of $$X_R$$. Similar to the previous step, $$X_j$$, the vector of nodal coordinates, is sampled from the truncated normal distribution. The centre of mutation is $$X_R$$, the standard deviation is $$\sigma_{Xj}$$ and the search range of shape variables is the truncated range.

3.5 Mutating Size Variables

Having determined the topology and shape, the size of the structure is determined in this step. Strategy parameters are mutated first:

$$\sigma_{Aj} = \exp(\tau_0 N_j)(\sigma_{AR} \otimes [\exp(\tau \times m_1 \times N(0,1)) \ldots \exp(\tau \times m_n \times N(0,1))])$$  (3)

where $$\sigma_{Aj}$$ is the vector of step sizes for size variables of $$j$$-th individual, and $$N_j$$ is identical to its value in Sections 3.3 and 3.4. Step sizes and areas of passive members remain equal to their values in the recombinant design. Similar to the previous steps, $$A_j$$, the vector of cross sections of the candidate design, is sampled from the truncated normal distribution. The centre of mutation is $$A_R$$, $$\sigma_{Aj}$$ is the standard deviation and the search range of size variables is the truncated range. Now the rounding strategy used in Section 3.3 is employed to round the element values of $$A_j$$. For example if $$a_{ij}=a_0$$, and the nearest lower and upper values in the given discrete set are $$l_i$$ and $$u_i (l_i \leq a_0 \leq u_i)$$, then $$a_{ij}$$ is rounded to the upper value ($$u_i$$) with a probability
of \((a_{ji}-l)/(u_{ji}-l))\) and to the lower value \((l_i)\) otherwise.

3.6 Evaluation

In this step, the generated design is evaluated and member stresses and nodal deflections are computed:

\[
G_j = [g_{ji}], g_{ji} = \begin{cases} 
\left| \frac{\sigma_{ji}}{\sigma_{ji}^a} \right| & \text{if } m_{ji} = 1 \\
0 & \text{if } m_{ji} = 0
\end{cases}, \quad i = 1, 2, \ldots, e
\]

\[
H_j = [h_{ji}], h_{ji} = \begin{cases} 
\left| \frac{f_{ji}}{f_{ji}^a} \right| & \text{if } m_{ji} = 1 \text{ and } \sigma_{ji} < 0 \\
0 & \text{otherwise}
\end{cases}, \quad i = 1, 2, \ldots, e
\]  

(4)

\[
U_j = [u_{jk}], u_{jk} = \begin{cases} 
\left| \frac{\delta_{jk}}{\delta_{jk}^a} \right| & \text{if coordinate } k \text{ of design } j \text{ is active} \\
0 & \text{otherwise}
\end{cases}, \quad k = 1, 2, \ldots, 3n
\]

Vectors \(G_j, H_j\) and \(U_j\) store the ratios of calculated stress \((\sigma_{ij})\), square root of the buckling load \((f_{ij})\), and nodal deflection \((\delta_{ij})\) to their allowable limits respectively. For a feasible design, all elements are equal to or less than one, otherwise some nodes or members have violated some constraints. For constraint violations, a penalty term is used knowing that the optimal design falls on or very close to the boundary of the feasible region. In comparison with death penalty that has been used in some previous studies (Deb and Gulati 2001, Luh and Lin 2008; 2011) an adaptive penalty term enables the search process to approximate this area from both sides (Ebenau \textit{et al.} 2005). Following this goal and utilization of engineering knowledge on truss analysis, a penalty term that is tailored for truss optimization is defined. For any arbitrary positive number \((\alpha > 0)\), the following assumptions are utilized:

- Nodal deflections are divided by \(\alpha\) if all areas are multiplied by \(\alpha\).
- Member stress is divided by \(\alpha\) if the area of that element is multiplied by \(\alpha\)
- Member critical buckling load is divided by \(\alpha^2\), if the area of that element is multiplied by \(\alpha\).

The first assumption is valid for both determinate and indeterminate trusses while the second
and third assumptions are valid only for determinate trusses. Nevertheless, they can still be partially utilized knowing that the optimal topology would have a low degree of indeterminacy. Using these assumptions, the required amount of increase in member area such that all constraints are satisfied can be computed. For example, if $g_{j2}=1.4$, $a_{j2}$ should be multiplied by 1.4 so that the 2nd member of the $j$-th design satisfies the stress constraint. If so, the overall volume of the structure is increased by $0.4a_{j2}l_{j2}$, where $l_{j2}$ is the length of the $j$-th member. The penalized objective function is defined as follows:

$$f_j = W_j + \rho \sum_{i=1}^{e} c_{Pi} a_{ji} l_{ji} (q_{ji}^2 - 1), \quad q_{ji} = \max \left\{ 1, g_{ji}, h_{ji}, \max\{U_j\} \right\}$$  \hspace{1cm} (5)

This equation implies that if areas of all members are multiplied by the corresponding $q_{ji}$, the resultant truss supposedly satisfies all constraints. This leads to an increase of $\sum_{i=1}^{e} l_{ji} a_{ji} (q_{ji} - 1)$ in the volume of design $j$, which depends not only on the constraint violation amount, but also on the current length and cross section of that member. This means similar constraint violation for a larger member results in a larger penalty term. Parameter $c_{Pi} \geq 0.5$ intensifies the penalty and controls the resizing rate, which is explained in Step 3.7.

### 3.7 Resizing

Following the deterministic method of Fully Stressed Design, a resizing step is performed to generate new individuals. In this step, vectors $G_j$, $H_j$, and $U_j$ calculated in the previous step are utilized to generate a supposedly near-boundary design from the $j$-th design by changing only the size variables. Member forces are assumed to be constant, and each member section is resized such that the stress or buckling constraints become activated, which means the member should be loaded up to its maximum capacity. However, reduction of member areas takes place more conservatively since, as discussed earlier, not all members may reach their stress limit in the optimum design. The shrinking rate of the member area is controlled by $c_{Pi}$. For $c_{Pi}=0.5$ (minimum value), each member area is shrunk so that stress or buckling constraints activate.
For $c_{P_i} > 0.5$, the shrinking rate lowers:

$$a_{j+\lambda,i} = \begin{cases} a_{ji} \times \max\{g_{ji}, h_{ji}\} & \text{if } \max\{g_{ji}, h_{ji}\} \geq 1 \\ a_{ji} \times \left(1 + (\max\{g_{ji}, h_{ji}\} - 1) \times \exp(1 - 2c_{P_i})\right) & \text{if } \max\{g_{ji}, h_{ji}\} < 1 \end{cases}$$

(6)

If the given set of areas is discrete, each member area is rounded to its upper value.

Since vector $A_{j+\lambda} - A_R$ is now different from $A_j - A_R$, it is logical to adapt the corresponding strategy parameter to increase the chance of sampling around $A_{j+1}$ in the subsequent iterations:

$$\sigma_{a_{j+\lambda,i}} = \begin{cases} (1.9\sigma_{a_{ji}}|A_{R\ell} - A_{j+\lambda,i}|)^{0.5} & \text{if } m_{ji} = 1 \\ \sigma_{a_{ji}} & \text{if } m_{ji} = 0 \end{cases}$$

(7)

The fitness of the recently generated individuals is calculated according to equation (5). This process is repeated until $(\lambda+\lambda)$ candidate solutions are generated.

3.8 Recombination

Global weighted recombination is performed to update the recombinant point. Having sorted the offspring, the $\mu$-best individuals ($\mu = \lambda/2$) are selected to form the recombinant point. The new values of $X_R, A_R, \sigma_{MR}, \sigma_{XR}$ and $\sigma_{AR}$ are specified by weighted average (arithmetic mean for design variables and geometric mean for strategy variables) of the selected individuals. The weights of each individual decrease logarithmically with their ranking.

Updating $M_R$ is more complicated. Increasing or reducing $i$-th component of $M_R$ depends on the fraction of individuals in the whole population and in the $\mu$-best individuals in which the $i$-th member is active. If the latter value is greater, presence of the $i$-th member is considered advantageous, and hence the $i$-th component of $M_R$ should increase to make presence of this member more likely in the next iteration. Accordingly, the following relation for updating the topology vector of the recombinant point is suggested:
\[ \mathbf{M}_R \leftarrow \mathbf{M}_R + c_M \left( \sum_{j=1}^{\mu} w_j \mathbf{M}_j - \frac{1}{2\lambda} \sum_{j=1}^{2\lambda} \mathbf{M}_j \right) \]  

(8)

where \( \mathbf{M}_j \) denotes the vector of topology variables of the \( j \)-th individual. Parameter \( c_M \) specifies the learning rate for topology variables, which depends on the population size, \( N_{VAR} \) and \( N_{top} \):

\[
c_M = \frac{\mu_{eff} + 2}{\mu_{eff} + 5 + \sqrt{N_{VAR} \times N_{top}}} \quad \mu_{eff} = \left( \frac{\sum_{i=1}^{\mu} w_j}{\left( \sum_{i=1}^{\mu} w_j^2 \right)^{0.5}} \right)
\]

(9)

where \( w_j \) is the weight of the \( j \)-th parent.

### 3.9 Update of parameters

The penalty coefficients are updated in this section. If less than 50\% of parents or offspring are feasible, the penalty coefficient, \( c_{Pi} \), is increased, otherwise decreased:

\[
\psi_i^{(\mu)} \leftarrow (1 - \tau)\psi_i^{(\mu)} + \tau \sum_{j=1}^{\mu} \text{sgn}(q_{ji} - 1)w_j, \quad \psi_i^{(\lambda)} \leftarrow (1 - \tau)\psi_i^{(\lambda)} + \tau \sum_{j=1}^{\lambda} \text{sgn}(q_{ji} - 1)
\]

\[
c_{Pi} \leftarrow \max \left\{ 0.5, c_{Pi} \sqrt{0.5 + \psi_i^{(\mu)}}, c_{Pi} \sqrt{0.5 + \psi_i^{(\lambda)}} \right\}
\]

(10)

where \( \psi_i^{(\mu)} \) and \( \psi_i^{(\lambda)} \) represent the fraction of parents and offspring in which the \( i \)-th member has violated a constraint. Cumulative information with exponential decay of the past information is utilized to compute the ratio of infeasible parents or offspring.

As the optimization process converges a considerable proportion of acceptable topologies will no longer be produced. Accordingly, some nodes or members may exist or vanish in almost all generated topologies in the next iteration. Accordingly, the effective number of topology variables \( (\bar{N}_{top}) \) is updated iteratively:

\[
\bar{N}_{top} = \left( 1 - 4 \sum_{i=1}^{e} \left( \left( \frac{1}{2\lambda} \sum_{j=1}^{2\lambda} m_{ji} \right) - 0.5 \right) \right) N_{top}
\]

(11)
Similarly, coordinates of passive nodes and the cross sectional area of passive members are not modified, which reduces the effective number of design variables. The effective number of shape variables ($\tilde{N}_{\text{shape}}$) and the effective number of size variables ($\tilde{N}_{\text{size}}$) are set to the average number of independent active coordinates and size variables respectively. Since the effective number of design variables has changed, the learning rates for the next iteration are updated:

$$
\tau_0 \leftarrow \frac{1}{2\sqrt{\tilde{N}_{\text{VAR}}}}, \tau \leftarrow \frac{1}{2\sqrt{\tilde{N}_{\text{VAR}}}}, \quad c_M \leftarrow \frac{\mu_{\text{eff}} + 2}{\mu_{\text{eff}} + 5 + \sqrt{\tilde{N}_{\text{VAR}} \times \tilde{N}_{\text{top}}}}
$$

where $\tilde{N}_{\text{VAR}} = \tilde{N}_{\text{top}} + \tilde{N}_{\text{shape}} + \tilde{N}_{\text{size}}$ is the updated value for the effective number of design variables.

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**Figure 1. Flowchart for FSD-ES.**

- **Start**
  - Initialize the recombinant design and the corresponding strategy parameters.
  - Generate a candidate topology by mutation of the recombinant topology.
  - Is the topology acceptable?
    - No
      - Determine the candidate shape by mutation.
    - Yes
      - Determine the candidate size by mutation.
      - Calculate the objective function and constraints.
      - Is the topology acceptable?
        - No
          - Have 2$\lambda$ designs?
            - Yes
              - Select the best $\mu$ individuals and update design and strategy parameters.
            - No
              - Form a new individual by resizing the recently generated candidate design.
        - Yes
          - Form a new individual by resizing the recently generated candidate design.
  - Has the algorithm converged?
    - Yes
      - End
    - No
      - Select the best $\mu$ individuals and update design and strategy parameters.

---

17
3.10 Convergence Check

Several effective convergence checks based on statistical criteria has been proposed (Hansen 2009), yet according to performance measures employed in this study, the algorithm terminates when it reached the maximum number of iterations, otherwise it is resumed from Section 3.3. FSD-ES does not use any information on the iteration number or maximum number of iterations to adjust strategy parameters. The flowchart for FSD-ES is provided in Figure 1.

4 Empirical evaluation

In this section, a number of qualitatively distinct problems are selected, on which FSD-ES is evaluated and compared with the most competent methods in the literature.

4.1 Test problems

Test problems considering only one aspect of truss optimization are excluded as the concentration of this article is on TSS optimization problems. Some prevalent, albeit too simple, test problems are excluded and a brief explanation that highlights distinctive features of each selected test problem is presented.

The first problem is a 45-bar 2D truss employed by Deb and Gulati (2001). The structure weight is minimized while symmetry is ignored. The ground structure, depicted in Figure 2a, has an all pair-wise interconnection (all-to-all scheme). A vertical load of 10 kips is exerted on nodes 7, 8 and 9. Member areas may take any values within [0.09,1] in\(^2\). The maximum deflection and axial stress are limited to 2 in. and 25 ksi. The density and modulus of elasticity are 0.1 lb/in\(^3\) and 10000 ksi respectively.

The second test is a TSS optimization problem adapted from (Rahami et al. 2008). The 15-bar ground structure is depicted in Figure 2b. This is quite a simple TSS problem since node 4 is intuitively redundant and other nodes may not be eliminated. Data required for simulation of this problem is presented in Table 1.
Table 1. Simulation data for the 15-bar truss problem.

<table>
<thead>
<tr>
<th>Design Variables</th>
<th>Shape (8)</th>
<th>Size (15)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X[2,3,4,...,12]</td>
<td>Ai</td>
<td>i=1,2,....,15</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Constraints</th>
<th>Stress</th>
<th>Displacement</th>
<th>Buckling</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(σc)i ≤ 172.4 MPa (25 ksi); (σt)i ≤ 172.4 MPa (25 ksi), i=1,2,...,15</td>
<td>None</td>
<td>None</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Shape Variables</th>
<th>Size Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shape Variables</td>
<td>Size Variables</td>
</tr>
<tr>
<td>254 cm (100 in) ≤ x0 ≤ 355.6 cm (140 in); 558.8 cm (220 in) ≤ x1 ≤ 660.4 cm (260 in); 254 cm (100 in) ≤ y0 ≤ 355.6 cm (140 in); 254 cm (100 in) ≤ y1 ≤ 355.6 cm (140 in); 127 cm (50 in) ≤ y2 ≤ 228.6 cm (90 in); −50.8 cm (−20 in) ≤ y3 ≤ 50.8 cm (20 in); −50.8 cm (−20 in) ≤ y4 ≤ 50.8 cm (20 in); 127 cm (50 in) ≤ y5 ≤ 228.6 cm (90 in); −50.8 cm (−20 in) ≤ y6 ≤ 50.8 cm (20 in); 127 cm (50 in) ≤ y7 ≤ 228.6 cm (90 in); −50.8 cm (−20 in) ≤ y8 ≤ 50.8 cm (20 in); 254 cm (100 in) ≤ x2 ≤ 355.6 cm (140 in); 558.8 cm (220 in) ≤ x3 ≤ 660.4 cm (260 in); 254 cm (100 in) ≤ y2 ≤ 355.6 cm (140 in); 254 cm (100 in) ≤ y3 ≤ 355.6 cm (140 in); 127 cm (50 in) ≤ y4 ≤ 228.6 cm (90 in); −50.8 cm (−20 in) ≤ y5 ≤ 50.8 cm (20 in); −50.8 cm (−20 in) ≤ y6 ≤ 50.8 cm (20 in); 127 cm (50 in) ≤ y7 ≤ 228.6 cm (90 in); −50.8 cm (−20 in) ≤ y8 ≤ 50.8 cm (20 in);</td>
<td>Ai ∈ S, i = 1, . . . , 15</td>
</tr>
<tr>
<td>Loading</td>
<td>Nodes</td>
</tr>
<tr>
<td>----------</td>
<td>-------</td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

Mechanical Properties

- Modulus of elasticity: E=68.95 GPa (1.0×10^4 ksi)
- Density of the material: ρ=0.0272 N/cm^3 (0.1 lb/in.^3)

The third problem is a simultaneous TSS optimization of a 25-bar spatial truss adapted from (Rahami et al. 2008). Front and left views of the ground structure are depicted in Figure 2c. The structure, but not the loading, is symmetric with respect to the xz and yz planes, which reduces the number of independent sections and coordinates to 8 and 5 respectively. Table 2 presents data required for simulation of this problem.

The fourth test is a two-tier 39-bar truss proposed by Deb and Gulati (2001) and subsequently used in many recent studies (Luh and Lin 2008, Wu and Tseng 2010, Luh and Lin 2011) as a more comprehensive TSS optimization problem. The symmetric ground structure is depicted in Figure 2d in which overlapping members are illustrated with curved line segments. The coordinates of nodes 1, 2, 3, 4 and 5 are fixed and node 11 is allowed to move vertically. The coordinate variables may vary up to ±120 inch with respect to their value in the ground structure. Structural symmetry is exploited to shrink the number of design variables. The range of available section areas is assumed to be continuous, where 0.05 and 2.25 in^2 are the lower and upper limits of this range. A vertical load of P=20 kips is exerted on nodes 2, 3 and 4. In comparison with the previous test, the number of design variables has increased. The problem has a low-fitness local optimum (W~214 lb) with minimal number of active nodes (7), which can trap algorithms that inherently favour simpler topologies.
Figure 2. Ground structure for test problems 1 to 4: a) 45-bar truss, b) 15-bar truss, c) 25-bar spatial truss (front and left view), d) 39-bar truss, e) 47-bar truss and f) 68-bar truss test problems.
Table 2. Simulation data for the 25-bar spatial truss problem.

<table>
<thead>
<tr>
<th>Design Variables</th>
<th>Shape (5)</th>
<th>Size (8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constraints</td>
<td>Stress</td>
<td>Displacement</td>
</tr>
<tr>
<td></td>
<td>(R) ≤ 275.8 MPa (40 ksi); (R) ≤ 275.8 MPa (40 ksi); R=1,2,…,25</td>
<td>u;i=0.089 cm (0.35 in)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Search Range</th>
<th>Shape Variables</th>
<th>Size Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>50.8 cm (20 in) ≤ x,y ≤ 152.4 cm (60 in); 101.6 cm (40 in) ≤ x,y ≤ 203.2 cm (80 in); 101.6 cm (40 in) ≤ y ≤ 203.2 cm (80 in); 254 cm (100 in) ≤ y ≤ 355.6 cm (140 in); 228.6 cm (90 in) ≤ z ≤ 330.2 cm (130 in)</td>
<td>A;i=1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Loading</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1: 4.541 (1.0) -44.537 (-10.0) -44.537 (-10.0)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2: 0.0 -44.537 (-10.0) -44.537 (-10.0)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3: 2.227 (0.5) 0.0 0.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6: 2.672 (0.6) 0.0 0.0</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mechanical Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of elasticity: E=60.95 GPa (1.0×10^6 ksi)</td>
</tr>
<tr>
<td>Density of the material: ρ=0.0272 N/cm^2 (0.1 lb/in.^3)</td>
</tr>
</tbody>
</table>

The fifth problem is size and shape optimization of a 47-bar power line truss adapted from the work of Hasançebi and Erbatur (2002b). Problem specifications are presented in Table 3 and the ground structure is depicted in Figure 2e. The structure is supposed to be able to carry three load cases. External load is not symmetric, but the structure design including nodal coordinates and member cross sections are symmetric. Coordinates of nodes 15, 16, 17 and 18 are fixed. The ranges of coordinate variables were not quoted in the referenced paper and hence, a logical range was selected for this study.

Table 3. Data for simulation of the 47-bar truss problem.

<table>
<thead>
<tr>
<th>Design Variables</th>
<th>Shape (17)</th>
<th>Size (27)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constraints</td>
<td>Stress</td>
<td>Displacement</td>
</tr>
<tr>
<td></td>
<td>(R) ≤ 103.4 MPa (15 ksi); (R) ≤ 137.9 MPa (20 ksi); r=1,2,…,47</td>
<td>u;i=0.089 cm (0.35 in)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Search Range</th>
<th>Shape Variables</th>
<th>Size Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 ≤ x,y ≤ 381.0 cm (150 in); 0 ≤ x,y ≤ 190.5 cm (75 in); 0 ≤ x,y ≤ 381.0 cm (150 in); 0 ≤ y ≤ 609.6 cm (240 in); 304.8 cm (120 in) ≤ y ≤ 914.4 cm (360 in); 1066.8 cm (420 in) ≤ y ≤ 1371.6 cm (540 in) ≤ y ≤ 1219.2 cm (480 in); 1371.6 cm (540 in) ≤ y ≤ 1524.0 cm (600 in)</td>
<td>S;i=1,2,…,47</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Loading</th>
<th>Case I</th>
<th>Case II</th>
<th>Case III</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>17.18 26.689 KN (6 kips)</td>
<td>17 26.689 KN (6 kips)</td>
<td>18 26.689 KN (6 kips)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mechanical Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of elasticity: E=206.84 GPa (3.0×10^6 ksi)</td>
</tr>
<tr>
<td>Density of the material: ρ=0.081434 N/cm^3 (0.3 lb/in.^3)</td>
</tr>
</tbody>
</table>

The sixth test is TSS optimization of a 68-bar truss subjected to two load cases at the end.

Data required for simulation of this problem is presented in Table 4. Horizontal and vertical
coordinates of the nodes may vary within 120 and 40 in. of the initial configuration respectively (Figure 2f). The ground structure allows elimination of most members and nodes except nodes 1, 3, and 17. This test problem is introduced in this study and has a significantly larger number of design variables compared to the previous test problems.

Table 4. Data for simulation of the 68-bar truss problem.

<table>
<thead>
<tr>
<th>Design Variables</th>
<th>Shape (31)</th>
<th>Size (68)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$y_{10}$, $x_{p}$, $y_{p}=2, 4, 5, 6, ..., 14, 15, 16, 18$</td>
<td>$a_{r}$, $r=1, 2, ..., 68$</td>
</tr>
<tr>
<td>Constraints</td>
<td>Stress Displacement Buckling</td>
<td></td>
</tr>
<tr>
<td>$(e_{i})$ ≤137.9 MPa (20 ksi); $(e_{i})$ ≤137.9 MPa (20 ksi), $i=1, 2, ..., 68$</td>
<td>$u_{j}$$\leq$2.5 in., $j=1, 2, 3, ..., 68$</td>
<td>$(e_{i})$</td>
</tr>
<tr>
<td></td>
<td>Search Range</td>
<td>Shape Variables</td>
</tr>
<tr>
<td></td>
<td>Size Variables</td>
<td>$A_{i}$$\in$S, $i=1, ..., 68$</td>
</tr>
<tr>
<td></td>
<td>Loading</td>
<td>Case I</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Case II</td>
</tr>
<tr>
<td>Mechanical Properties</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Density of the material: $\rho=0.081434$ N/cm$^3$ (0.3 lb/in.$^3$);</td>
<td></td>
</tr>
</tbody>
</table>

4.2 Evaluation method

The performance measure employed in this article is based on the Expected Running Time (ERT) to reach a predefined tolerance of the global minimum, as discussed and advocated by Hansen et al. (2010):

$$ERT(f_{\text{target}})=((SR)(FE_{S})+(1-SR)(FE_{US}))/SR$$

(13)

where SR is the success rate, which is the fraction of independent runs in which the algorithm could reach $f_{\text{target}}$, $FE_{S}$ is the average number of function evaluations of successful runs to reach $f_{\text{target}}$, and $FE_{US}$ represents the average number of function evaluations of unsuccessful runs. ERT facilitates comparison of optimization methods since it considers the overall effects of reliability and convergence speed. To utilize equation (13), the algorithm should stop when it reaches a certain tolerance of the objective function or when the convergence criterion is met. However, in truss optimization, as with most practical problems, the global optimum is not
known and besides, the performance measure should discriminate between converging to a high/low fitness minimum. Accordingly, the optimization algorithm is run for a long time and the required number of evaluations to reach different structural weights is recorded. $FE_{US} \sim FE_S$ is a logical assumption (Auger and Hansen 2005), which simplifies equation (13) to the following:

$$ERT(f_{target}) = \frac{FE_S}{SR}.$$  \hfill (14)

In this study, the above equation is used to calculate ERT from $FE_S$ and SR at any arbitrary target function value. The ERT plots in this case can discriminate and compare both the short and long term performance of algorithms. Such discrimination is of great practical interest as the budget of function evaluations determines whether short or long-term success is desired.

**4.3 Parameter tuning**

All control parameters of FSD-ES are set to their recommended values as described in section 3 except the population size, which should ideally be proportional to the problem complexity. The population size ($\lambda$) is set using the following relation, which considers the number of design variables and constraints:

$$K_1 = N_{top} + N_{shape} + N_{size} + (N_{top} \times N_{shape})^{0.5} + (N_{shape} \times N_{size})^{0.5} + (N_{size} \times N_{top})^{0.5} + (N_{top} \times N_{shape} \times N_{size})^{0.333}$$

$$K_2 = (z_{def} \times e \times N_{size} + z_{stress} \times n \times d) \times N_{load} \times d$$

$$\lambda = [(0.2 \times K_1 \times K_2)^{0.5} + 5.5]$$  \hfill (15)

where $N_{size} = N_{size} - 0.5 \times N_{top}$ is the adjusted number of size variables. $K_1$ and $K_2$ are based on the complexity caused by the number of design variables and constraints respectively. $d$ is the space dimension, which is 2 for planar and 3 for spatial structures, and $N_{load}$ represents the number of load cases. $e$ and $n$ are the number of members and nodes in the ground structure and $z_{def}$ is 1 if nodal deflections are constrained and 0 otherwise. Similarly, $z_{stress}$ is 1 if members are subject to stress or buckling constraints and 0 otherwise. Following the procedure
explained in section 3.3, the estimated value for $N_{\text{top}}$ is 43.4, 8.25, 3.55, 18.94, 0, and 65.5, which leads to the population size of 35, 22, 39, 44, 66, and 136 for problems 1 to 6 respectively. The optimization process is terminated after $20\lambda+100$ iterations; however, as emphasized earlier, ERT, SR and FE$S$ for specific target weights are to be compared. Accordingly, FSD-ES requires no ad hoc tuning effort, since all parameters are set based on known features of the problem.

4.4 Results and Discussion

Results from FSD-ES on the selected test suite are presented in this section. Each test is repeated 100 times independently and the calculated ERT accompanied by SR and FE$S$ is plotted versus the target weight. These plots are provided in Figure 3. Only feasible designs that satisfy all constraints are considered in calculating FE$s$ and SR. The best feasible solution found in this study is also reported and compared to those found in previous studies. Based on the results, the following conclusions can be made:

- Reaching lighter structures logically needs more evaluations, however, the ERT grows much faster when lighter structures are desired. This is due to the fact that not all independent runs could reach some desired weights. Consequently, the gap between the FE$s$ and ERT lines increases when the target weight decreases. When only a few independent runs can reach a target weight, the computed value of ERT is prone to unreliability due to the stochastic nature of the runs.

- For the 45-bar truss problem, FSD-ES can reach a structural weight of 44.000 lb after 19,656 function evaluations (FE), provided that the run is successful. 90% of independent runs have converged to this weight and hence there is only a slight difference between the ERT and FE$S$ for this problem. To the authors’ knowledge, the best solution for this problem is 43.99 lb, reported by Wu and Tseng (2010), which was reached after 30,800 evaluations. The best design of FSD-ES is quite similar to theirs,
but the required number of evaluations for the proposed FSD-ES is comparatively smaller.

Figure 3. ERT, SR and FE₃ to reach arbitrary structural weights for the employed test problems: a) 45-bar truss, b) 15-bar truss, c) 25-bar spatial truss, d) 39-bar truss, e) 47-bar truss and f) 68-bar truss.
Figure 4. The best final design for a) 45-bar truss b) 15-bar truss c) 39-bar truss d) 47-bar truss and e) 68-bar truss problem.

- For the 15-bar truss problem, FSD-ES needs 3,859 FE to reach $W=72.50$ lb, however, the ERT is more than five times larger. A rapid deterioration of SR starts at the target weight of 75 lb, resulting in fast growth of the ERT. The best design found in this study weighs 69.585 lb, and 5% of independent runs could reach $W=70$ lb after an average of 8,508 FE. The best solution of SCPSO (Gholaizadeh 2013) weighs $W=72.49$ lb and was reached after 12,500 FE, almost 4.2% heavier than the best solution of the proposed FSD-ES. Moreover, SCPSO requires tuning of several control parameters and selecting a grid. The best solution of FA (Miguel et al. 2013) weighs 74.33 lb, reached after 8,000
FE. The GA proposed in Rahami et al. (2008) reached the best solution of 75.10 lb after 8,000 FE, which is about 7.9% heavier than the best solution found by the proposed FSD-ES. For this problem, FSD-ES surpasses the best existing algorithms both in efficiency and quality of the best final solution.

- For the 25-bar spatial truss, FSD-ES requires on average 8,660 FE to reach W=114.42 lb, however, the corresponding SR is pretty low. To the authors’ knowledge, the best reported solution for this problem is 114.36 lb (Rahami et al. 2008), reached after 10,000 FE. A GA by Tang et al. (2005) and FA by Miguel et al. (2013) could reach W=114.74 lb and W=116.58 lb after 6,000 FE respectively. The best solution of SCPSO (Gholaizadeh 2013) weighs 117.23 lb, reached after 8,000 FE. For this problem, performance of FED-ES is quite similar to the best available results and superior to SCPSO and FA.

- For the 39-bar truss problem, a gradual drop in SR initiates at W=190 lb, caused by the existence of several topologically distinct high fitness designs at this zone. However, 16% of independent runs could still reach W=182 lb, on average after 40,256 FE. For this problem, Luh and Lin (2008) reached W=188.73 lb after 453,600 FE, Luh and Lin (2011) reached W=188.86 lb after 262,500 FE, and Wu and Tseng (2010) could reach W=188.45 lb after 137,200 FE. The FA of Miguel et al. (2013) could reach W=191.30 lb after 50,000 FE. At the target weight of 188.0 lb, the FE\textsubscript{5} and ERT of FSD-ES are 20,686 and 26,185, spectacularly smaller than the number of evaluations of the cited studies. The best feasible solution found in this study weighs 180.98 lb, which is about 3.7% lighter than and topologically different from the best reported solution in the literature.
Table 5. The best solutions found for the test problems. Coordinates and areas are in inch and inch square, respectively. The normalized constraint violation and the structure weight are provided in the four last rows.

<table>
<thead>
<tr>
<th></th>
<th>45-h truss</th>
<th>15-h truss</th>
<th>25-h truss</th>
<th>39-h truss</th>
<th>47-h truss</th>
<th>68-h truss</th>
</tr>
</thead>
<tbody>
<tr>
<td>x1</td>
<td>0.95659</td>
<td>0.95659</td>
<td>0.95659</td>
<td>0.95659</td>
<td>0.95659</td>
<td>0.95659</td>
</tr>
<tr>
<td>x2</td>
<td>72.5982</td>
<td>72.5982</td>
<td>72.5982</td>
<td>72.5982</td>
<td>72.5982</td>
<td>72.5982</td>
</tr>
<tr>
<td>A1</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>A2</td>
<td>0.539</td>
<td>0.539</td>
<td>0.539</td>
<td>0.539</td>
<td>0.539</td>
<td>0.539</td>
</tr>
<tr>
<td>A3</td>
<td>0.954</td>
<td>0.954</td>
<td>0.954</td>
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</tr>
<tr>
<td>A4</td>
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<td>0.021479</td>
<td>0.021479</td>
<td>0.021479</td>
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</tbody>
</table>

<table>
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<tr>
<th></th>
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<th>1.0000</th>
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</tr>
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<tbody>
<tr>
<td></td>
<td>Buckling</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td></td>
<td>Def.</td>
<td>0.6250</td>
<td>1.0000</td>
<td>0.7708</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>Weight</td>
<td>44.000 lb</td>
<td>69.585 lb</td>
<td>114.417 lb</td>
<td>180.983 lb</td>
<td>1846.52 lb</td>
<td>1203.51 lb</td>
</tr>
</tbody>
</table>

- There are some notable points regarding this problem. First, several topologically distinct high-fitness designs exist for this problem. In 100 independent runs, FSD-ES converged to many distinct topologies. A number of these distinct final designs that weigh less than 188 lb are illustrated in Figure 5. To the authors’ knowledge, only topology #18 is reported in literature, meaning the previous algorithms couldn’t detect topologies that potentially lead to lighter structures. Second, the best previously reported designs have 13–15 active members, however, topologies that could reach structures lighter than 187.5 lb generally have more members. This can pose a challenge for truss optimizers that are biased toward simpler structures. Finally, topologies of the best designs can hardly be guessed by engineering intuition, which highlights benefits of using a reliable truss optimizer. Consequently, the proposed FSD-ES significantly
outperforms available optimizers for this problem, both in efficiency and quality of the final designs.

- For the 47-bar truss problem, the best design, to the authors’ knowledge, weighs 1,861.1 lb (Gholai zadeh 2013), reached after 49,000 evaluations. However, the SR was not reported and hence the ERT cannot be calculated. SA of Hasancibi and Erbatur (2002b) could reach $W=1,871.7$ lb, but the FE was not reported. The best solution of the proposed FSD-ES weighs 1,846.5 lb, relatively lighter than the best available results. 86% of FSD-ES runs reached the target weight of 1861 lb, on average after 55,802 FE. The convergence speed to reach this weight is quite similar to that of SCPSO (Gholai zadeh 2013), but FSD-ES could produce a relatively lighter structure.

![Figure 5. Some selected final designs for the 39-bar truss problem which have distinct topology: Topology #2: $W=181.02$ lb, Topology #3: $W=181.38$ lb, Topology #4: $W=181.60$ lb, Topology #7: $W=182.37$ lb, Topology #8: $W=183.34$ lb, Topology #10: $W=183.89$ lb, Topology #13: $W=186.91$ lb, Topology #15: $W=186.96$ lb, Topology #18: $W=187.30$ lb.](image-url)
- For the 68-bar truss problem, independent runs of FSD-ES converged to a variety of different designs ranging from 1,203.5 lb up to 1,555 lb. This is probably due to the complexity of the ground structure and various reasonable topologies which may challenge truss optimizers. The best final solution (Figure 4e) has a reasonable topology where all three types of constraints were activated; however, no comparison with other specialized algorithms can be made.

5 Summary and conclusions

Optimization of truss structures is a demanding task where problem complexity, prompted by existence of various types of variables and constraints, require specialized variants of optimization algorithms. In this article, an optimization algorithm based on evolution strategies, called Fully Stressed Design based on Evolution Strategy (FSD-ES), was extended to handle simultaneous topology, shape, and size truss optimization problems.

Based on the results of previous research on truss structure optimization, an illuminative, discriminative and practically reasonable performance measure has been employed to analyse, assess, and compare the obtained results. The calculation of the Expected Running Time (ERT) has enabled us to not only compare efficiency of different algorithms quantitatively, but also assess the short and long-term success of each algorithm. Such statistical evaluation and presentation of results facilitate the comparison of optimization algorithms, especially when the actual global minimum of the evaluated problem is not known.

FSD-ES exhibited several advantages. First, unlike most available stochastic truss optimizers that demand ad hoc tuning of several strategy parameters, FSD-ES is a quasi-parameter-free method. In our experiments, all strategy parameters were set based on the a priori known features of the problem. This makes the method a robust and practically useful optimizer. Second, the proposed FSD-ES can be used for simultaneous optimization of topology, shape, and size of truss structures. Third, in three test problems (15-bar, 39-bar, and
47-bar problems), the proposed FSD-ES method outperformed the best available solutions in the literature. This is particularly true for the 39-bar truss, where it could detect several topologically distinct designs that were lighter than the best available results in the literature. For the 45-bar truss problem, the final solution of the existing best algorithms and the proposed FSD-ES are quite similar, but FSD-ES converged relatively faster. For the 25-bar spatial truss, the performance of FSD-ES is quite similar to the best available optimizers when the final solutions and the required number of evaluations are compared. However, the success rate when target weighs less than 117 lb are demanded is rather small, probably due to the fact that the resizing step of FSD-ES focuses on the stress and buckling constraints while only the deflection constraints were activated in this problem. The last problem was introduced in this study as a significantly more complicated benchmark problem, especially when considering that all three types of constraints were activated in the best final design. Assessing and comparing prospective truss optimizers on this test problem provides differentiating results in more complicated structures.

Based on the extensive results reported in this paper, the proposed methodology seems to be an extremely competitive optimization algorithm. It is now ready to be applied to more complex truss structure optimization problems involving topology, shape, and size variables.

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References


