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Proton polarizability effect in the Lamb shift of the hydrogen atom

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The proton polarizability correction to the Lamb shift of electronic and muonic hydrogen is calculated on the basis of isobar model and experimental data on the structure functions of deep inelastic lepton-nucleon scattering. The contributions of the Born terms, vector-meson exchanges and nucleon resonances are taken into account in the construction of the photoabsorption cross sections for transversely and longitudinally polarized virtual photons $\sigma_{T,L}$.

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I. INTRODUCTION

The precise investigation of the energy levels of hydrogenic atoms (muonium, positronium, hydrogen atom, deuterium, helium ions et al.) allows to obtain more exact values for many fundamental physical constants such as the lepton masses, the ratio of lepton and proton masses, the fine structure constant, the Rydberg constant which are used for creating standards of units [1, 2]. The insertion of new simple atomic systems in the range of experimental investigation can lead to significant progress in solving of these problems. Muonic hydrogen (μp) is just one of a number of simple atoms which has attracted considerable attention in the last years. Since the muon is heavier than the electron by a factor of 206, the energy structure of (μp) is extremely sensitive to the effect of the electron vacuum polarization, recoil effect, the proton structure and polarizability corrections [3, 4, 5, 6]. In the Lamb shift ($2P-2S$) of (μp) the value of the proton structure correction of order $(Z\alpha)^4$ increases essentially as compared with electronic hydrogen. Considering that this contribution is determined also by the proton charge radius r_p, the experimental investigation of the energy spectrum of muonic hydrogen can play the key role in a more precise study of the proton structure along with the experimental data on the electron-nucleon scattering [2, 7, 8]. At present time the Lamb shift ($2P-2S$) experiment in muonic hydrogen at PSI (Paul Sherrer Institute) with a precision of 30 ppm entered the closing stage [9, 10]. The suggested aim is to determine the root-mean-square (rms) charge radius of the proton to the accuracy 10^{-3}, about a factor 20 times better than presently known from electron scattering experiments. It opens the possibility to check bound state QED predictions toward a level

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of 10^{-7} precision. Because the Lamb shift experiment in (μp) measure the energy difference between the $2^3S_{1/2}$ and $2^5P_{3/2}$ atomic levels, it is important to know the hyperfine structure of $2S$ and $2P$ energy levels [5, 11]. In this connection it is important to consider again several theoretical contributions which have essential role in the calculation of the total theoretical value of the $(2P-2S)$ Lamb shift in the atom (μp) with the necessary accuracy. The proton polarizability contribution is among important effects [12, 13, 14, 15, 16, 17, 18], which was calculated earlier in muonic hydrogen on the basis of experimental data on the structure functions of the lepton-nucleon scattering. Slightly different values for this contribution occur to the present [14, 16, 17, 18].

The study of electromagnetic excitations of baryonic resonances which is carried out at CEBAF (Continuous Electron Beam Accelerator Facility), entered currently a new phase of development because new data with unprecedented precision have become available. The aims of the CEBAF investigations are in the measurements of the nucleon transition form factors to nucleon resonances N^* at different photon virtualities Q^2 in the resonance region, in the study of the gluon content of baryonic resonances and the helicity amplitudes $A_{1/2}$, $A_{3/2}$, $S_{1/2}$ for different states N^* [19]. Recently, precise measurements for exclusive electroproduction of π^0, π^+, η mesons on protons in the resonance region were obtained at Jefferson Lab using the CEBAF Large Acceptance Spectrometer (CLAS) [20]. The obtained experimental data include measurements of the cross sections, angular distributions for the $n\pi^+$, $p\eta$ final states. These data allow also to investigate the contributions of resonances to π, η electroproduction in detail. The performed experiments permitted already to refine the values of a number of theoretical parameters determining the production of the nucleon resonances N^* in the reaction $\gamma^*+N\to N^*$. The goal of the present investigation is to perform new calculation of the proton polarizability contribution to the Lamb shift in hydrogen atom based on theoretical model (unitary isobar model), describing the processes of photo- and electroproduction of π,η mesons, nucleon resonances on the nucleon in the resonance region, and on evolution equations for the parton distributions in deep inelastic region.

II. GENERAL FORMALISM

The proton polarizability contribution to the Lamb shift of order $(Z\alpha)^5$ is determined by the amplitude of virtual Compton forward scattering $\gamma^*+p\to\gamma^*+p$, presented on the Feynman diagrams in Fig.1. It's parameterization has the following form [21, 22]:

$$M^{(p)}_{\mu\nu}=\bar v(p_2)\Bigg\{\frac{1}{2}C_1\left(-g_{\mu\nu}+\frac{k_\mu k_\nu}{k^2}\right)+\frac{1}{2m_2^2}C_2\left(p_{2\mu}-\frac{m_2\nu}{k^2}k_\mu\right)\left(p_{2\nu}-\frac{m_2\nu}{k^2}k_\nu\right)+ \tag{1}$$

$$+\frac{1}{2m_2}H_1\left([\gamma_\nu,\hat k]p_{2\mu}-[\gamma_\mu,\hat k]p_{2\nu}+[\gamma_\mu,\gamma_\nu]\right)+\frac{1}{2}H_2\left([\gamma_\nu,\hat k]k_\mu-[\gamma_\mu,\hat k]k_\nu+[\gamma_\mu,\gamma_\nu]k^2\right)\Bigg\}v(q_2),$$

where k is the four-momentum of the virtual photon, $\nu=k_0$ is the virtual photon energy, m_2 is the proton mass. Symmetrical part of the tensor (1) gives the contribution to the Lamb shift (the structure functions $C_{1,2}(\nu,k^2)$) and antisymmetric part contributes to the hyperfine structure (the structure functions $H_{1,2}(\nu,k^2)$). The lepton tensor is the following:

$$M^{(l)}_{\mu\nu}=\bar u(q_1)\left[\gamma_\mu\frac{\hat{p_1}+\hat k+m_1}{(p_1+k)^2-m_1^2+i\epsilon}\gamma_\nu+\gamma_\nu\frac{\hat{p_1}-\hat k+m_1}{(p_1-k)^2-m_1^2+i\epsilon}\gamma_\mu\right]u(p_1), \tag{2}$$

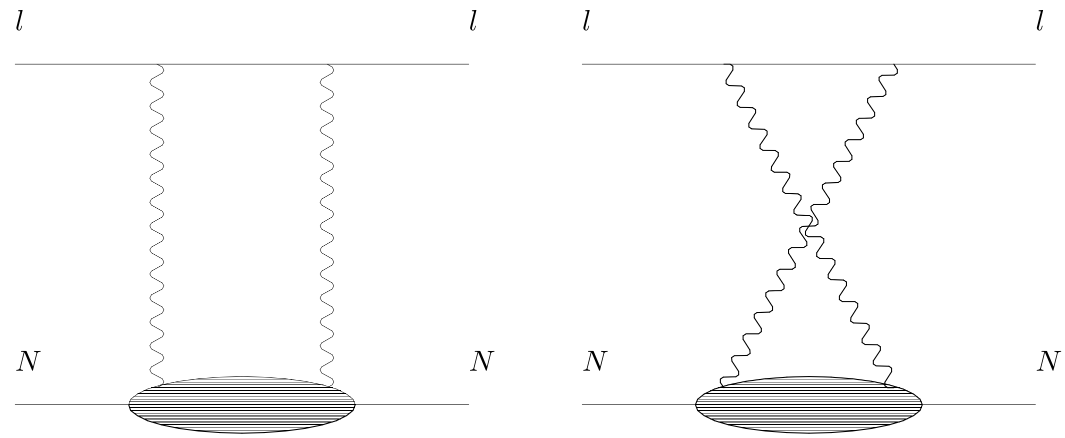

FIG. 1: Two-photon Feynman amplitudes determining the correction on the proton polarizability in the Lamb shift of hydrogen atom.

where $p_{1,2}$, $q_{1,2}$ are four-momenta of the lepton and proton in the initial and final states, m_1 is the lepton mass.

Taking the product of the amplitudes (1) and (2), we can extract the part of the interaction operator which is not dependent on the particle spins and contributes to the Lamb shift (LS):

$$\left[M^{(\mu)}_{\mu\nu}M^{(p)}_{\mu\nu}\right]^{LS}=\frac{2m_1}{k^4-4k_0^2m_1^2}\left[-(2k_0^2+k^2)C_1+(k^2-k_0^2)C_2\right]. \quad (3)$$

The structure functions $C_i(k_0,k^2)$ satisfy a following dispersion relations [23]:

$$C_1(k_0,k^2)=C_1(0,k^2)+\frac{1}{\pi}k_0^2\int_{\nu_0}^{\infty}\frac{d\nu^2}{\nu^2(\nu^2-k_0^2)}ImC_1(\nu,k^2), \quad (4)$$

$$C_2(k_0,k^2)=\frac{1}{\pi}\int_{\nu_0}^{\infty}\frac{d\nu^2}{(\nu^2-k_0^2)}ImC_2(\nu,k^2), \quad (5)$$

$$\nu_0=m_\pi+\frac{1}{2m_2}(Q^2+m_\pi^2),\quad Q^2=-k^2.$$

The threshold value of the photon energy ν_0 represents the minimal energy needed for the production of the π-meson in the reaction $\gamma^*+p\to\pi^0+p$. Let us to point out that reliable data on the subtraction term in the first dispersion integral (4) are absent. But in the limit of small values of k^2 this term is connected with the proton magnetic polarizability:

$$\lim_{k^2\to 0}\frac{C_1(0,k^2)}{k^2}=\frac{m_2}{\alpha}\beta_M, \quad (6)$$

where $\beta_M=1.9(0.5)\times 10^{-4}\ fm^3$ [24]. The dipole parameterization for $\beta_M(k^2)$ was suggested in Ref.[18]:

$$\beta_M(k^2)=\beta_M\frac{\Lambda^8}{(\Lambda^2+k^2)^4}, \quad (7)$$

where $\Lambda^2=0.71\ GeV^2$ as for the elastic nucleon form factor. Imaginary parts of the amplitudes $C_i(k_0,k^2)$ are expressed in terms of the structure functions $F_i(x,Q^2)$ for deep inelastic scattering as

$$\frac{1}{\pi}ImC_1(x,Q^2)=\frac{F_1(x,Q^2)}{m_2},\quad \frac{1}{\pi}ImC_2(x,Q^2)=\frac{F_2(x,Q^2)}{\nu},\quad x=\frac{Q^2}{2m_2\nu}. \tag{8}$$

So, to obtain numerical value of the proton polarizability correction we can use experimental data for the functions $F_{1,2}(\nu,k^2)$ and different parameterizations for it prepared on the basis of these results [25, 26, 27]. Making use of relations (5)-(7) and transforming the integration in the loop amplitudes to four-dimensional Euclidean space with the aid of the formula

$$\int d^4k=4\pi\int_0^\infty k^3dk\int_0^\pi \sin^2\phi\cdot d\phi,\quad k^0=k\cos\phi, \tag{9}$$

we can perform the integration over the angle variable ϕ and represent the proton polarizability contribution to the Lamb shift of hydrogen atom in the form:

$$\Delta E^{LS}_{pol}=-\frac{2\mu^3(Z\alpha)^5}{\pi n^3m_1^4}\int_0^\infty dk\int_{\nu_0}^\infty dy\ \mathcal{F}(y,k)+\frac{2\mu^3(Z\alpha)^4}{\pi n^3m_1}\int_0^\infty h(k^2)\beta_M(k^2)kdk, \tag{10}$$

$$\mathcal{F}(y,k)=\frac{1}{(R+1)st^2(4s-t)}\Big\{-8\sqrt{s}(1+s)^{3/2}\sqrt{t}(2s+R)- \tag{11}$$

$$-\sqrt{t}(t-4s)[t+s(6+2R+4s+t)]+\sqrt{4+t}[(t-2)t+s\left(8+t^2+2R(t+4)\right)]\Big\}F_2(y,k^2),$$

$$h(k^2)=1+\left(1-\frac{t}{2}\right)\left(\sqrt{\frac{4}{t}+1}-1\right),\quad t=\frac{k^2}{m_1^2},\quad s=\frac{y^2}{k^2}, \tag{12}$$

where $R(y,k^2)=\sigma_L/\sigma_T$ is the ratio of the cross sections for the absorption of longitudinally and transversely polarized photons by hadrons. In such a way, the correction ΔE^{LS}_{pol} is expressed in terms of two structure functions $F_2(\nu,k^2)$ and $R(\nu,k^2)$, describing unpolarized lepton-nucleon scattering.

III. STRUCTURE FUNCTIONS OF UNPOLARIZED LEPTON-NUCLEON SCATTERING

The greatest contribution to the integral (10) is given by the region of the variable k^2: $0\div 1\ GeV^2$ and near the threshold values of a photon energy ν. So, the exact construction of the structure functions F_2, R in this region is very important to obtain reliable estimation of the proton polarizability effect. Deep inelastic lepton-nucleon scattering is described by the following reaction:

$$l+N\to l'+X, \tag{13}$$

where X denotes the sum over all final particles. This reaction represents the inclusive lepton (l) production with the measurement of their energy and scattering angle. It is assumed that this process occurs due to one-photon exchange. An important kinematical variable of the reaction (13) is the invariant mass of electroproduced hadronic system W:

$$W^2=m_2^2-Q^2+2m_2\nu,\quad k^2=-Q^2. \tag{14}$$

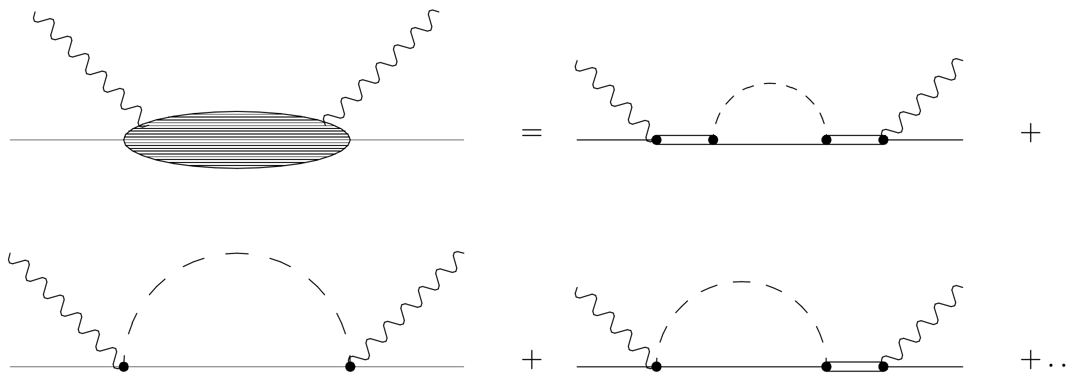

FIG. 2: The proton polarizability correction in the resonance region. On the Feynman diagram solid, double solid, wave and dashed lines correspond to the nucleon, baryon resonance, photon and pion respectively.

Using the variable (14) we can divide the total integration region in Eq.(10) on the resonance region $W \leq 2$ GeV where the production of low-lying nucleon resonances occurs and deep inelastic region when $W > 2$ GeV.

In the resonance region the proton polarizability contribution to the Lamb shift is determined by the processes of a π-, η-meson production on nucleons and the production of basic low-lying nucleon resonances. Several amplitudes of such reactions are presented in Fig.2. To calculate the contributions of separate resonances to the cross sections $\sigma_{T,L}$ in the isobar model we used the Breit-Wigner parameterization suggested in Refs.[28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39]. In the considered region of the variables k^2, W the most contribution is given by five resonances: $P_{33}(1232)$, $S_{11}(1535)$, $D_{13}(1520)$, $P_{11}(1440)$, $F_{15}(1680)$. Accounting the resonance decays to the $N\pi$- and $N\eta$-states we can express the absorption cross sections $\sigma_{1/2}$ and $\sigma_{3/2}$ as follows:

$$\sigma_{1/2,3/2} = \left(\frac{k_R}{k}\right)^2 \frac{W^2\Gamma_\gamma\Gamma_{R\to N\pi}}{(W^2-M_R^2)^2+W^2\Gamma_{tot}^2}\frac{4m_p}{M_R\Gamma_R}|A_{1/2,3/2}|^2, \tag{15}$$

where $\mathrm{A}_{1/2,3/2}$ are transverse electromagnetic helicity amplitudes,

$$\Gamma_\gamma = \Gamma_R\left(\frac{k}{k_R}\right)^{j_1}\left(\frac{k_R^2+X^2}{k^2+X^2}\right)^{j_2}, \quad X = 0.3\ GeV. \tag{16}$$

The resonance parameters Γ_R, M_R, j_1, j_2, Γ_{tot} are taken from Refs.[24, 31, 35, 36]. In accordance with Refs.[30, 32, 36] the parameterization of a one-pion decay width is

$$\Gamma_{R\to N\pi}(q) = \Gamma_R\frac{M_R}{M}\left(\frac{q}{q_R}\right)^3\left(\frac{q_R^2+C^2}{q^2+C^2}\right)^2, \quad C = 0.3\ GeV \tag{17}$$

for the resonance $P_{33}(1232)$ and

$$\Gamma_{R\to N\pi}(q) = \Gamma_R\left(\frac{q}{q_R}\right)^{2l+1}\left(\frac{q_R^2+\delta^2}{q^2+\delta^2}\right)^{l+1}, \tag{18}$$

for $D_{13}(1520)$, $P_{11}(1440)$, $F_{15}(1680)$. l is the pion angular momentum, $\delta^2 = (M_R - m_p - m_\pi)^2 + \Gamma_R^2/4$. Here q (k) and q_R (k_R) denote the c.m.s. pion (photon) momenta of resonances with the mass M and M_R respectively. In the case of $S_{11}(1535)$ we take into account πN and ηN decay modes [32, 36]:

$$\Gamma_{R\to N\pi,N\eta} = \frac{q_{\pi,\eta}}{q} b_{\pi,\eta} \Gamma_R \frac{q^2_{\pi,\eta} + C^2_{\pi,\eta}}{q^2 + C^2_{\pi,\eta}}, \tag{19}$$

where $b_{\pi,\eta}$ is the π, η branching ratio. The cross section σ_L is determined by an expression similar to Eq.(15) where we must change $A_{1/2,3/2}$ on the longitudinal amplitude $S_{1/2}$. The calculation of helicity amplitudes $A_{1/2}$, $A_{3/2}$, $S_{1/2}$ as functions of Q^2 was done on the basis of the oscillator quark model in Refs.[37, 40, 41, 42, 43, 44].

The two-pion decay modes of the higher nucleon resonances $S_{11}(1535)$, $D_{13}(1520)$, $P_{11}(1440)$, $F_{15}(1680)$ are described phenomenologically using the two-step process as in Ref.[36]. The high-lying nucleon resonance R can decay first into N^* (P_{33} or P_{11}) and a pion or into a nucleon and ρ, σ meson. Then the new resonances decay into a nucleon and a pion or two pions:

$$R \to r + a = \begin{cases} N^* + \pi \to N + \pi + \pi, \\ \rho(\sigma) + N \to N + \pi + \pi. \end{cases} \tag{20}$$

The total decay width of such processes can be presented as a phase-space weight integral over the mass distribution of the intermediate resonance $r = N^*, \rho, \sigma$ $(a = \pi, N)$:

$$\Gamma_{R\to r+a}(W) = \frac{P_{2\pi}}{W} \int_0^{W-m_a} d\mu \cdot p_f \frac{2}{\pi} \frac{\mu^2 \Gamma_{r,tot}(\mu)}{(\mu^2 - m_r^2)^2 + \mu^2 \Gamma^2_{r,tot}(\mu)} \frac{(M_R - m_2 - 2m_\pi)^2 + C^2}{(W - m_2 - 2m_\pi)^2 + C^2}, \tag{21}$$

where $C = 0.3\ GeV$. The factor $P_{2\pi}$ must be taken from the constraint condition: $\Gamma_{R\to r+a}(W_R)$ coincides with the experimental data in the resonance point. p_f is the three-momentum of the resonance r in the rest frame of R. $\Gamma_{r,tot}$ is the total width of the resonance r. The decay width of the meson resonance in Eq.(21) is parameterized similarly to that of the $P_{33}(1232)$:

$$\Gamma(\mu) = \Gamma_r \frac{m_r}{\mu} \left(\frac{q}{q_r}\right)^{2J_r+1} \frac{q_r^2 + \delta^2}{q^2 + \delta^2}, \quad \delta = 0.3\ GeV, \tag{22}$$

where m_r and μ are the mean mass and the actual mass of the meson resonance, q and q_r are the pion three momenta in the rest frame of the resonance with masses μ and m_r. J_r and Γ_r are the spin and decay width of the resonance with the mass m_r.

Main nonresonant contribution to the cross sections $\sigma_{T,L}$ in the resonance region is determined by the Born terms constructed on the basis of Lagrangians of γNN, $\gamma\pi\pi$, πNN interactions. Another part of nonresonant background comprises the t- channel contributions of ρ, ω mesons obtained by means of effective Lagrangians $\gamma\pi V$, VNN interactions ($V = \rho, \omega$) [34, 45]. In the unitary isobar model accounting the Born terms, the vector meson, nucleon resonance contributions and the interference terms we calculated the cross sections $\sigma_{T,L}$ by means of numerical program MAID (http://www.kph-uni-mainz.de/MAID) in the resonance region as the functions of two variables W and Q^2. Then the structure function $F_2(W, Q^2)$ can be presented as follows:

$$F_2(W, Q^2) = \frac{Q^2}{4\pi^2\alpha} (\sigma_T + \sigma_L) \frac{K\nu}{(Q^2 + \nu^2)}, \tag{23}$$

where K is the flux factor of virtual photons. There are two definitions for the quantity K: the Gilman definition $K_G = \sqrt{Q^2 + \nu^2}$, and the Hand definition $K_H = \nu(1 - Q^2/2m_2\nu) =$

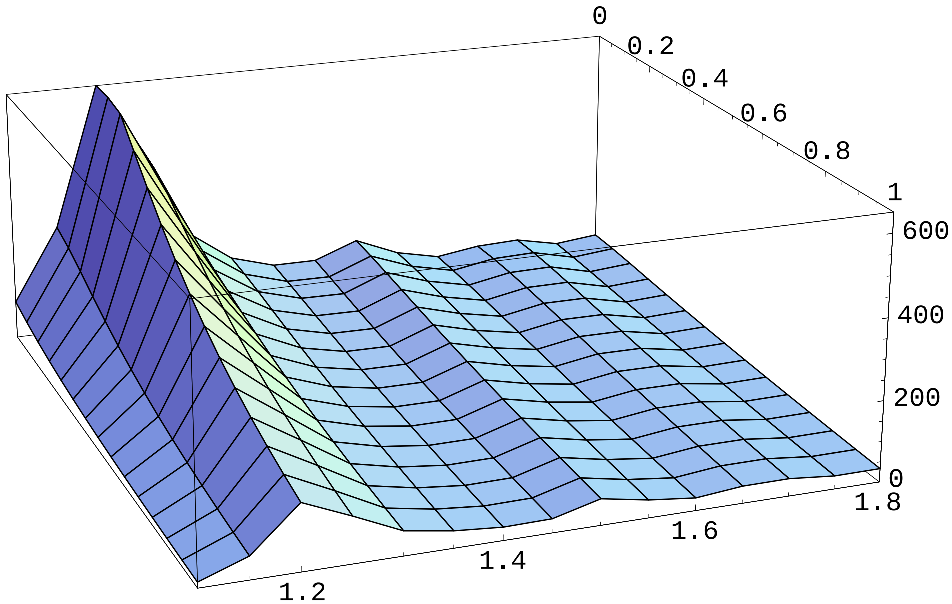

FIG. 3: Total photoabsorption cross section $\sigma_{tot}(W,Q^2) = (\sigma_T + \sigma_L)$ in μb as the function of variables Q^2 $(0 \div 1)$ GeV^2 and W $(1.1 \div 1.8)$ GeV.

$(W^2 - m_2^2)/2m_2$ [46]. Our results are presented in Fig.3 for the total cross section $\sigma_{tot}(W,Q^2) = (\sigma_T + \sigma_L)$. It contains three clear peaks corresponding to resonances $P_{33}(1232)$, $D_{13}(1520)$, $F_{15}(1680)$. The photoabsorption cross section $\sigma_{tot}(W,Q^2=0)$ differs from experimental data in the range $1.5 \le W \le 2$ GeV obtained in Refs.[47, 48]. So, theoretical model must be improved by the account of two-pion resonance decays as described in Eq.(21).

In the nonresonant region there exist several parameterizations for the function $F_2(Q^2,W)$ [25] obtained on the basis of experimental data on deep inelastic lepton-nucleon scattering. The structure function F_2 can be expressed in terms of parton distributions

$$F_2(x,Q^2) = \sum_i e_i^2 x q_i(x,Q^2). \tag{24}$$

So, to construct it in deep inelastic region we can use the Q^2 evolution equations for the quark and gluon distributions [49]:

$$\frac{dq_i(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\left[q_i(y,Q^2)P_{qq}\left(\frac{x}{y}\right) + g(y,Q^2)P_{qg}\left(\frac{x}{y}\right)\right], \tag{25}$$

$$\frac{dg(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\left[\sum_i q_i(y,Q^2)P_{gq}\left(\frac{x}{y}\right) + g(y,Q^2)P_{gg}\left(\frac{x}{y}\right)\right], \tag{26}$$

where the sum is considered over all quarks and antiquarks. P_{qq}, P_{gq}, P_{qg}, P_{gg} are the quark-gluon splitting functions [50]. Numerical solution of the integrodifferential evolution equations (25), (26) by means of the method suggested in Ref.[51] allows to obtain the parton distributions and the structure function $F_2(x,Q^2)$ for different values of a photon momentum squared Q^2. Corresponding numerical results are in good agreement with the world experimental data. On the other hand, the 23 parameter model for the structure function

$F_2(x,Q^2)$ based on experimental data was proposed in Ref.[27]. Here it was expressed as a sum of the Pomeron $F_2^{\mathcal{P}}$ and the Reggeon $F_2^{\mathcal{R}}$ term contributions:

$$F_2(x,Q^2)=\frac{Q^2}{Q^2+m_0^2}\left[F_2^{\mathcal{R}}(x,Q^2)+F_2^{\mathcal{P}}(x,Q^2)\right], \tag{27}$$

$$F_2^{\mathcal{R}}(x,Q^2)=C_{\mathcal{R}}(t)x_{\mathcal{R}}^{a_{\mathcal{R}}(t)}(1-x)^{b_{\mathcal{R}}(t)},\quad F_2^{\mathcal{P}}(x,Q^2)=C_{\mathcal{P}}(t)x_{\mathcal{P}}^{a_{\mathcal{P}}(t)}(1-x)^{b_{\mathcal{P}}(t)}, \tag{28}$$

where x is the Bjorken variable,

$$\frac{1}{x_{\mathcal{R}}}=1+\frac{W^2-m_2^2}{Q^2+m_{\mathcal{R}}^2},\quad \frac{1}{x_{\mathcal{P}}}=1+\frac{W^2-m_2^2}{Q^2+m_{\mathcal{P}}^2}, \tag{29}$$

More accurate numerical values of the model parameters are presented in Ref.[27] (version 2 (2004)).

For the second structure function $R(Q^2,W)$ in nonresonant region we used also the parameterization obtained with the aid of experimental data [52]. In the resonance region there are no experimental data for the quantity $R(Q^2,W)$. In the most important part of the resonance region $\sigma_L \ll \sigma_T$. So, to perform the numerical calculation of the correction ΔE^{LS}_{pol} we supposed that $R(Q^2,W)\approx 0$.

IV. NUMERICAL RESULTS

In this paper we calculate the proton polarizability correction to the Lamb shift of electronic and muonic hydrogen on the basis of the isobar model describing the processes of low-energy scattering of virtual photons on nucleons and the evolution equations for the parton distributions. These two significant ingredients of the calculation allow to construct the absorption cross sections for transversely and longitudinally polarized photons by nucleons $\sigma_{T,L}$ and to express the structure function $F_2(Q^2,W)$ (23), (27), which determines required contribution (10). Numerical results are presented in Table 1. We investigated contributions to the correction (10) which have numerical value of order 1 μeV for the $1S$ state in muonic hydrogen and 1 Hz for the $1S$ state in electronic hydrogen. It is inferred from these results that the basic contribution to the polarizability effect is given by processes of the π-meson production on nucleons in the reaction $\gamma^*+N\rightarrow\pi+N$ including the resonant reactions (2 line in Table 1). In the isobar model we kept also processes of the η-meson production on nucleons [45] (3 line in Table 1) and the production processes of the K-mesons (4 line in Table 1). Declared accuracy of the calculation calls for further consideration of the two-pion final states in the reaction $\gamma^*+N\rightarrow N+\pi+\pi$ (5 line in Table 1). Indeed, the comparison of the total photoabsorption cross section, derived from the isobar model, with experimental data shows that in the resonance region at $2\ GeV\geq W\geq 1.5\ GeV$ theoretical photoabsorption cross section is less than experimental one by the value 100 μb approximately. Two-pion final states in the resonant reactions of the form (20) are taken into account in the construction of the cross sections $\sigma_{T,L}$ through the use of a two-stage model (21). Corresponding contribution to the correction (10) is equal to (-6 μeV). But there exists nonresonant contribution of two-pion final states to the total cross section $\sigma_{tot}(Q^2,W)$. We included in Table 1 approximate estimate of this contribution equal to (-6 μeV). It is based on the assumption that in the whole range of variables Q^2, W: $0\leq Q^2\leq 1$, $1.5\leq W\leq 2$ determining the value of the correction (10) the total cross

TABLE I: Proton polarizability correction in the Lamb shift of electronic and muonic hydrogen.

Contribution of the reaction (13) to the correction ΔE^{LS}_{pol}	(e^-p^+) Hz		(μ^-p^+) μeV	
	$1S$	$2S$	$1S$	$2S$
Contribution of $N\pi$-states	- 86.81	-10.851	-103.3	-12.91
Contribution of $N\eta$-states	-0.02	-0.003	-0.6	-0.08
Contribution of K-mesons	-0.03	-0.004	-1.1	-0.14
Contribution of $N\pi\pi$-states	- 0.54	-0.068	-12.0	-1.50
Nonresonant contribution	-0.44	-0.055	-12.0	-1.50
Contribution of subtraction term	0.99	0.124	18.4	2.30
Summary contribution	-86.85	-10.86	-110.6	-13.83

section of virtual photoabsorption is less than experimental data by the value of order 100 μeV as for the cross section at $Q^2 = 0$.

There exists a number of theoretical uncertainties connected with quantities entering in the correction (10). In the improved isobar model [34, 45] containing 14 resonances, we can omit theoretical error which arises due to the insertion of other high-lying nucleon resonances. On our sight the main theoretical error is closely related with the calculation of the helicity amplitudes $A_{1/2}(Q^2)$, $A_{3/2}(Q^2)$, $S_{1/2}(Q^2)$ in the quark model based on the oscillator potential [46]. Only systematical experimental data for the helicity amplitudes of the photoproduction on the nucleons $A_{1/2}(0)$, $A_{3/2}(0)$ are known with sufficiently high accuracy to the present [24]. In the case of amplitudes for the electroproduction of the nucleon resonances experimental data contain only their values at several points Q^2. So, we have no consistent check for the predictions of the oscillator model. Possible theoretical uncertainty connected with the calculation of amplitudes $A_{1/2}(Q^2)$, $A_{3/2}(Q^2)$, $S_{1/2}(Q^2)$ with the account of relativistic corrections can attain the value of order 10 %. Then the theoretical error for the correction (10) in the resonance region comprises 20 % from the obtained value, that is ± 2 μeV for the energy level $2S$ in muonic hydrogen. There is theoretical uncertainty in the contribution due to two-pion nonresonant processes which is presented above. The error in this case can constitute no less than 30 % from corresponding contribution that is ± 2 μeV. The essential part of the theoretical error is connected with the subtraction term in the dispersion integral (4). Indeed, the increase of the world average value for the proton magnetic polarizability from 1.6×10^{-4} fm^3 to 1.9×10^{-4} fm^3 [24] during last years leads to the decrease of the summary contribution to the Lamb shift $(2P-2S)$ by 0.4 μeV. The error of β_M indicated in Ref.[24], gives the uncertainty ± 0.6 μeV to the theoretical result for the shift $\Delta E^{LS}_{pol}(2P-2S)$. The obtained value of the proton polarizability contribution to the Lamb shift $(2P-2S)$ in muonic hydrogen is equal to (13.8 ± 2.9) μeV. It is intermediate in the value between the result $(16 \div 17)$ μeV calculated in Refs.[16, 17] and 12 μeV, obtained in Ref.[18]. Our theoretical uncertainty is slightly higher than in Refs.[16, 17, 18], because of the presence of a number of additional theoretical quantities in the isobar model with the definite theoretical uncertainties. In the case of electronic hydrogen the value of the proton polarizability contribution to the Lamb shift is determined for the most part by the process $\gamma^* + N \to \pi + N$ including the resonance contribution. The obtained shift (-87) Hz of the $1S$ energy level in (e^-p^+) is also in the agreement with the previously derived results

(-72) Hz in Ref.[14], (-71) Hz in Ref.[15] and (-95) Hz in Ref.[16].

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