On complexity of one maximum cut problem

Alexander Kel’mannov, Artem Pyatkin

1 Sobolev Institute of Mathematics, Novosibirsk, Russia; kelm@math.nsc.ru
2 Sobolev Institute of Mathematics, Novosibirsk, Russia; artem@math.nsc.ru

The following problem is considerd:

**Problem Max-Cut**($\mathbb{R}^q$)-SD (Max-Cut in Euclidean space, the case for Squared Distances). **Input**: A set $X = \{x_1, \ldots, x_N\}$ of Euclidean vectors $\mathbb{R}^q$ and positive number $A$. **Question**: Is there a partition of the set $X$ into two subsets $Y$ and $Z$ such that $\sum_{y \in Y} \sum_{z \in Z} \|y - z\|^2 \geq A$?

The optimization variant of this problem corresponds to the search for a cut of maximum weight in the complete undirected weighted graph whose vertices are Euclidean points and the weights of the edges are equal to the squared distances between these points. It is known that in the general (non-Euclidean) case when the weights of the edges in the graph are positive the problem is strongly NP-complete [1]. The complexity status of the considered (Euclidean) case was not earlier stated. This was the motivation of our research. Besides, this problem is actual, in particular, for solving noise proof clustering problems in data analysis.

The main result of the work is:

**Theorem 1.** Problem Max-Cut($\mathbb{R}^q$)-SD is strongly NP-complete.

It is shown in the proof that the classical strongly NP-complete problem Max-Cut can be reduced to the partial case of the considered problem with the numerical parameters bounded by a polynomial on the input size.

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