Kerr-Schild Geometry from Cosmology to Microworld and Space-Time Structure.

Alexander BURINSKII
Theor. Phys. Lab., NSI, Russian Academy of Sciences,
Moscow RUSSIA

December 27, 2012

Abstract

The Kerr-Schild (KS) geometry is linked tightly with the auxiliary flat Minkowski background. Nevertheless, it describes many curved space-times and the related physical models, starting from cosmology and black holes to the microworld of the spinning elementary particles and the pre-quantum structure of vacuum fluctuations. We consider here a KS model of the Bubble Universe – a semi-closed Universe with a rotating de Sitter (or anti-de Sitter) space embedded in an external flat space-time. When the solution has two horizons, it may also be interpreted as an Universe inside a black hole. In micro-world the KS geometry yields a model of the spinning particle consistent with gravity and describes a pre-quantum twistorial structure of space-time with the beam-like fluctuations of metric consistent with the beamlike fluctuations of electromagnetic vacuum. These light-like twistor-beams excitations are consistent with gravity and generalize the known pp-wave solutions. Following Wheeler’s estimations of the density of vacuum fluctuations we arrive at the general conclusion that Universe should be flat and have a zero cosmological constant. It contradicts predominant doctrine of the Big Bang and expanding Universe, and enforces us to return to an ‘effective flat geometry’ filled by the electromagnetic background radiation.

Key words: Kerr-Schild metric, twistors, semi-closed Universe, bubble models, Higgs field, solitons, vacuum, extended electron.

1 Introduction

General covariance is the main merit of General Relativity and the main reason of its misinterpretation. The use of different coordinate systems allows one to treat different embeddings of the models into the real world which leads to different physical interpretations of the same solution.
A typical example is the freedom in choice of the radial coordinate $r$ of the Schwarzschild black hole (BH) solutions, in which the radial coordinate determines the position of the black hole horizon, which allows one to use diverse analytic extensions of this coordinate through the BH horizon. This leads to the diverse maximal analytic extension (MAE) of the BH geometry and the appearance of the ‘black’ and ‘white’ holes and many other topological possibilities which “... do not have any relation to the real physics of the black holes.” [1]).

The Kerr-Schild (KS) class of metrics has a rigidly fixed coordinate system related with the flat Minkowski background and to some extra imprinted causal twistorial structure – the congruence of principal null directions. In spite of the extreme rigidity and the tight relation to the flat background, the KS metrics include many important curved space-times of General Relativity, in particular, they allow one to describe:

- rotating black holes and rotating stars (without horizon),
- de Sitter and Anti de Sitter space-times, and their rotating analogues,
- a de Sitter (or AdS) background with an embedded black hole,
- opposite situations: dS (or AdS) space embedded inside a black hole,
- charged and rotating black holes and stars,
- bubble models with a domain wall separating the inner and external space-times.

The Kerr-Schild metric is given in the Cartesian coordinates of the Minkowski spacetime $x = x^\mu = (t, x, y, z) \in M^4$, and is represented by the simple form $g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_\mu k_\nu$, where $\eta_{\mu\nu}$ is the auxiliary Minkowski metric, and the vector field $k^\mu(x)$ is a null field, $k_\mu k^\mu = 0$, with respect to the both metrics $g_{\mu\nu}$ and $\eta_{\mu\nu}$.

Quantum theory works in flat spacetime and unambiguously suggests us that the world is flat. Principal grounds for this statement follow from the microstructure of the quantum vacuum – extremely strong vacuum fluctuations forming the zero-point field. This huge value of the quantum vacuum energy is predominate over all other matter and should determine the structure, and in particular, the curvature of our Universe. The reality of this field is exhibited by the Lamb shift and by the experimentally confirmed Casimir effect. As it is argued by Wheeler in [1], the nuclear densities are $10^{14} g/cm^3$, while estimations of the energy density for vacuum fluctuations yield $10^{94} g/cm^3$. The ratio of these densities may be compared with the ratio of the energy density of the images on the cinema screen with respect to the energy density of the screen itself. The natural assumption is that the energy of ‘pictures’ should not curve the screen!

\footnote{We use the signature \((++++)\).}
The KS class of metrics gives us the possibility to combine consistently the quantum point of view with general relativity: quantum theory should live on the auxiliary Minkowski background, while the resulting KS space may be curved in agreement with the General Relativity. Twistor structure of the KS geometry, determined by the discussed below Kerr theorem, results in many other examples of the consistency KS gravity with quantum theory. In particular, the pp-wave analog of plane waves allows one to realize in the KS curved spaces the twistorial Fourier transform [2] and creates a pre-quantum KS geometry [3]. Conformal properties of the KS geometry provide relations with superstring theory [4, 5], which allows us to build a Kerr-Newman soliton model of spinning particle consistent with gravity [6]. We note that twistorial structure of quantum vacuum conflicts with the model of a closed and expanding Universe. Recall, that the idea of closed Universe was created by Einstein in 1917, and the principal motivation for it were the problems in Newton theory – infiniteness of the gravitational potential for the homogenous matter density [7]. At present, this motivation could seem to be superficial for two reasons: 1) many infinities are known now for other effectively working and physically accepted theories, the most important example being QED which is full of the diverse divergencies, and 2) the real matter distribution in the Universe is not homogenous, and the potential of the localized sources should be regularized locally by gravity.

KS metrics allows one to consider the ‘curved’ de Sitter Universe on the flat Minkowski background with metric $\eta_{\mu\nu}$. This fact was mentioned first by Gürses and Gürsay (GG) in [8]. In this paper we exploit a generalizations of this representation.

In sec.2 we consider basic properties and peculiarities of the KS geometry and also the Kerr Theorem which determines its twistorial structure. In sec.3. we use the GG form of metric for regularization of the Kerr-Newman BH solution, and in sec.4 we consider the based on GG metric model of domain wall bubble, which corresponds to our interpretation of the semi-closed Universe.

In sec.5 we consider nontrivial consequences of the twistorial KS structure – the beamlike exact KS solutions for the electromagnetic (EM) excitations on the KS background which indicate a fluctuating twistor-beam structure of the vacuum and the EM excitations, and finally, in sec.6 we consider the bubble model of spinning elementary particle based on the regularized Kerr-Newman (KN) solution.

2 Structure of the Kerr-Newman solution

The KN metric reads

$$g_{\mu\nu} = \eta_{\mu\nu} + 2H k_\mu k_\nu, \quad H = \frac{mr - e^2/2}{\Sigma},$$

(1)
where
\[ \Sigma = r^2 + a^2 \cos^2 \theta, \]  
(2)
and \( k^\mu \) is a null vector field determined by eq. (4) below. The electromagnetic (EM) vector potential has the form
\[ A_{KN}^\mu = \text{Re} \frac{e}{r + ia \cos \theta} k^\mu, \]  
(3)
which is aligned with the vector field \( k^\mu \), forming the Principal Null Congruence (PNC), or the Kerr congruence. Gravitational and EM fields are concentrated near the Kerr singular ring \( r = \cos \theta = 0 \), where \( r, \theta \) are the oblate spheroidal coordinates (see Fig.1).

In the black hole solutions the Kerr ring is hidden beyond the horizon. For the large values of angular momentum, in particular, in the model of spinning particle (sec.6), the Kerr ring is open and forms a sort of waveguide, or a closed string which generate zitterbewegung of the Dirac electron. The Kerr ring is a branch line of the Kerr-Schild geometry into two sheets corresponding to \( r > 0 \) and \( r < 0 \).

In the KS representation [9], a few coordinate systems are used simultaneously. In particular, the null Cartesian coordinates
\[ \zeta = (x + iy)/\sqrt{2}, \quad \bar{\zeta} = (x - iy)/\sqrt{2}, \quad u = (z - t)/\sqrt{2}, \quad v = (z + t)/\sqrt{2} \]
are used for description of the Kerr congruence in the differential form
\[ k_\mu dx^\mu = P^{-1}(du + \tilde{Y} d\zeta + Y d\bar{\zeta} - Y \tilde{Y} dv), \]  
(4)
via the complex function \( Y(x^\mu) = e^{i\phi} \tan \frac{\theta}{2} \), which is a projective angular coordinate on the celestial sphere,
\[ Y(x^\mu) = e^{i\phi} \tan \frac{\theta}{2}. \]  
(5)
2.1 Twosheetedness of the KS space-time and the Kerr theorem

The Kerr congruence (PNC) is controlled by the

**Kerr theorem** [9, 10, 11, 3, 2]: The geodesic and shear-free Principal Null Congruences (PNC) (type D metrics) are determined by holomorphic function $Y(x)$ which is analytic solution of the equation

$$F(T^a) = 0,$$

where $F$ is an arbitrary analytic function of the three projective twistor coordinates

$$T^a = \{Y, \, \zeta - Yv, \, u + Y\tilde{z}\}.$$  

The Kerr theorem is a practical tool for obtaining exact Kerr-Schild solutions, which is performed by the following sequence of steps:

$$F(T^a) = 0 \Rightarrow F(Y, x^\mu) = 0 \Rightarrow Y(x^\mu) \Rightarrow k^\mu(x^\mu) \Rightarrow g^{\mu\nu}$$

For the Kerr-Newman solution function $F$ is quadratic in $Y$, which yields two roots $Y^\pm(x^\mu)$ corresponding to two congruences.

As a result the two (IN and OUT) congruences determine two sheets of the Kerr solution: the “negative (−)” and “positive (+)” sheet, where the fields change their directions.

In particular, two different congruences $k^\mu(+)$ $\neq k^\mu(−)$ determine two different KS metrics $g_{\mu\nu}^{(+)} \neq g_{\mu\nu}^{(−)}$ on the same Minkowski background. As shown in Fig.2, the Kerr congruence propagates analytically from the IN to the OUT-
sheet via the disk $r = 0$, and therefore, the two KS sheets are linked analyti-
cally, which is conveniently described by the oblate spheroidal coordinate system
$r, \theta, \phi$, which asymptotically, for large $r$ goes to the usual spherical coordinate
system. Twosheetedness is the long-term mystery of the Kerr solution.

The extremely simple form of the Kerr-Schild (KS) metric (1) is related with
complicate form of the Kerr congruence, which represents a type of deformed
(twisted) hedgehog. In the rotating BH solutions the usual pointlike singularity
inside the BH turns into a closed singular ring, which is interpreted as a closed
string in the corresponding models of the spinning elementary particles [6, 14,
5, 4].

The Kerr singular ring represents some important extra argument against
the singular initiate state of the Big Bang model, since the pointlike singularity
is defocused in the Kerr solution by the rotation. The singular initiate state
should disappear for the rotating Big Bang model. ²

The Kerr-Schild geometry gives a new look at the BH horizon. The sensitiv-
ity of the horizon to electromagnetic fields is well known [15]. In particular, the
horizons disappear for $|e| > m$, which is the point of bifurcation. By $|e| \geq m$, the
KN solution is not changed, while all Carter’s diagrams (MAE) disappear.

The Kerr coordinates are rigidly linked with a background Minkowski spacetime,
and therefore, the coordinate system of the KS geometry is decoupled from the
position of the horizon. It allows one to analyze influence of the electromagnetic
(EM) field on the horizon and its deformations by the EM field [15]. The basic
KS solutions turn out to be independent of the position of the horizon and even
of its existence.

Similar arguments where given by Misner-Thorne-Wheeler (Sec. 33.2 of [1]
– the complicate spacetime of the “maximal analytic extension” (MAE) of the
Kerr-Newman (KN) black hole spacetimes does not have any relation to the
question on black holes for two reasons:

a) the most part of the internal KN spacetime takes the already collapsed
star, and

b) even the external KN geometry does not give an adequate description of
the real geometry due to the nonstationary processes around BHs.

³ Phase transition as a regulator of the black hole singularity.

Smooth and regular rotating sources of the Kerr-Schild class were obtained in
[16, 17]. They are based on a generalization of the KN geometry suggested by
the Gürses and Gürsey (GG) in [8]. Starting from the KS form of metric

$$g_{\mu \nu} = \eta_{\mu \nu} + 2Hk_{\mu}k_{\nu},$$

²An additional argument is that there is a quantum limit for the superdense state due to
the vacuum volume Casimir effect (superdense matter goes into a pseudovacuum state.) [12].
GG used the generalized form of the function $H$,

$$H = f(r)/\Sigma, \quad \Sigma = (r^2 + a^2 \cos^2 \theta), \quad (8)$$

which allows one to suppress the Kerr singular ring ($r = \cos \theta = 0$) by a special choice of the function $f(r)$.

It is assumed that by this deformation, the Kerr congruence, determined by the vector field $k^\mu(x^\nu)$ remains lightlike, $k_\mu k^\mu = 0$, and retains the usual KS form (4).

The GG-form of the metric describes the Kerr-Newman BH solution, and also the rotating de Sitter and Anti de Sitter solutions. Besides, it allows one to match smoothly the rotating metrics of different types!

The regularized solutions have tree regions:

i) the Kerr-Newman exterior, $r > r_0$, where $f(r) = mr - e^2/2$,

ii) the source region, $r < r_0 - \delta$, where $f(r) = f_{\text{int}}$ suppresses the KN singularity, providing smoothness of the metric up to second derivatives.

iii) intermediate region (domain wall) providing a smooth phase transition interpolating between the regions i) and ii).

To remove the Kerr-Newman singularity, one has to set for the internal region

$$f_{\text{int}} = \alpha r^4.$$ 

In this case, the Kerr singularity is replaced by a regular rotating source, forming internal space-time with a constant curvature, $R = -24\alpha$, [16].

The functions

$$D = \frac{f''}{\Sigma}, \quad (9)$$

$$G = \frac{f' r - f}{\Sigma^2}. \quad (10)$$

determine stress-energy tensor in the orthonormal tetrad \{u, l, m, n\} connected with the Boyer-Lindquist coordinates,

$$T_{ik} = (8\pi)^{-1}[(D + 2G)g_{ik} - (D + 4G)(l_i l_k - u_i u_k)]. \quad (11)$$

In the above formula, $u^i$ is a timelike vector field given by

$$u^i = \frac{1}{\sqrt{\Delta \Sigma}}(r^2 + a^2, 0, 0, a),$$

where $\Delta = r^2 + a^2 - 2f$, [16, 17].

This expression shows that the matter inside the source is separated into ellipsoidal layers corresponding to constant values of the coordinate $r$, and each layer rotates with angular velocity $\omega(r) = \frac{u^\phi}{\Sigma} = a/(a^2 + r^2)$. This rotation becomes rigid only in the thin shell approximation, $r = r_0$. The linear velocity of the matter w.r.t. the auxiliary Minkowski space is $v = \frac{a \sin \theta}{\sqrt{a^2 + r^2}}$, so that on
the equatorial plane $\theta = \pi/2$, for small values of $r$ ($r \ll a$), one has $v \approx 1$, that corresponds to an oblate, relativistically rotating disk.

The energy density $\rho$ of the material satisfies $T^i_k u^k = -\rho u^i$ and is, therefore, given by

$$\rho = \frac{1}{8\pi} 2G. \quad (12)$$

Two distinct spacelike eigenvalues, corresponding to the radial and tangential pressures of the non rotating case are

$$p_{\text{rad}} = -\frac{1}{8\pi} 2G = -\rho, \quad (13)$$

$$p_{\text{tan}} = \frac{1}{8\pi} (D + 2G) = \rho + \frac{D}{8\pi}. \quad (14)$$

In the exterior region function $f$ must coincide with Kerr-Newman solution, $f_{KN} = mr - e^2/2$.

There appears a "domain wall" – a transition region placed in between the boundary of the matter object (the de Sitter interior) and the external KN gravi-electro-vacuum solution. This transition region is described by a smooth function $f(r)$ interpolating between the functions $f_0(r)$ and $f_{KN}(r)$.

4 Domain wall bubble with two horizons as a regularized black hole and as a cosmological model

The resulting source may be considered as a bubble or bag filled by a special matter with positive ($\alpha > 0$) (or negative $\alpha < 0$) energy density. This bubble source may be naked or covered by the horizon. If such a source is covered by the horizon, it will be seen by external observer as an usual black hole, since the external observer cannot check presence of the source behind the horizon. On the other hand this black hole is regularized, since its singular source is replaced by the domain wall bubble.\(^3\)

Graphical analysis for the case $\alpha > 0$

The case $\alpha > 0$ corresponds to de Sitter interior and uncharged source. There is only one intersection between $f_0(r) = \alpha r^4$ and $f_{KN}(r) = mr$. The position of the transition layer will be $r_0 = (m/\alpha)^{-1/3}$. The second derivative of the corresponding interpolating function will be negative at this point, yielding an extra contribution to the positive tangential pressure in the transition region.

Positions of the horizons

The external region is described by the KN electro-vacuum solution, and the transfer from the external KN solution to the internal region (source) may

\(^3\)The shell bubble source of the KN solution with flat interior was first suggested by López [18].
be considered as a phase transition from ‘true’ to ‘false’ vacuum. The point of phase transition $r_0$ is determined by the equation $f_{\text{int}}(r_0) = f_{KN}(r_0)$. The point of intersection $r_0$ corresponds to balance of the mass.

Consider first a non-rotating case, $\Sigma = r^2$: $\alpha = 8\pi \Lambda / 6$. The inner space-time has a constant curvature $R = -24\alpha$. There is a de Sitter interior for $\alpha > 0$, and anti de Sitter interior for $\alpha < 0$. The interior is flat if $\alpha = 0$. The energy density of the source is $\rho = \frac{1}{4\pi}(f'r - f)/\Sigma^2$, the tangential and radial pressures are $p_{\text{rad}} = -\rho$, $p_{\text{tan}} = \rho - \frac{1}{4\pi}f''/\Sigma$.

Transfer to rotating case. One has to set $\Sigma = r^2 + a^2 \cos^2 \theta$, and consider $r$ and $\theta$ as the oblate spheroidal coordinates.

The Kerr source represents a disk with the boundary $r = r_0$ which rotates rigidly. In the coordinate system corotating with the disk, the matter of the disk looks homogenously distributed; however, because of the relativistic effects the energy-momentum tensor increases strongly in equatorial plane near the boundary of the disk.

The positions of the horizons for the usual charged and rotating black hole solution are determined by the relation $r_\pm = m \pm \sqrt{m^2 - e^2 - a^2}$. The positions of the horizons for the considered bubble-source are determined by graphical analysis represented in Fig.5. One sees that similar to the case of the charged BH, there appear two horizons around the position of the domain wall $r_0$: the external horizon $r_+$ is just similar to the usual BH horizon and hides the domain wall from external observer. At the same time, the internal horizon

Figure 3: Position of phase transition $r_0$ as an intersection of plots $f_0(r)$ and $f_{KN}(r)$. Uncharged source, $\alpha > 0$, arbitrary units.
is similar to cosmological horizon and hides the domain wall from the observer living inside the domain wall. It gives the model of an Universe inside a BH.

If the rotation increases, the horizons disappear and the domain wall bubble is exhibited as a rotating stars or another object consistent with general relativity. There appears principal new feature that the KN singularity is regularized and the naked rotating solution does not break the principle of cosmic censorship. In particular, the model of such a source was considered in [6, 16, 18, 19, 20, 21, 22, 23, 24] by the treatment of the Kerr-Newman (KN) solution as a model of spinning particle consistent with gravity. It was shown that the system of chiral fields (involving the Higgs field) is able to form a positive potential interpolating the corresponding phase transition from internal to external supersymmetric states. Therefore, the bubble is formed by a domain wall separating the external and internal regions of the bubble.

We are going to the principal point of our interpretation of the regularized black hole solutions with two horizons $r_+$ and $r_-$. The observer, who is positioned outside the external horizon $r_+$, may consider this horizon as the usual horizon related with this black hole, which hides the interior of the black hole from him. At the same time, the observer, who is positioned inside the internal horizon of the regularized black hole solution $r_-$, sees the usual almost flat world, and in fact he lives in a de Sitter space for which the horizon $r_-$ is the ‘cosmological’ horizon. He will never know what happens behind this horizon, and what is the structure of the full solution beyond the horizon. Therefore we

Figure 4: The phase transfer from the external KN solution $f(r) = f_{KN}$ to the internal solution $f(r) = f_{int}$ for $r < r_0$. 
Figure 5: “De Sitter” source of the external Kerr-Newman field. The dotted line $f(r) = (r^2 + a^2)/2$ corresponds to the allowed positions of the horizons.

arrive at the model of the *Universe inside the BH solution, or inside of an elementary particle*. Ideas of this type on the junction of the de Sitter-type interior through the transition layer to the Schwarzschild geometry, or about the geometry of Universe isomorphic to a semi-closed world, were considered earlier by Klein (1961), Zeldovich-Novikov (see [25] and refs. therein) and were expressed the most close to our point of view by Frolov, Markov and Mukhanov in [26]. However, note that the considered above KS representation results in the model which is rotating and connected with the flat background, and also, it describes the smooth phase transition without an appeal to quantum arguments.

5 Fluctuating twistor structure of the KS geometry.

Analysis of the exact electromagnetic solutions on the KS background confirms that gravity conflicts with the usual plane waves. It has been shown that there are no smooth harmonic solutions on the KS background, and elementary exci-
tations of the black-holes and their consistent back-reaction on the KS metric form twistor-beams – singular beams supported by twistor null lines of the Kerr congruence, [3, 2, 15]. The twistor-beams are similar to laser beams, and turn asymptotically into singular pp-waves – a type of fundamental strings – gravitational analog of the fundamental heterotic strings of the string field theory [5]. The beam-like basic excitations of the BH have important consequences for the physics of the BHs. The BH horizons are extra sensitive to the electromagnetic (EM) field [15]4. The electromagnetic beams have very strong back reaction to metric and deform topologically the horizon. It has been shown [15, 3] that the EM excitations of the BH solutions create the EM beams which penetrate the horizon, creating there the holes which connect the BH interior with external region, [27].

Since the horizon is extra sensitive to electromagnetic excitations, it should also be sensitive to the vacuum electromagnetic excitations (which are exhibited classically as a Casimir effect), and therefore, the created by vacuum excitations twistor-beam pulses shall also perforate horizon, producing a fine-grained structure of fluctuating microholes which allow radiation to escape from the interior of the black-hole, Fig.3. It yields a semiclassical mechanism of the BH evaporation.

Figure 6: Excitations of a black hole by a weak electromagnetic field create twistor-beams perforating the BH horizon by fluctuating micro-holes.

Another important effect related with the twistor-beams is exhibited for the multiparticle Kerr-Schild solutions, [28, 29]. In agreement with the Kerr theorem, the generating function $F(Y, x^\mu)$ of the Kerr congruence takes for the $n$–particle Kerr-Schild solutions the form of the product of partial functions $F_i$.

---

4In particular, it is known that position of the BH horizons depends on the value of the BH charge and the horizons disappear at all for the strongly charged BH solutions.
for the $i$-th particle,

$$F(Y, x^\mu) = \prod_i F_i(Y, x^\mu).$$

(15)

Since each $F_i$ is quadratic in $Y$, the resulting solution of the basic equation

$$F(Y) = 0,$$

(16)

has $2n$ roots $Y^\pm_i(x^\mu)$, $i = 1, 2, ..n$, and the KS geometry turns out to be multi-valued and multisheeted twistorial space-time, having independent congruences of the twistor null lines on different sheets. In general, these congruences “do not interact and do not feel each other”. However, if $Y^\pm_i(x^\mu) = Y^\pm_k(x^\mu)$ there appear the multiple roots in the basic equation (16), which create extra null singular lines which join the $i$-th and $k$-th particles. For each two particles $i$ and $k$, there are exactly two such twistor null lines - one is directed from $i$-th to $k$-th ($Y^+_i(x^\mu) = Y^-_k(x^\mu)$) and another one from $k$-th to $i$-th ($Y^+_k(x^\mu) = Y^-_i(x^\mu)$).

As a consequence, each two particles in the Universe turn out to be connected by two singular lines (pp-wave strings), supported by twistor lines which are common for these particles. The electromagnetic excitations consistent with gravity form pp-waves propagating along these twistor lines, realizing a type of “photon exchange”. The complex analyticity of these pp-waves breaks down at the points of their emanation (which corresponds to their creation) and at the points of their termination (corresponding to their annihilation).\footnote{The creations and annihilations occur at the matter sources, like the discussed above domain wall bubbles.}

Figure 7: Four sheets of the Kerr space formed by the null congruences of two particles. The sheets are analytically matched via a common twistor null line.
have a very strong back reaction to the metric and the space-time which turns out to be covered by fluctuating twistor-beams (twistorial alternative to graviton). Therefore, the pp-wave twistor-beams turn out to be the basic elementary excitations of the KS space-time, representing an analog of the electromagnetic vacuum fluctuations in configuration space consistent with gravity. The twistorial structure of the KS geometry forms a multisheeted background of the fluctuating twistor-beams, creating a fluctuating pre-quantum KS geometry [2, 15]. Due to the conformal structure of the KS geometry, there appears a way to derive procedure of quantization from the formalism of the operator product expansion (OPE).

6 Regularized KS geometry as a background of the electron

Quantum theory states that electron is pointlike and structureless. For example, Frank Wilczek writes: “...There’s no evidence that electrons have internal structure (and a lot of evidence against it)” . Leonard Susskind writes similarly: the electron radius is “...most probably not much bigger and not much smaller than the Planck length...”, i.e. \( l_p = m_pG/c^2 = 1.6 \cdot 10^{-33}\text{cm} \).

However, the observable parameters of the electron, mass \( m \), charge \( e \), spin \( s \) and the gyromagnetic ratio \( g = 2 \) ([30]) indicate unambiguously that its gravitational background should be the KN solution. Because of the very large spin of the electron, \( a = J/m >> m \), the BH horizons disappear, so, the KN background with the electron parameters is not a black hole! It is not flat and has the above discussed non-trivial twosheeted topology created by the Kerr singular ring. For parameters of the electron the KN metric is almost flat everywhere and gravitational field is concentrated near the Kerr singular ring. This ring can be identified with a closed gravitational string [5, 31, 32] which is very similar to the Sen solution for the closed heterotic string. Elementary excitations of the Kerr string form the lightlike pp-wave traveling waves [5, 4, 15, 20] accompanied by axial twistor-beams along twistor lines of the Kerr congruence. The Kerr closed string has the Compton radius, and traveling waves along the string create the known effect of zitterbewegung giving rise to the Dirac theory of electron [5].

The singular stringlike source of the KN metric can be regularized by the considered above mechanism of a phase transition, which turns the KN solution into a gravitating soliton model, forming a rotating bubble source covering the former KN singular ring [6, 5]. The interior of the bubble is filled with the Higgs field which regularizes the EM field, expelling it from the bubble.

---

6The field around the Kerr-Sen solution to low energy string theory [13] is similar to the Sen solution for fundamental heterotic string [14].

7The bubble source turns out to be highly oblate because of the relativistic rotation.
The twisted EM field forms a closed Wilson (Bohm-Aharonov) loop along the sharp boundary of the oblate bubble, which interacts with the phase of Higgs field, producing the quantized angular momentum \([6, 5]\). It is remarkable that quantization of the angular momentum appears as a consequence of the semi-classical Higgs mechanism of the broken symmetry.

The pointlike experimental exhibition of the electron may be explained as a result of its relativistic rotation \([4]\). The lightlike twistor null lines of the Kerr congruence are tangent to the Kerr ring forming a heterotic string with the light-like circular current. This string is relativistically sliding along itself and may look as a pointlike particle because of the relativistic contraction. One can show that scattering of this string by the real photons should exhibit a pointlike structure because the real photons (with the lightlike interval \(ds^2 = 0\)) will have only one point of the incidence with the lightlike string. First of all we note, that the lightlike interval, \(s_{12}^2 = 0\), between a position of the EM source \(x_1\) and the point of its incidence with the Kerr string \(x_2\) exists only in equatorial plane of the Kerr spacetime\(^8\). Then, one sees that there exists only one incident point in the equatorial plane. This point is the tangent point to the lightlike direction of the Kerr string.

However, it was argued in \([4, 5]\) that the Compton size of this string may apparently be observed in the experiments with the deeply virtual Compton scattering, which paid great attention recently as a novel regime of the ”nonforward” Compton scattering \([33, 34]\). On the other hand, as it was recently noticed in \([35]\), the relativistic invariance of Kerr’s angular momentum, \(J = ma\), should result in the abrupt shrinking of the Compton size \(a\) for relativistic electrons.

7 Conclusion

The based on twistors KS geometry displays wonderful universality penetrating all the regions of theoretical physics from cosmology and black holes to the structure of elementary particles and twistorial microstructure of spacetime. Twistorial structure of the KS geometry creates new solutions and new effects which remove contradictions between gravity and quantum systems, representing a new perspective to the problems of quantum gravity.

Acknowledgements

I am very thankful to Prof. Ruggero Santilli for invitation to attend this remarkable conference and for financial support. I am also thankful to Prof. Theo Nieuwenhuizen for the thorough reading the manuscript and worth remarks, and also to Prof. Dirk Bouwmeester for invitation to Leiden University where this paper was finished.

\(^8\)It is well known from the analysis of the photon trajectories in Kerr geometry.
Appendix A. Supersymmetry and cosmological constant

Supersymmetry appears in many different contexts in theoretical physics. The recent experiments at LHC rejected supersymmetry in its primitive interpretation. In our opinion, the idea that each particle has its superpartner is incorrect, since it is based on the wrong idea that in a theory with unbroken supersymmetry, for every type of the boson there exists a corresponding fermion with the same mass and internal quantum numbers, and vice-versa. In fact, theory of supersymmetry [36] states it only on the field level for a supermultiplet of the fields. Supersymmetry states only that there should be equal number of the bosonic and fermionic degrees of freedom. On the same grounds we believe that the Higgs boson will never be obtained, since the Higgs field is only one of the fields of supermultiplet - it is not particle, although some kind of the special type of soliton formed from the Higgs field may exist and may be obtained in future. In this respect I consulted Prof. Santilli, and his opinion is that inconsistency of supersymmetry (conflict with unitarity) may be retained on the level of field theory too.\(^9\)

Meanwhile the theory of superfields is based on the Hermitian Lagrangians and is extremely elegant, possessing exclusive effectiveness for description of the models with broken symmetry including the supergravity models.

Relationship of the zero cosmological constant with supersymmetry was demonstrated by Zumino in [37]. The zero point energy is formed of the mutually compensating bosonic and fermion vacuum modes. The contribution is

\[
N \frac{1}{2(2\pi)^3} \int \sqrt{p^2 + m^2} d^3 p,
\]

with \(N\) the number of one-particle states of excitations. It holds that \(N > 0\) for tensor fields and \(N < 0\) for spinor fields.

Each particle is formed by a supermultiplet of the fields, and vacuum energies compensate among the various physical fields of a supermultiplet – not among the particles! In particular, electron is not a pure fermion. It contains the bosonic EM field and the mass creating gravitational fields. In the corresponding field theory it is formed as a soliton built of a supermultiplet of the fields including the electromagnetic field, spinor field, gravity and a system of scalar fields, see sec.6.

\(^9\)The Santilli theory generalizes algebraic grounds of supersymmetry including generalization to irreversible processes.
Appendix B. Field model of phase transition

An elegant analytic description of the phase transition can be obtained in the frame of Domain Wall model based on the supersymmetric system of the chiral fields. The considered in [6] model represents a generalization of the Abelian Higgs model used by Abrikosov, Nielsen and Olesen (ANO) for description of the vortex string in superconducting media [38]. The ANO Lagrangian is

\[ L_{\text{NO}} = \frac{-1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} (D_\mu \Phi)(D^\mu \Phi)^* + V(r). \]  

(17)

For the field model coupled with gravity, the derivatives \( D_\mu \) have to be considered as covariant ones. In [6] the system of chiral scalar fields \( \Phi^i = \{ \Phi, Z, \Sigma \}, i = 1, 2, 3 \) is considered, and the Lagrangian takes the form

\[ L_{\text{matter}} = \frac{-1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{1}{2} \sum_i (D_\mu^{(i)} \Phi^i)(D^{(i)*} \Phi^i) + V, \]  

(18)

where \( F_{\mu\nu} = A_{\mu\nu} - \Lambda_{\nu\mu}, \ B_{\mu\nu} = B_{\mu\nu} - B_{\nu\mu} \). The covariant derivatives are

\[ D_\mu^{(1)} = \nabla_\mu + ie A_\mu, \quad D_\mu^{(3)} = D_\mu^{(3)} = \nabla_\mu + ie B_\mu, \quad D_\mu^{(2)} = \nabla_\mu. \]  

(19)

Adapting the scalar multiplet \( \Phi^i = \{ \Phi, Z, \Sigma \} \) to the bubble source, we consider the first field \( \Phi^1(x) \) as the usual Higgs field, which should vanish outside the bubble and acquire a nonzero vacuum expectation value (vev) inside the bubble, giving a mass to the electromagnetic field \( A^\mu \), which enforce to vanish the EM field inside the bubble. The scalar field \( \Phi^3(x) \) is considered as another (dual) Higgs field which has opposite behavior, taking zero vev inside the bubble and a nonzero vev outside. The related with \( \Phi^3(x) \) gauge field \( B^\mu \) should acquire a mass outside the bubble, which keeps it confined inside the bubble. In the bubble-soliton model [6] the field \( A^\mu \) forms a closed string concentrating near the sharp boarder of the bubble in the equatorial plane. By introduction of the second gauge field \( B_\mu \), this model approaches the Weinberg-Salam theory, and there is expected the appearance of the second ‘electroweak’ closed string related with the field \( B^\mu \). The scalar field \( \Phi^2(x) = Z \) is assumed to be uncharged and used for synchronization of the phase transition. This very special mechanism of the phase transition should be arranged by a special behavior of the potential \( V(x) \), which is provided by the algorithm known from theory of supersymmetry [36].

**Supersymmetric phase transition.** In accord with theory of supersymmetry [36], the potential \( V \) which determines the character of phase transition and the vacuum states are described analytically from a superpotential \( W \), which is analytical function of the chiral fields, \( \Phi^i(x) \). The potential is determined by the relations

\[ V(x) = \sum_i |\partial_i W|^2. \]  

(20)
Vacuum states, which provide minimum of the potential $V$, are determined by the condition $\partial_i W = 0$.

The form of the superpotential used in [6] is
\[ W = Z(\Sigma \bar{\Sigma} - \eta^2) + (cZ + \mu) \Phi \bar{\Phi}. \] (21)

where $c$, $\mu$, $\eta$ are real constants. It generates a domain wall with the described above necessary properties. The supersymmetric potential given by (20) and (21) is non-negative and interpolates between the external vacuum state $V_{(\text{ext})} = 0$ and internal pseudovacuum state $V_{(\text{int})} = 0$. There appear two supersymmetric vacuum states separated by a domain wall: the vacuum state inside the bubble, $V_{(\text{int})} = 0$ for $r < r_0$, with the vev solutions $\Phi^i(x) = \Phi^i_{(\text{int})}$, and the external vacuum state, $V_{(\text{ext})} = 0$ for $r > r_0$ with the vev solutions $\Phi^i = \Phi^i_{(\text{ext})}$.

References


