

Direct Adaptive Tracking Control of Quadrotor Aerial Vehicles

Yannick Morel

Department of Mechanical, Materials and Aerospace
Engineering
University of Central Florida
Orlando, Florida 32816
ymorel@gmail.com

Alexander Leonessa

Department of Mechanical, Materials and Aerospace
Engineering
University of Central Florida
Orlando, Florida 32816
aleo@mail.ucf.com

ABSTRACT

This paper presents a novel adaptive control algorithm solving the trajectory tracking problem for quadrotor aerial vehicles. The control law is derived using a backstepping procedure. A technique derived from dynamic surface control is used to simplify the expression of the obtained control algorithm, with no significant loss in terms of performance. Proof of stability is obtained using Lyapunov stability theory. Results from numerical simulations illustrate the performance of the obtained controller.

1. INTRODUCTION

Numerous research groups have in recent years focused on control of Unmanned Aerial Vehicles (UAVs). This family of vehicles can be divided in two distinct classes. Airplanes can reach considerable speed, allowing them to cover long distances in a relatively short amount of time. These vehicles naturally lend themselves well to mission pertaining to surveillance of extensive areas. They can be distinguished from Vertical Take Off and Landing (VTOL) vehicles, as the latter are not able to reach comparable speeds, but are physically able to hover in the vicinity of an arbitrary entity. They are therefore particularly well suited for missions involving the monitoring of stationary or slow moving targets.

Focussing on the VTOL category, another distinction can be made between classical helicopters, which essentially use a single main rotor for lift in conjunction with an auxiliary tail rotor used to adjust the vehicle's attitude, and quadrotors, which use a combination of four rotors for both lift and attitude. Although quadrotors feature more rotors than classical helicopters, they are of a simpler mechanical design, as the latter requires a tilting mechanism to orient the main rotor to adjust pitch and roll, which is more simply accomplished, for quadrotors, using differential lift on pairs of rotors. This simplified mechanical design however puts the onus on the control system to provide levels of agility and maneuverability comparable to that of classical helicopters.

Controlling the motion of a quadrotor UAV is a chal-

lenging task. The interaction of the air flows generated by the four rotors contribute to complex aerodynamic forces affecting the vehicle's motion. The system's dynamics are not only nonlinear, but also difficult to satisfactorily characterize, due to the complexity of the system's aerodynamic properties.

Accordingly, due to the nonlinear nature of the system, linear control techniques are expected to perform relatively poorly ([1]), as they can not account nor compensate for nonlinear phenomenon affecting the vehicle's dynamics. In [2], a Proportional Integral Derivative (PID) controller and a Linear Quadratic (LQ) controller were implemented and proved capable of regulating the system. However, these designs rely on a linearized version of the vehicle's model. The controllers perform adequately if the vehicle is in a configuration close to that the model was linearized about, but the performance dramatically deteriorates if the vehicle state significantly differs from this desired equilibrium. As a result, these controllers have difficulties in handling perturbations, and seem unlikely to allow for trajectory tracking.

As stated above, a nonlinear controller should be better suited to handle and account for nonlinearities in the vehicle's model. Indeed, a backstepping technique ([3]) yielded a control algorithm that, when implemented on the same system as that considered in [2], outperformed the PID and LQ controllers ([4]). Similarly, in [5, 6], feedback linearization ([7]) and a different backstepping approach were used and allowed to satisfactorily control the vehicle's altitude and yaw.

As mentioned above, due to the presence of intricate aerodynamic forces, obtaining a mathematical model satisfactorily describing a quadrotor dynamical behavior is a difficult task. To handle uncertainties in the model, a robust feedback linearization approach was used in [8]. The obtained control law proved to be robust to parameter uncertainties in simulation.

Classically, model uncertainties are handled using adaptive techniques. In [9], a Single Hidden Layer Neural Network (SHL-NN) is used to compensate for unknown dynamics and allows implementation of the obtained controller with almost no knowledge of the system's dynamics. The ap-

proach was extended in [10], allowing the control algorithm to satisfy control input saturation constraints, and successfully implemented on a physical system. The algorithm is limited to control of the vehicle's pitch.

In this paper, we propose a novel adaptive control algorithm, solving the attitude and altitude tracking control problem for quadrotor aerial vehicles. A direct adaptive control law is derived, using an integrator backstepping approach ([3]). Section 2 details the considered mathematical model. A step by step description of the controller derivation is presented in Section 3. Results of numerical simulations illustrate the performance of the controller.

2. MATHEMATICAL MODEL

2.1 Kinematics

Considering motion in six degrees of freedom, the vehicle's position is described by

$$\eta_1 \triangleq [x_N \ y_E \ z]^T, \quad (1)$$

where (x_N, y_E) denotes the position of the vehicle in a two dimensional frame, the x_N axis is arbitrarily chosen to point North, the y_E axis East, while the z axis is pointing downward. The vector η_1 is expressed in an Earth Fixed Frame (EFF) of arbitrary origin. The vehicle's attitude is described by

$$\eta_2 \triangleq [\phi \ \theta \ \psi]^T, \quad (2)$$

where ϕ, θ and ψ denote the vehicle's roll, pitch, and yaw, respectively. Position and attitude are regrouped in $\eta \triangleq [\eta_1^T \ \eta_2^T]^T$. The body-fixed velocities are introduced with the following notation,

$$v_1 \triangleq [u \ v \ w]^T, \quad (3)$$

$$v_2 \triangleq [p \ q \ r]^T, \quad (4)$$

where u, v and w are the forward, lateral and downward velocities, expressed in the Body Fixed Frame (BFF), while p, q and r are the angular velocities in roll, pitch and yaw, respectively. Linear and angular velocities are regrouped in the velocity vector $v \triangleq [v_1^T \ v_2^T]^T$. The overall state of the vehicle is described by the state vector $x \triangleq [\eta^T \ v^T]^T$.

The time derivatives of (1) and (2) are related to (3) and (4) as follows ([11]),

$$\dot{\eta}_1 = J_1(\eta_2)v_1, \quad (5)$$

$$\dot{\eta}_2 = J_2(\eta_2)v_2, \quad (6)$$

where

$$J_1(\eta_2) \triangleq \begin{bmatrix} c_\theta c_\psi & s_\theta s_\theta c_\psi - c_\phi s_\psi & c_\phi s_\theta c_\psi + s_\phi s_\psi \\ c_\theta s_\psi & s_\theta s_\theta s_\psi + c_\phi c_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi \\ -s_\theta & s_\theta c_\theta & c_\phi c_\theta \end{bmatrix}, \quad (7)$$

$$J_2(\eta_2) \triangleq \begin{bmatrix} 1 & s_\phi t_\theta & c_\phi t_\theta \\ 0 & c_\phi & -s_\phi \\ 0 & s_\phi/c_\theta & c_\phi/c_\theta \end{bmatrix}, \quad (8)$$

and s_ζ, c_ζ , and t_ζ denote the sine, cosine and tangent of angle ζ . Equations (5) and (6) constitute the kinematic equations of the considered system.

2.2 Dynamics

The equations characterizing the dynamic behavior of the system are of the form ([11])

$$\dot{v}_1(t) = \frac{1}{m}C_1(v_2(t))v_1(t) + f_g(\phi(t), \theta(t)) + \frac{1}{m} [0 \ 0 \ f_z(t)]^T, \quad (9)$$

$$v_1(0) = v_{10}, \ t \geq 0,$$

$$\dot{v}_2(t) = I^{-1}C_2(v_2(t))v_2(t) + I^{-1}\tau(t), \quad v_2(0) = v_{20}, \quad (10)$$

where m is the mass of the vehicle,

$$I \triangleq \begin{bmatrix} i_x & 0 & 0 \\ 0 & i_y & 0 \\ 0 & 0 & i_z \end{bmatrix}, \quad (11)$$

is the rotational inertia matrix,

$$C_1(\eta_2) \triangleq \begin{bmatrix} 0 & r & -q \\ -r & 0 & p \\ q & -p & 0 \end{bmatrix}, \quad C_2(\eta_2) \triangleq \begin{bmatrix} 0 & -i_z r & i_y q \\ i_z r & 0 & -i_x p \\ -i_y q & i_x p & 0 \end{bmatrix}, \quad (12)$$

are Coriolis matrices, $f_g(\phi, \theta) \triangleq g [-s_\theta \ c_\theta s_\phi \ c_\theta c_\phi]^T$ is the vector of restoring forces, g is the gravity acceleration, F_z is the thrust provided by the propulsion system, and τ regroups the torques and moments generated by the propellers.

We augment the dynamical system (9)-(10) with the following equation, which models the delay between a change in a Pulse Width Modulation (PWM) signal provided to the motors and the time at which it affects the remaining dynamics of the vehicle,

$$\dot{\tilde{\tau}}(t) = T^{-1}(-\tilde{\tau}(t) + g_m^2 \tilde{B} \Lambda^{-1} p_2(t)), \quad \tilde{\tau}(0) = \tilde{\tau}_0, \ t \geq 0, \quad (13)$$

where $T \in \mathbb{R}^{4 \times 4}$ is a diagonal matrix of time constants, $T >$

$0, \tilde{\tau} \triangleq [f_z \ \tau^T]^T, g_m \in \mathbb{R},$

$$\tilde{B} \triangleq \begin{bmatrix} k & 0 & 0 & 0 \\ 0 & lk & 0 & 0 \\ 0 & 0 & lk & 0 \\ 0 & 0 & 0 & d \end{bmatrix}, \quad \Lambda \triangleq \frac{1}{4} \begin{bmatrix} 1 & 0 & 2 & 1 \\ 1 & -2 & 0 & -1 \\ 1 & 0 & -2 & 1 \\ 1 & 2 & 0 & -1 \end{bmatrix}, \quad (14)$$

where k, l and $d \in \mathbb{R}$, and, finally, $p_2 \in \mathbb{R}^4$ regroups the squares of the PWM signals provided to the four motors.

3. DIRECT ADAPTIVE CONTROL ALGORITHM

In this Section, we will introduce the considered tracking errors. Then, following an approach similar to that presented in [12], we will derive a nonlinear control law which solves the considered tracking problem. Note that the vehicle does not directly track a desired trajectory, but that of a reference system corresponding to a virtual vehicle. This reference system, composed of third order oscillators ([13]), will track the desired trajectory. The state of this reference system is noted $x_r \in \mathbb{R}^{12}$.

3.1 Tracking Error

We define $\eta_s \triangleq [\eta_2^T \ k_{sz}z]^T$, where $k_{sz} \in \mathbb{R}$ is a scaling constant. The position of the reference system is given by $x_{r1} \triangleq [x_{ra}^T \ k_{sz}x_{rz}]^T$, where $x_{ra} \in \mathbb{R}^3$ corresponds to three reference angles, and $x_{rz} \in \mathbb{R}$ is a reference altitude. The considered position error is of the form

$$e_p(x_{r1}, \eta_s) \triangleq x_{r1} - \eta_s. \quad (15)$$

3.2 First Lyapunov Function Candidate

Consider the following function of e_p ,

$$V_1(e_p) \triangleq \frac{1}{2} e_p^T e_p, \quad (16)$$

note that $V_1(e_p) \in C^1$ for all $e_p \in \mathbb{R}^4$, $V_1(e_p) > 0$ for all $e_p \in \mathbb{R}^4 \setminus \{0\}$, and $V_1(0) = 0$. The time derivative of $V_1(e_p)$ is given by

$$\dot{V}_1(x_r, x) = e_p^T \left(x_{r2} - J_3(\eta_2) v_s + k_{sz} [0_{1 \times 3} \ u s_\theta - v s_\phi c_\theta]^T \right), \quad (17)$$

where $x_{r2} \triangleq \dot{x}_{r1}$, $v_s \triangleq [v_2^T \ k_{rz} w]^T$, $0_{i \times j}$ is the $i \times j$ dimensional zero matrix, and

$$J_3(\eta_2) \triangleq \begin{bmatrix} J_2(\eta_2) & 0_{3 \times 1} \\ 0_{1 \times 3} & c_\phi c_\theta \end{bmatrix}. \quad (18)$$

Now consider the following filter,

$$T_1 \dot{\chi}_1(t) + \chi_1(t) = J_3^{-1}(\eta_2(t)) \left(k_{sz} \begin{bmatrix} 0_{3 \times 1} \\ u(t) s_\theta(t) - v(t) s_\phi(t) c_\theta(t) \end{bmatrix}^T + G_1 e_p(t) \right) + \frac{1}{2} J_3^T(\eta_2(t)) e_p(t), \quad t \geq 0, \quad (19)$$

where $G_1 \in \mathbb{R}^{4 \times 4}$, $G_1 > 0$, with initial condition $\chi_1(0) = \chi_{10}$ and associated filtering error

$$\alpha_1(x_{r1}, x, \chi_1) \triangleq J_3^{-1}(\eta_2) \left(k_{sz} \begin{bmatrix} 0_{3 \times 1} \\ u s_\theta - v s_\phi c_\theta \end{bmatrix}^T + G_1 e_p(x_{r1}, \eta_s) \right) + \frac{1}{2} J_3^T(\eta_2) e_p(x_{r1}, \eta_s) - \chi_1, \quad (20)$$

and define the velocity error

$$e_v(x_{r2}, x, \chi_1) \triangleq J_3^{-1}(\eta_2) x_{r2} + \chi_1 - v_s. \quad (21)$$

Using (20) and (21), we can rewrite (17) as

$$\dot{V}_1(e_p, e_v, \eta_2, \alpha_1) = -e_p^T G_1 e_p + e_p J_3(\eta_2) \left(\alpha_1 - \frac{1}{2} J_3^T(\eta_2) e_p \right) + e_v^T J_3^T(\eta_2) e_p, \quad (22)$$

the first term of which is negative quadratic, the second is due to the filtering error, and third is due to the velocity error.

3.3 Second Lyapunov Function Candidate

Consider the following function of e_p and e_v ,

$$V_2(e_p, e_v) \triangleq \frac{1}{2} e_p^T e_p + \frac{1}{2} e_v^T e_v, \quad (23)$$

which is C^1 for all $e_p, e_v \in \mathbb{R}^4$, $V_2(e_p, e_v) > 0$ for all $e_p, e_v \in \mathbb{R}^4 \setminus \{0\}$, and $V_2(0, 0) = 0$. The time derivative of $V_2(e_p, e_v)$ is given by

$$\dot{V}_2(e_{pv}, x_r, x, \alpha_1, \tilde{\tau}) = -e_p^T G_1 e_p + e_p J_3(\eta_2) \left(\alpha_1 - J_3^T(\eta_2) \frac{e_p}{2} \right) + e_v^T \left(J_3^T(\eta_2) e_p + \dot{e}_v(x_r, x, \alpha_1, \tilde{\tau}) \right), \quad (24)$$

where $e_{pv} \triangleq [e_p^T \ e_v^T]^T$, and

$$\dot{e}_v(x_r, x, \tilde{\tau}, \alpha_1) = M^{-1} \Theta_1^* f_{e_v}(x_r, x, \alpha_1) + J_3^{-1}(\eta_2) x_{r3} - M^{-1} \hat{B} \tilde{\tau}, \quad (25)$$

with

$$f_{e_v}(x_r, x, \alpha_1) \triangleq \begin{bmatrix} [qr \ pr \ pq \ pv - qu]^T \\ k_{sz}g [0_{1 \times 3} \ -c_\phi c_\theta]^T + \left(\frac{d}{dt}J_3^{-1}(\eta_2)\right)x_{r2} + T_1^{-1}\alpha_1 \end{bmatrix}, \quad (26)$$

$$M \triangleq \begin{bmatrix} I & 0_{3 \times 1} \\ 0_{1 \times 3} & \frac{m}{k_{sz}} \end{bmatrix}, \quad \Theta_1^* \triangleq [\text{diag}[i_z - i_y \ i_x - i_z \ i_y - i_x \ 1] \ M]. \quad (27)$$

We now define a second filter as follows,

$$T_2\dot{\chi}_2(t) + \chi_2(t) = f_{e_v}(t) + \begin{bmatrix} 0_{4 \times 1} \\ J_3^{-1}(\eta_2(t))e_p(t) + G_2e_v(t) \end{bmatrix},$$

$$\chi_2(0) = \chi_{20}, \quad t \geq 0, \quad (28)$$

where $G_2 \in \mathbb{R}^{4 \times 4}$, $G_2 > 0$, with associated filtering error

$$\alpha_2(e_{pv}, x_r, x, \chi_1) \triangleq f_{e_v}(x_r, x, \alpha_1) + \begin{bmatrix} 0_{4 \times 1} \\ J_3^{-1}(\eta_2)e_p + G_2e_v \end{bmatrix} - \chi_2, \quad (29)$$

and define the new tracking error

$$e_\tau(x_r, x, \chi_2, \tilde{\tau}, \Theta_1) \triangleq \hat{B}^T \Theta_1 \phi_1(x_{r3}, \eta_2, \chi_2) - \tilde{\tau}, \quad (30)$$

where Θ_1 is an estimate of Θ_1^* and

$$\phi_1(x_{r3}, \eta_2, \chi_2) \triangleq \left(\chi_2 + \begin{bmatrix} 0_{4 \times 1} \\ J_3^{-1}(\eta_2)x_{r3} \end{bmatrix} \right). \quad (31)$$

Using (29) and (30), we can rewrite (25) as

$$\dot{e}_v(e_{pv\tau}, x_r, x, \chi_2, \alpha_2, \tilde{\Theta}_1) = -J_3^T(\eta_2)e_p - G_2e_v + M^{-1}(\Theta_1^*\alpha_2 - \tilde{\Theta}_1\phi_1(x_{r3}, \eta_2, \chi_2) + \hat{B}e_\tau), \quad (32)$$

where $\tilde{\Theta}_1 \triangleq \Theta_1 - \Theta_1^*$, and $e_{pv\tau} \triangleq [e_{pv}^T \ e_\tau^T]^T$. Combining (32) and (24), we obtain

$$\dot{V}_2(e_{pv\tau}, x_r, x, \alpha, \tilde{\Theta}_1) = -e_v M^{-1} \tilde{\Theta}_1 \phi_1(x_{r3}, \eta_2, \chi_2) + e_\tau^T \hat{B}^T M^{-1} e_v - e_{pv}^T G_{12} e_{pv} + e_{pv}^T P(\eta_2) \alpha - \frac{1}{2} e_p^T J_3(\eta_2) J_3^T(\eta_2) e_p, \quad (33)$$

where

$$G_{12} \triangleq \begin{bmatrix} G_1 & 0_{4 \times 4} \\ 0_{4 \times 4} & G_2 \end{bmatrix}, \quad P(\eta_2) \triangleq \begin{bmatrix} J_3(\eta_2) & 0_{4 \times 8} \\ 0_{4 \times 4} & M^{-1} \Theta_1^* \end{bmatrix}. \quad (34)$$

The first term in (33) is due to the estimation error $\tilde{\Theta}_1$, the second one to the tracking error e_τ , the third term is negative quadratic, the fourth and fifth terms are related to filtering errors.

3.4 Third Lyapunov Function Candidate

Consider the following function of e_p , e_v and $\tilde{\Theta}_1$,

$$V_3(e_{pv}, \tilde{\Theta}_1) \triangleq \frac{1}{2} e_{pv}^T e_{pv} + \frac{1}{2} \text{tr}(M^{-1} \tilde{\Theta}_1 \Gamma_1^{-1} \tilde{\Theta}_1^T), \quad (35)$$

where $\Gamma_1 \in \mathbb{R}^{8 \times 8}$, $\Gamma_1 > 0$. Note that $V_3(e_{pv}, \tilde{\Theta}_1)$ is C^1 for all $e_{pv} \in \mathbb{R}^8$ and $\tilde{\Theta}_1 \in \mathbb{R}^{4 \times 8}$, $V_3(e_{pv}, \tilde{\Theta}_1) > 0$ for all $(e_{pv}, \tilde{\Theta}_1) \in \mathbb{R}^8 \cup \mathbb{R}^{4 \times 8} \setminus \{0\}$, and $V_3(0, 0) = 0$. The time derivative of $V_3(e_{pv}, \tilde{\Theta}_1)$ is given by

$$\dot{V}_3(e_{pv\tau}, x_r, x, \alpha, \tilde{\Theta}_1, t) = -e_{pv}^T G_{12} e_{pv} + e_\tau^T \hat{B}^T M^{-1} e_v + e_{pv}^T P(\eta_2) \alpha - \frac{1}{2} e_p^T J_3(\eta_2) J_3^T(\eta_2) e_p - \text{tr}(M^{-1} \tilde{\Theta}_1 (\phi_1(x_{r3}, \eta_2, \chi_2) e_v - \Gamma_1^{-1} \dot{\tilde{\Theta}}_1(t)^T)). \quad (36)$$

Choosing

$$\dot{\Theta}_1(t) = e_v(t) \phi_1^T(t) \Gamma_1 - \sigma_1 \|e_v(t)\|^2 \Theta_1(t),$$

$$\Theta_1(0) = \Theta_{10}, \quad t \geq 0, \quad (37)$$

where $\sigma_1 \in \mathbb{R}$, $\sigma_1 > 0$, we obtain

$$\dot{V}_3(e_{pv\tau}, x_r, x, \alpha, \tilde{\Theta}_1) = -e_{pv}^T G_{12} e_{pv} + e_\tau^T \hat{B}^T M^{-1} e_v + e_{pv}^T P(\eta_2) \alpha - \frac{1}{2} e_p^T J_3(\eta_2) J_3^T(\eta_2) e_p - \sigma_1 \|e_v\|^2 \text{tr}(M^{-1} \tilde{\Theta}_1 \Gamma_1^{-1} \tilde{\Theta}_1^T). \quad (38)$$

3.5 Fourth Lyapunov Function Candidate

Consider the following function of $e_{pv\tau}$ and $\tilde{\Theta}_1$,

$$V_4(e_{pv\tau}, \tilde{\Theta}_1) \triangleq \frac{1}{2} e_{pv\tau}^T e_{pv\tau} + \frac{1}{2} \text{tr}(M^{-1} \tilde{\Theta}_1 \Gamma_1^{-1} \tilde{\Theta}_1^T), \quad (39)$$

which is C^1 for all $e_{pv\tau} \in \mathbb{R}^{12}$ and $\tilde{\Theta}_1 \in \mathbb{R}^{4 \times 8}$, $V_4(e_{pv\tau}, \tilde{\Theta}_1) > 0$ for all $(e_{pv\tau}, \tilde{\Theta}_1) \in \mathbb{R}^{12} \cup \mathbb{R}^{4 \times 8} \setminus \{0\}$, and $V_4(0, 0) = 0$. The time derivative of $V_4(e_{pv\tau}, \tilde{\Theta}_1)$ is given by

$$\dot{V}_4(e_{pv\tau}, x_r, x, \alpha, \tilde{\Theta}_1, t) = -e_{pv}^T G_{12} e_{pv} + e_{pv}^T P(\eta_2) \alpha - \frac{1}{2} e_p^T J_3(\eta_2) J_3^T(\eta_2) e_p - \sigma_1 \|e_v\|^2 \text{tr}(M^{-1} \tilde{\Theta}_1 \Gamma_1^{-1} \tilde{\Theta}_1^T) + e_\tau^T (\hat{B}^T M^{-1} e_v + \dot{e}_\tau(t)), \quad (40)$$

where

$$\begin{aligned} \dot{e}_\tau(x_r, x, \Theta_1, \chi_2, \alpha_2, \hat{r}, \tilde{\tau}, p_2) = & \tilde{B}^T (\dot{\Theta}_1(x_r, x, \Theta_1) \varphi_1(x_{r3}, \eta_2, \chi_2) \\ & + \Theta_1 T_2^{-1} \alpha_2 + \Theta_{12} \left(\frac{d}{dt} J_3^{-1}(\eta_2) \right) x_{r3} \\ & + \Theta_{12} J_3^{-1}(\eta_2) \dot{x}_{r3}(x_r, \hat{r})) + T^{-1} \tilde{\tau} - \bar{B} \Lambda^{-1} p_2, \end{aligned} \quad (41)$$

where Θ_{12} is the $(1, 4) \times (5, 8)$ block of Θ_1 . Now consider the control command

$$p_2(t) \triangleq \Lambda \Theta_2(t) \varphi_2(t), \quad t \geq 0, \quad (42)$$

where $\Theta_2 \in \mathbb{R}^{4 \times 12}$ is an estimate of $\Theta_2^* \triangleq \frac{1}{2} \tilde{B}^{-1} T [T^{-1} \tilde{B}^T M^{-1} I_4]$, I_j is the j^{th} dimensional identity matrix, and

$$\begin{aligned} \varphi_2(t) \triangleq & [\tilde{\tau}^T(t) \ e_v^T(t) \ \tilde{\varphi}_2^T(t)]^T, \quad t \geq 0, \quad (43) \\ \tilde{\varphi}_2(x_r, x, \Theta_1, \chi_2, \alpha_2, \hat{r}, e_\tau) \triangleq & \tilde{B}^T \left(\dot{\Theta}_1(x_r, x, \Theta_1) \varphi_1(x_{r3}, \eta_2, \chi_2) \right. \\ & + \Theta_1 T_2^{-1} \alpha_2 + \Theta_{12} \left(\frac{d}{dt} J_3^{-1}(\eta_2) \right) x_{r3} \\ & \left. + \Theta_{12} J_3^{-1}(\eta_2) \dot{x}_{r3}(x_r, \hat{r}) \right) + G_3 e_\tau, \end{aligned} \quad (44)$$

Combining (41) and (42), we obtain

$$\dot{e}_\tau(t) = -\tilde{B}^T M^{-1} e_v(t) - G_3 e_\tau(t) - \bar{B} \tilde{\Theta}_2(t) \varphi_2(t), \quad (45)$$

where $\tilde{\Theta}_2 = \Theta_2 - \Theta_2^*$, and $\bar{B} = g_m^2 T^{-1} \tilde{B}$. Hence,

$$\begin{aligned} \dot{V}_4(e_{pvt}, x_r, x, \alpha, \tilde{\Theta}, t) = & -e_{pvt}^T G_{123} e_{pvt} + e_{pv}^T P(\eta_2) \alpha \\ & - \frac{1}{2} e_p^T J_3(\eta_2) J_3^T(\eta_2) e_p - \sigma_1 \|e_v\|^2 \text{tr}(M^{-1} \tilde{\Theta}_1 \Gamma_1^{-1} \Theta_1^T) \\ & - e_\tau^T \bar{B} \tilde{\Theta}_2 \varphi_2(t), \end{aligned} \quad (46)$$

where $\tilde{\Theta} \triangleq [\tilde{\Theta}_1 \ \tilde{\Theta}_2]$, and $G_{123} \triangleq \text{diag}[G_1 \ G_2 \ G_3]$.

3.6 Fifth Lyapunov Function Candidate

Consider the following function of e_{pvt} and $\tilde{\Theta}$,

$$V_5(e_{pvt}, \tilde{\Theta}) \triangleq \frac{1}{2} e_{pvt}^T e_{pvt} + \frac{1}{2} \sum_{i=1}^2 \text{tr}(M^{-1} \tilde{\Theta}_i \Gamma_i^{-1} \tilde{\Theta}_i^T), \quad (47)$$

which is \mathcal{C}^1 for all $e_{pvt} \in \mathbb{R}^{12}$ and $\tilde{\Theta} \in \mathbb{R}^{4 \times 20}$, $V_5(e_{pvt}, \tilde{\Theta}) > 0$ for all $(e_{pvt}, \tilde{\Theta}) \in \mathbb{R}^{12} \cup \mathbb{R}^{4 \times 20} \setminus \{0\}$, and $V_5(0, 0) = 0$. The

time derivative of $V_5(e_{pvt}, \tilde{\Theta})$ is given by

$$\begin{aligned} \dot{V}_5(e_{pvt}, x_r, x, \alpha, \tilde{\Theta}, t) = & -e_{pvt}^T G_{123} e_{pvt} + e_{pv}^T P(\eta_2) \alpha \\ & - \frac{1}{2} e_p^T J_3(\eta_2) J_3^T(\eta_2) e_p - \sigma_1 \|e_v\|^2 \text{tr}(M^{-1} \tilde{\Theta}_1 \Gamma_1^{-1} \Theta_1^T) \\ & - \text{tr}(\bar{B} \tilde{\Theta}_2 (\varphi_2(t) e_\tau^T - \Gamma_2^{-1} \dot{\Theta}_2(t)^T)). \end{aligned} \quad (48)$$

Choosing

$$\begin{aligned} \dot{\Theta}_2(t) = & e_\tau(t) \varphi_2^T(t) \Gamma_2 - \sigma_2 \|e_\tau(t)\|^2 \Theta_2(t), \\ \Theta_2(0) = & \Theta_{20}, \quad t \geq 0, \end{aligned} \quad (49)$$

where $\sigma_2 \in \mathbb{R}$, $\sigma_2 > 0$, we obtain

$$\begin{aligned} \dot{V}_5(e_{pvt}, x_r, x, \alpha, \tilde{\Theta}) = & -e_{pvt}^T G_{123} e_{pvt} + e_{pv}^T P(\eta_2) \alpha \\ & - \frac{1}{2} e_p^T J_3(\eta_2) J_3^T(\eta_2) e_p - \sigma_1 \|e_v\|^2 \text{tr}(M^{-1} \tilde{\Theta}_1 \Gamma_1^{-1} \Theta_1^T) \\ & - \sigma_2 \|e_\tau\|^2 \text{tr}(\bar{B} \tilde{\Theta}_2 \Gamma_2^{-1} \Theta_2^T). \end{aligned} \quad (50)$$

Note that, by completion of the square,

$$\begin{aligned} e_p^T J_3(\eta_2) \alpha_1 = & -\frac{1}{2} (J_3^T(\eta_2) e_p - \alpha_1)^T (J_3^T(\eta_2) e_p - \alpha_1) \\ & + \frac{1}{2} e_p^T J_3(\eta_2) J_3^T(\eta_2) e_p + \frac{1}{2} \|\alpha_1\|^2. \end{aligned} \quad (51)$$

In addition, for $\|T_i\|$ sufficiently small, $\|\alpha_i(t)\| < \bar{\alpha}_i$, $t \geq 0$, $i = 1, 2$, and we obtain the following upper bound on \dot{V}_5 ,

$$\begin{aligned} \dot{V}_5(e_{pvt}, x_r, x, \alpha, \tilde{\Theta}) \leq & -e_{pvt}^T G_{123} e_{pvt} + \frac{1}{2} \bar{\alpha}_1^2 + \|e_v\| M^{-1} \Theta_1^* \bar{\alpha}_2 \\ & + \frac{\sigma_1}{2} \|e_v\|^2 (\text{tr}(M^{-1} \Theta_1^* \Gamma_1^{-1} \Theta_1^{*T}) - \text{tr}(M^{-1} \tilde{\Theta}_1 \Gamma_1^{-1} \tilde{\Theta}_1^T)) \\ & + \frac{\sigma_2}{2} \|e_\tau\|^2 (\text{tr}(\bar{B} \Theta_2^* \Gamma_2^{-1} \Theta_2^{*T}) - \text{tr}(\bar{B} \tilde{\Theta}_2 \Gamma_2^{-1} \tilde{\Theta}_2^T)), \end{aligned} \quad (52)$$

Which guarantees convergence of $(e_{pvt}, \tilde{\Theta})$ ([14]) to the compact set

$$\begin{aligned} \mathcal{M} \triangleq \{ & (e_{pvt}, \tilde{\Theta}) : e_{pvt}^T G_{123} e_{pvt} \leq \frac{1}{2} \bar{\alpha}_1^2 + \|e_v\| M^{-1} \Theta_1^* \bar{\alpha}_2 \\ & \text{tr}(M^{-1} \tilde{\Theta}_1 \Gamma_1^{-1} \tilde{\Theta}_1^T) \leq \text{tr}(M^{-1} \Theta_1^* \Gamma_1^{-1} \Theta_1^{*T}) \\ & \text{tr}(\bar{B} \tilde{\Theta}_2 \Gamma_2^{-1} \tilde{\Theta}_2^T) \leq \text{tr}(\bar{B} \Theta_2^* \Gamma_2^{-1} \Theta_2^{*T}) \}. \end{aligned} \quad (53)$$

4. NUMERICAL SIMULATION

The control law (42) presented in Section 3 was tested through numerical simulation. The reference system used is

of the form

$$\begin{bmatrix} \dot{x}_{r1}(t) \\ \dot{x}_{r2}(t) \\ \dot{x}_{r3}(t) \end{bmatrix} = \begin{bmatrix} 0_{4 \times 4} & I_4 & 0_{4 \times 4} \\ 0_{4 \times 4} & 0_{4 \times 4} & I_4 \\ -\omega_0^2 & -\omega_0^2 & -A_{r4} \end{bmatrix} \begin{bmatrix} x_{r1}(t) \\ x_{r2}(t) \\ x_{r3}(t) \end{bmatrix} + \begin{bmatrix} 0_{4 \times 4} \\ 0_{4 \times 4} \\ \omega_0^2 \end{bmatrix} \hat{r}(t),$$

$$x_r(0) = x_{r0}, \quad t \geq 0, \quad (54)$$

where $\omega_0 = 10.2 I_4$, and $A_{r4} = 38.8 I_4$. In addition, the following values were used in the vehicle's model, $m = 12$, $i_x = 20$, $i_y = 10$, $i_z = 1$, $g_m = 10$, $l = 0.25$, $k = 2$, $d = 0.1$, $g = 9.81$, $T = 0.01 I_4$. The gain were arbitrarily chosen to be $G_1 = G_2 = G_3 = 10 I_4$, and the learning coefficients $\Gamma_1 = I_{12}$, $\Gamma_2 = I_{12}$. The initial conditions for the presented simulation are as follows, $\eta_{10} = [0 \ 0 \ 10]^T$, $\eta_{20} = \frac{\pi}{8} [1 \ 1 \ 1]^T$, $v = 0_{6 \times 1}$, $x_{r0} = \frac{\pi}{16} [1 \ 1 \ 1 \ 0_{1 \times 9}]^T$, and the desired trajectory that x_{r1} is tracking is of the form $x_d(t) = [a \sin(bt) - a \sin(bt) \ \frac{a}{2} \sin(bt) - 10]^T$, $t \geq 0$, with $a = \frac{\pi}{12}$, $b = \frac{2\pi}{10}$. This reference trajectory has the vehicle elevating in altitude while its orientation is oscillating.

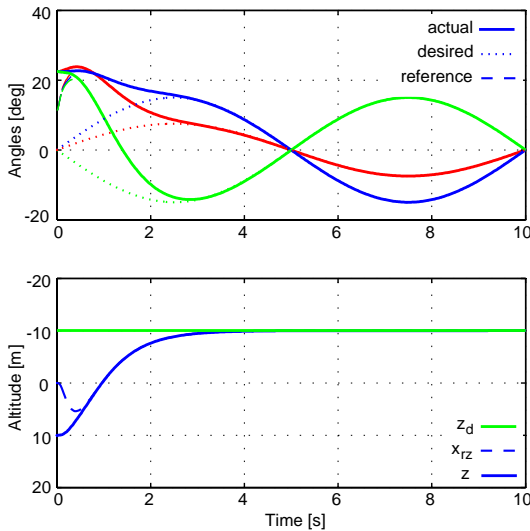


Figure 1. Orientation and altitude.

As seen on Figure 1 (top), the actual roll (in blue), pitch (green) and yaw (red) converge rapidly and smoothly to their corresponding desired trajectory. Similarly, the altitude converges to its desired value (bottom).

5. CONCLUSION

We presented a novel direct adaptive controller solving the tracking control problem for quadrotor UAVs. A step by step description of the algorithm derivation was provided, and the obtained control law was tested through numerical simulation. Future improvement on the presented result include the addition of a SHL-NN allowing for increased ro-

bustness to unmodelled or mismodelled dynamics, and compensation for external perturbations such as winds.

REFERENCES

- [1] Waslander, S. L., Hoffmann, G. M., Jang, J. S., and Tomlin, C. J., 2005. "Multi-agent quadrotor testbed control design: Integral and sliding mode vs. reinforcement learning". In Proc. 2005 IEEE/RSJ Int. Conf. Intelligent Robots and Systems, pp. 3712–3717.
- [2] Bouadballah, S., Noth, A., and Siegwart, R., 2004. "PID vs LQ control techniques applied to an indoor micro quadrotor". In Proc. 2004 IEEE/RSJ Int. Conf. Intelligent Robots and Systems, pp. 2451–2456.
- [3] Krstić, M., Kanellakopoulos, I., and Kokotović, P., 1995. *Nonlinear and Adaptive Control Design*. John Wiley and Sons, Inc, New York, NY.
- [4] Bouadballah, S., and Siegwart, R., 2005. "Backstepping and sliding-mode techniques applied to an indoor micro quadrotor". In Proc. 2005 IEEE Int. Conf. on Robotics and Automation, pp. 2247–2252.
- [5] Altug, E., Ostrowski, J. P., and Mahony, R., 2002. "Control of a quadrotor helicopter using visual feedback". In Proc. 2002 IEEE Int. Conf. on Robotics and Automation, pp. 72–77.
- [6] Altug, E., Ostrowski, J. P., and Taylor, C. J., 2003. "Quadrotor control using dual camera visual feedback". In Proc. 2003 IEEE Int. Conf. on Robotics and Automation, pp. 4294–4299.
- [7] Isidori, A., 1995. *Nonlinear Control Systems*. Springer-Verlag, New York, NY.
- [8] Mokhtari, A., Benallegue, A., and Daachi, B., 2005. "Robust feedback linearization and GH_∞ controller for a quadrotor unmanned aerial vehicle". In Proc. 2005 IEEE/RSJ Int. Conf. Intelligent Robots and Systems, pp. 1198–1203.
- [9] Hovakimyan, N., Nardy, F., Calise, A., and Nakwan, K., 2002. "Adaptive output feedback control of uncertain nonlinear systems using single-hidden-layer neural networks". *IEEE Trans. on Neural Networks*, **13**, pp. 1420–1431.
- [10] Kutay, A. T., Calise, A., and Hovakimyan, N., 2005. "Experimental results on adaptive output feedback control using a laboratory model helicopter". *IEEE Trans. on Control Systems Technology*, **13**, pp. 196–202.
- [11] Nelson, R. C., 1997. *Flight Stability and Automatic Control*. McGraw-Hill.
- [12] Leonessa, A., VanZwieten, T. S., and Morel, Y., 2004. "Neural network model reference adaptive control of marine vehicles". *Current Trends in Nonlinear Systems and Control*.
- [13] Morel, Y., 2002. "Design of an adaptive nonlinear controller for an autonomous underwater vehicle equipped with a vectored thruster". *Master Thesis, Florida Atlantic University*.
- [14] LaSalle, J., and Lefschets, S., 1961. *Stability by Liapunov's Direct Method with Applications*. Academic Press.