Abstract—When energy is harnessed by wireless nodes from renewable sources, its availability becomes uncertain and its use for communications must be carefully designed. While the optimal power allocation has been derived in previous works when energy availability is fully (a priori) known, practical algorithms are needed when only causal and statistical information is available. In this paper, we study the optimal transmission policy when only the statistical distribution of the energy arrival intervals is known and no information is available on the amount of energy that will be harvested. We first obtain an exact solution for the case of a step-wise transmission power profile. This result is then extended to the time-continuous case. Within energy arrival intervals, the obtained power profile is shown to be non increasing as a function of time and non decreasing as a function of the residual energy. Numerical results are finally provided focusing on an exponentially distributed energy arrival process as a case study.

I. INTRODUCTION

Recent technical advances are making energy harvesting systems a reality [1], [2]. Harnessing solar, mechanical, electromagnetic, or thermal energy and converting it into an electrical form, can dramatically increase the life time of a variety of devices, such as sensor nodes and, more in general, energy constrained appliances, even increasing transmission range, sensing reliability, etc. [3]–[6]. With reference, in particular, to wireless sensors, an increasing interest has arisen in transmission policies aimed at optimizing the energy utilization and management [7], [8].

Transmit-power allocation strategies for energy harvesting systems are classified into offline and online policies. Offline policies require that the system has a full (non causal) knowledge of the energy arrival instants as well as of the amount of energy harvested. Given this a priori information, the optimal time profile of the transmit-power was shown in [9] to be a non decreasing step-wise function. Theoretical results on the transmit power profile are also shown in [10], where the objective was to minimize the time needed to send a given number of bits.

The main limitation of offline policies is the need of non causal information, which is not available in the most of cases. For this reason, online policies are generally used as performance benchmarks.

Online policies are designed, on the contrary, considering only causal information and some statistical description of energy arrival instants and amounts. Under these hypotheses, the optimal transmit-power allocation is studied, for instance, in [11] assuming that the amount of harvested energy is modeled as a set of independent, identically distributed (i.i.d.) random variables (r.v.s) for a discrete-time energy arrival model. In [11], it is shown that the power allocation strategy that maximizes the overall throughput is a non-decreasing function of the residual energy available at a given time instant. Online policies for a discrete-time energy arrival model with an additive white Gaussian noise (AWGN) channel have been also proposed in [9] and [12], and two low-complexity approaches, save-and-transmit and best-effort-transmit, both requiring the knowledge of the (statistical) average value of the harvested energy, have been proposed.

Limits on the achievable rate in the presence of fading and energy storage inefficiencies are studied in [13] with reference to a discrete-time energy arrival model with knowledge of the average amount of harvested energy. In this case it is shown that the optimal power allocation follows the conventional water-filling strategy.

Online transmission policies aimed at maximizing the number of bits transmitted before a given deadline have been studied in [14], under the assumption that the energy arrivals can be modeled as a compound Poisson process and considering a fading channel. The optimal transmission policy is derived following an approach based on dynamic programming. However, owing to the excessive complexity of the algorithm, some other sub-optimal but simpler online policies, based on water-filling, are discussed. For all policies the power level has a stepwise behavior, with transitions in correspondence of fading and energy arrival events.

The optimal power allocation for an energy harvesting system when the number of energy arrivals follow a Poisson process and the amount of energy is modeled as an exponential r.v. is studied in [15]. However, the derivation of the optimal strategy requires the evaluation of a non-linear ordinary differential equation.

The case of no information available about the energy arrival events (neither in terms of time intervals nor with reference to the amount of harvested energy) is developed in [16] by using competitive ratio analysis. In [16], an algorithm is proposed to minimize the time spent to transmit a given number of bits.
where having no knowledge on future arrivals, the algorithm sets a power level that depends only on \( B \), and keeps it constant until all bits are delivered or a new event occurs.

Motivated by the previous works, that in most cases are too optimistic (perfect knowledge of “energy-events” statistic), too pessimistic (no knowledge at all) or too complex for low cost applications, the aim of this paper is to design low-complexity online transmit power policies under the realistic hypothesis of partial statistical knowledge of energy arrival events. More specifically, we assume that transmitters have a causal knowledge of the stored energy and a statistical description of the energy arrival time instants, but no information (not even statistical) about the amount of energy available in the subsequent time instants. Under such assumptions, instead of addressing globally optimal transmit-power profiles, we focus on locally optimal policies whose decision on the power allocation is taken after an energy arrival and is driven by the amount of available energy and the statistic of energy arrival instants. We first find the transmit power profile among the class of step-wise functions and then we extend the result to any shape. The resulting profile, which is shown non-increasing in time between two consecutive energy arrivals, will be discussed and compared to an optimal ideal allocation.

Throughout the paper, the functions \( f_X(x) \) and \( F_X(x) \) denote the probability density function (PDF) and the cumulative distribution function (CDF) of the r.v. \( X \), respectively, and \( \mathbb{E}_X \{ \cdot \} \) denotes the expectation operator with respect to the random variable \( X \). We will use bold notation for vectors, so that \( x \in \mathbb{R}^{m \times 1} \) denotes a vector of \( m \) real elements. We will use \( ^T \) to denote the transposition operator, and \((x)^+ \) to denote \( \max\{x, 0\} \).

II. SYSTEM MODEL

We consider a node with energy harvesting capabilities and assume that the amount of energy, \( Y_j \), with \( j = 0, 1, 2, \ldots \), is harvested by the node at the random time instants \( \tau_j \), with \( \tau_n < \tau_m \) for \( n < m \) and \( \tau_0 = 0 \). The time interval between two consecutive energy arrival events is called epoch and its duration is \( \ell_j \triangleq \tau_j - \tau_{j-1} \) (\( j \geq 1 \)) (see Fig. 1a). Epoch duration \( \ell_1, \ell_2, \ldots \) are supposed to be i.i.d., with PDF given by \( f_{\ell}(\ell) \).

We assume that most of the energy is consumed by the device during the transmission phase; the other causes of energy consumption (i.e., processing and data reception) are neglected [9]. The amount of energy available at the node at time \( t \) is \( E(t) \) (see Fig. 1b), which is given by

\[
E(t) = A_0 + \sum_{j=0}^{J(t)-1} Y_j - \int_0^t P(\xi)d\xi \quad t \geq 0
\]

where \( A_0 \) is the residual energy just before \( t = 0 \), \( J(t) = \{ \min n : t \leq \tau_n \} \), and \( P(t) \) is the instantaneous transmit power (see Fig. 1c). Note that at \( t = 0 \) the amount of available energy at is \( E(0) = A_0 + Y_0 \). We also assume that for any time instant the energy spent cannot exceed the energy harvested (energy neutrality constraint), that is, \( E(t) \geq 0 \) for \( t > 0 \).

![Fig. 1. Energy arrival model: a) Energy arrival events and durations of the epochs; b) Transmit power profile; c) Available energy profile; d) Amount of information transmitted.](image)

The channel is assumed to be AWGN and the signal received at destination is given by \( y(t) = \sqrt{P(t)}x(t) + n(t) \), where \( x(t) \) is the signal transmitted by the source, with \( \mathbb{E}\{|x(t)|^2\} = 1 \), and \( n(t) \) is the thermal noise, with \( \mathbb{E}\{|n(t)|^2\} = \sigma^2 \). The instantaneous transmission rate \( r(t) \) (in bits per channel use) is

\[
r(t) = \frac{1}{2} \log_2 (1 + p(t))
\]

where \( p(t) \triangleq P(t)/\sigma^2 \). Using (2), the amount of information transmitted in the interval \([0, t)\) (see Fig. 1d) becomes

\[
G(t) = \frac{1}{2} \int_0^t \log_2 (1 + p(\xi))d\xi.
\]

The function \( p(t) \) is defined as the transmission policy.

III. TRANSMIT POWER OPTIMIZATION

The objective of this work is to define the optimal transmission policy \( p(t) \), given causal information on the energy amount and statistical information on the energy arrival. More specifically, the transmitter is assumed to know at any instant \( t \) only the residual energy \( E(t) \) and the statistic of the epoch length \( f_{\ell}(\ell) \). The node does not have any a priori information (neither deterministic nor statistical) on the future amount of energy harvested. As a consequence, a global optimization, in terms, for instance, of the average (long term) rate (as studied in [15]) cannot be applied. This scenario has been also investigated in [14, Sec. VI-B], where some sub-optimal online power allocations are proposed. The main difference between the approach followed in [14] and our analysis is that [14] assumes a constant power allocation in between two energy arrival events, whereas in our approach the power profile is arbitrary.

Under the previous assumptions, the transmit power at time \( t \in [\tau_{j-1}, \tau_j) \) is decided on the basis of the statistical knowledge of the epoch duration (through its PDF, \( f_{\ell}(\ell) \)) and the
energy available at the beginning of the epoch \( E(\tau_{j-1}) \). Since no additional information can be acquired by the transmitter during the epoch, the decision on the transmission policy \( p(t) \) can be taken at the beginning of the epoch. In the absence of any knowledge on the amount of energy received in the future, a realistic approach to choose \( p(t) \) is to focus on a given epoch and maximize the information transmitted in that time interval. Having this objective in mind, we rewrite (3) as

\[
G(t) = \sum_{j=1}^{J(t)} G_j(t - \tau_{j-1})
\]

with

\[
G_j(t) = \begin{cases} 
0 & t < 0 \\
\frac{1}{2} \int_0^t \log_2 (1 + p_j(\xi)) \, d\xi & t \in [0, \ell_j] \\
\frac{1}{2} \int_0^{\ell_j} \log_2 (1 + p_j(\xi)) \, d\xi & t \geq \ell_j
\end{cases}
\]

where \( p_j(t) \) for \( t \in [0, \ell_j] \) is the transmission policy at the epoch \( j \) and \( p(t) \) can be written as

\[
p(t) = p_J(t) - \tau_{J(t)-1}
\]

reminding that \( J(t) = \{ \min \, n : t \leq \tau_n \} \).

If we now consider an arbitrary epoch \([\tau_{j-1}, \tau_j)\), the optimization problem is restricted to the maximization of \( G_j(t) \) for \( t \in [0, \ell_j] \). Since the duration of the epoch is a r.v., the optimization problem can be formulated as

\[
\begin{aligned}
\{ p_j^*(t) = \arg \max & \int_0^\infty f_\ell(\xi) \int_0^{\xi} \ln (1 + p_j(t)) \, dt \, d\xi \\
& \text{s.t.} \quad p_j(t) \geq 0 \\
& \int_0^{\ell_j} p_j(\xi) d\xi \leq e_j
\end{aligned}
\]

where \( e_j = \frac{E(\tau_{j-1})}{\sigma^2} \), and the function \( \ln(\cdot) \) has been used in (7) instead of \( (1/2) \log_2(\cdot) \), since it does not have any influence on the solution of the optimization problem.

Unfortunately, (7) is a typical example of infinite dimensional optimization problem, whose direct solution can be rather cumbersome [17]. To make the problem tractable, we will first restrict the set of possible functions \( p(t) \) to the class of step-wise functions, and then extend the solution to continuous functions with arbitrary shape.

### A. Step-wise function optimization

We restrict the set of possible functions \( p_j(t) \) to the class of functions that are constant over defined time intervals. More specifically, starting from a set of \( K+1 \) time instants \( \{T_k\}_{k=0}^K \), with \( K \) and \( \{T_k\}_{k=0}^K \) arbitrary but deterministically chosen, \( T_m < T_n \) for \( m < n \), and \( T_0 = 0 \), we search \( p_j(t) \) among the functions that belong to the class \( \mathcal{P}_K = \{ p_j(t) : p_j(t) = p_j,k, \, t \in [T_{k-1}, T_k) \, k = 1, \ldots, K \} \), with \( p_j(t) = 0 \) for \( t > T_K \). Focusing on the solutions belonging to \( \mathcal{P}_K \), we obtain the following Theorem.

**Theorem 1:** Given the class \( \mathcal{P}_K \) of possible transmit power functions, and defining

\[
\delta_k \triangleq T_k - T_{k-1},
\]

\[
c_k \triangleq \delta_k [F_\ell(T_K) - F_\ell(T_k)] + \int_{T_{k-1}}^{T_k} f_\ell(\xi)(\xi - T_{k-1}) d\xi,
\]

\[
\alpha_k \triangleq c_k / \delta_k,
\]

the solutions \( p_{j,1}, \ldots, p_{j,K} \) of the optimization problem (7) can be written as

\[
p_{j,k}^* = \begin{cases} 
\frac{\alpha_k (e_j + T_{m_j})}{\sum_{i=1}^J c_i} - 1 & 1 \leq k \leq m_j \\
0 & m_j + 1 \leq k \leq K
\end{cases}
\]

where

\[
m_j = \{ \max i : \sum_{k=1}^i \delta_k (\frac{\alpha_k}{\alpha_i} - 1) \leq e_j \}.
\]

**Proof:** By considering the class \( \mathcal{P}_K \), the double integral in (7) simplifies as

\[
\int_0^\infty f_\ell(\xi) \int_0^\xi \ln (1 + p_j(t)) \, dt \, d\xi =
\]

\[
\sum_{k=1}^K \int_{T_{k-1}}^{T_k} f_\ell(\xi) \left( \sum_{i=1}^{k-1} \delta_i \ln (1 + p_{j,i}) \right) d\xi
\]

\[
= \sum_{k=1}^K \ln (1 + p_{j,k}) \, c_k
\]

where \( c_k \) is defined in (9). Now, the optimization problem given by (7) becomes

\[
\begin{aligned}
\{ p_j^* = \arg \max & \sum_{k=1}^K \ln (1 + p_{j,k}) \, c_k \\
& \text{s.t.} \quad p_{j,i} \geq 0 \quad i = 1, \ldots, K \\
& \sum_{k=1}^K p_{j,k} \delta_k \leq e_j
\end{aligned}
\]

where \( p_j = [p_{j,1}, \ldots, p_{j,K}]^T \). It is easy to show that problem (14) satisfies the Karush-Kuhn-Tucker (KKT) conditions for the existence of the solution. Furthermore, it is straightforward to show that the optimal solution of (14) can be obtained only when

\[
\sum_{k=1}^K p_{j,k} \delta_k = e_j
\]

which can be easily proved by contradiction observing that, if \( p_{j'} \) were the solution of (14), with \( \sum_{k=1}^K p_{j',k} \delta_k = e_j' < e_j \), the excess energy \( \epsilon \triangleq e_j - e_j' > 0 \) could be used to increase (for instance) \( p_{j,1} \) and obtain the new power profile \( p_{j''} = [p_{j,1} + \epsilon/T_1, p_{j,2}, \ldots, p_{j,K}]^T \). In this case we would obtain

\[
\sum_{k=1}^K \ln (1 + p_{j,k}) c_k < \sum_{k=1}^K \ln (1 + p_{j''}) c_k
\]

which contradicts the optimality of \( p_{j'} \).
By substituting (15) in (14), and after some algebra, we obtain the following equivalent system
\[
\begin{align*}
\frac{c_k}{1+\rho_{j,k}} - \lambda \delta_k &= 0 \quad k = 1, \ldots, K \\
p_{j,k} &\geq 0 \quad k = 1, \ldots, K \\
\sum_{k=1}^{m_j} p_{j,k} \delta_k &= e_j.
\end{align*}
\]
(17)
The first equation of (17) gives \(p_{j,k} = \alpha_k/\lambda - 1\), therefore, \(p_{j,k} \geq 0\) only if \(\lambda \leq \alpha_k\), or, equivalently,
\[
p_{j,k} = (\alpha_k/\lambda - 1)^+, \quad \text{for } k \geq 1.
\]
(18)
By substituting \(p_{j,k} = (\alpha_k/\lambda - 1)^+\) in (15), we obtain
\[
e_j = \sum_{k=1}^{m_j} \left(\frac{\alpha_k}{\lambda} - 1\right)^+ \delta_k = \sum_{k=1}^{m_j} \left(\frac{\alpha_k}{\lambda} - 1\right) \delta_k
\]
(19)
where \(m_j\) is the largest integer such that \(\alpha_{m_j} \geq \lambda\). Note that the last equality in (19) follows from the fact that \(\alpha_{k-1} \geq \alpha_k\) for \(k \geq 2\). This condition can be proved by observing that
\[
\alpha_{k-1} - \alpha_k = [F(t_{K}) - F(t_{K-1})] + \int_{T_{K-1}}^{T_k} \frac{\xi - T_{k-1}}{\delta_{k-1}} d\xi - [F(t_{K}) - F(t_{K-1})] - \int_{T_{K-2}}^{T_k} \frac{\xi - T_{k-1}}{\delta_{k-1}} d\xi
\]
\[
\geq [F(t_{K}) - F(t_{K-1})] + \int_{T_{K-2}}^{T_k} \frac{\xi - T_{k-2}}{\delta_{k-1}} d\xi - \int_{T_{K-2}}^{T_k} f_t(\xi) d\xi
\]
\[
= \int_{T_{K-2}}^{T_k} \frac{\xi - T_{k-2}}{\delta_{k-1}} d\xi \geq 0 \quad \text{for } k \geq 2
\]
(20)
where we have used the inequality
\[
\int_{T_{K-2}}^{T_k} f_t(\xi) d\xi \geq \int_{T_{K-2}}^{T_k} \frac{\xi - T_{k-2}}{\delta_{k-1}} d\xi.
\]
(21)
From (19) we can obtain \(\lambda\) as
\[
\lambda = \frac{\sum_{k=1}^{m_j} c_k}{e_j + \sum_{k=1}^{m_j} \delta_k}.
\]
(22)
Recalling that \(p_i = 0\) for \(i > m_j\), and by substituting (22) in (18), we obtain (11).

Finally, since \(\lambda \leq \alpha_{m_j}\), (22) becomes
\[
e_j \geq \sum_{k=1}^{m_j} \delta_k \left(\frac{\alpha_k}{\alpha_{m_j}} - 1\right)
\]
(23)
which leads to the iterative procedure given by (12) to evaluate \(m_j\).

### 2. Continuous function optimization

Theorem 1 holds for arbitrary values of \(\delta_k\) and \(K\). As a consequence, result also applies when \(\delta_t\) tends to 0 and \(K\) tends to infinity; hence, the following Theorem derives.

**Theorem 2:** The exact solution of the optimization problem in (7) can be written as
\[
p^*_j(t) = \begin{cases} 
\frac{e_j(1-F(t)) + f^{\hat{t}}_j F(\hat{t})}{1-F(t)} & 0 \leq t \leq \hat{t}_j \\
0 & t > \hat{t}_j
\end{cases}
\]
(24)
where \(\hat{t}_j\) is the solution of
\[
e_j [1 - F(\hat{t}_j)] - \hat{t}_j F(\hat{t}_j) + \int_0^{\hat{t}_j} f_t(\xi) d\xi = 0.
\]
(25)

**Proof:** Assuming \(K \to \infty\), \(\delta_t = dt\), and \(t = T_k\), the coefficients \(c_k\) and \(\alpha_k\) become \(dc(t)\) and \(\alpha(t)\), respectively, and can be written as
\[
dc(t) = [1 - F(t)] dt + \int_{t-dt}^{t} f_t(\xi) |t - (t - dt)| d\xi
\]
(26)
\[
\approx [1 - F(t)] dt
\]
(27)
where we have used the following result
\[
\int_{t-dt}^{t} f_t(\xi) |t - (t - dt)| d\xi \approx f_t(t-dt)[0] dx = 0
\]
(28)
and \(\alpha(t) = dc(t)/dt = 1 - F(t)\).

Denoting \(\hat{t}_j = T_{m_j}\), (11) becomes
\[
p^*_j(t) = \begin{cases} 
\frac{1-F(t)\ell_j}{\int_{\ell_j}^{\hat{t}_j} [1-F(t)] dt} - 1 & 0 \leq t \leq \hat{t}_j \\
0 & t > \hat{t}_j
\end{cases}
\]
(29)
that can be easily rearranged as in (24).

Starting from (23), the parameter \(\hat{t}_j\) can be obtained by solving
\[
e_j = \int_0^{\hat{t}_j} \left(\frac{\alpha(\xi)}{\alpha(\hat{t}_j)} - 1\right) dt = \int_{\ell_j}^{\hat{t}_j} \left(\frac{1-F(t)}{1-F(t_j)} - 1\right) dt
\]
(30)
which can be also written as (25). \(\square\)

### C. Remarks and discussion

The results of Theorems 1 and 2 lead to the following corollaries.

**Corollary 1:** The optimal transmission policies within an epoch, both in the step-wise and in the continuous case, are non increasing functions of time. This result differs from the optimum offline polices, which are proved to be non decreasing step-wise functions remaining constant during any epoch [9].

This behavior can be intuitively explained by observing that, in the absence of a deterministic knowledge of the next energy arrival instant, to minimize the remaining energy at the end of the epoch, it is better to use more power at the beginning and then reduce such power in the subsequent instants.

**Proof:** For the step-wise function case, the condition \(p_{j,k-1} \geq p_{j,k}\) can be proved by recalling that \(p_{j,k} = \alpha_k/\lambda - 1\) for \(k = 1, \ldots, m_j\) and that \(\alpha_{k-1} \geq \alpha_k\) (as shown in the proof of Theorem 1). For the continuous function case, (24) can be rewritten as
\[
p^*_j(t) = -F(t)A(\hat{t}_j) + B(\hat{t}_j) \quad 0 \leq t < \hat{t}_j
\]
(31)
Transmission power $p^*$ is sufficient to prove that $p^*$ and $p^*_j(t)$ are also non increasing functions of time (Remark 1) is sufficient to prove that $p^*_j$ and $p^*_j(t)$ are non decreasing functions of the residual energy.

Proof: Since the residual energy is a non increasing function of time (within an epoch), the condition that $p^*_j$ and $p^*_j(t)$ are also non decreasing functions of the residual energy, in agreement with the result obtained by [11, Th.1] for a time-slotted system where both fading and energy arrival events occur on a discrete-time axes.

IV. NUMERICAL EXAMPLES

A. Performance benchmarks

To evaluate the performance of the transmission policies given by (11) and (24), we consider an ideal scenario as a benchmark. In such scenario, for any given instant $t \in [\tau_k-1, \tau_k)$, the node knows both the available energy $E(t)$ and the arrival time of the next amount of energy, $\tau_k$. The value of the transmit power that maximizes the throughput in $[\tau_k-1, \tau_k)$ is constant and equal to $p^*_{j(\text{ideal})}(t) = e_j/\ell_k$ [10], and the amount of information transmitted at $\tau_k$ corresponds to

$$C^*_{j(\text{ideal})}(\tau_k) = \frac{\ell_k}{2} \log_2 \left( 1 + \frac{e_j}{\ell_k} \right).$$

(32)

An additional (lower bound) benchmark is also provided. We consider the case where the transmission power is kept constant till either the residual energy reaches zero or an energy recharge event occurs. In such case, the value of transmit power is calculated by assuming a transmission for a duration equal to the average energy inter-arrival time, $\eta$, that is, $p^*_{j(\text{constant})}(t) = e_j/\eta$ for $0 \leq t \leq \eta$, and zero otherwise.

In the numerical results we consider, as case study, a scenario with exponentially distributed energy arrival intervals, whose optimal power transmission profile is reported in the following subsection.

B. Case study: exponentially distributed energy arrival intervals

We now consider the special case that the energy arrival epochs have an exponentially distributed duration, that is

$$f^\text{exp}_k(\ell) = \frac{1}{\eta} e^{-\ell/\eta} \quad \ell \geq 0.$$

(33)

This model is usually considered in the literature to characterize the statistics of the energy arrival intervals [14], [15]. In such case, some parameters described in Section III can be written in closed form. In particular, $c_k$ in (9), and $p_j(t)$ in (24) can be rewritten as

$$c_k = \delta_k \left[ e^{-T_k/\eta} - e^{-T_k/\eta} + \eta e^{-T_{k-1}/\eta} - e^{-T_k/\eta} (\eta + T_k - T_{k-1}) \right].$$

(34)

We first consider the shape of the optimal transmission power profile under the hypothesis of exponentially distributed energy arrival intervals, Fig. 2 shows $p^*(t)$ as a function of $t$ for different values of the residual energy $e_j$, when $\eta = 1$ s. As proved in Remark 1, the power profile is a non increasing function of time. As expected, the duration of the power profile increases with the initial amount of energy; this can be easily shown by observing that the coefficient $\hat{t}_j$ in (36), which defines the duration of the power profile, depends on the value of energy. In particular, (36) can be rewritten as $e_j = \eta(e^x - x - 1)$ with $x = \hat{t}_j/\eta$, which shows that $\hat{t}_j/\eta$ (and therefore $\hat{t}_j$) is a monotonic non decreasing function of $e_j$.\hfill \square
Fig. 3 shows the optimal transmission policy at a given time instant as a function of the current value of the residual energy, for different values of the energy inter-arrival time \( \eta \). As discussed in Corollary 2, the transmission power is a monotonic non-decreasing function of the residual energy. As expected, when \( \eta \) increases, the algorithm tends to allocate less power to compensate for the larger average energy inter-arrival time.

Finally, Fig. 4 shows the value of \( G_j(t) \) averaged with respect to the duration of the epoch, that is, \( G_j \triangleq \mathbb{E}_{t}[G_j(t)] = \int_0^\infty G_j(x) f_j(x) dx \) for different cases: step-wise, continuous, ideal, and constant transmission power profiles, for \( \eta = 1 \) s. For the step-wise function case, different values of \( K \) are considered, and the duration of the time intervals are fixed at \( \delta \triangleq \delta_k = T_K/K \) with \( T_K = 5\eta \); this latter value has been chosen so that the probability that \( p_j(t) = 0 \) is negligible \((1/e^5)\). The comparison between the step-wise and the continuous case show that the case with \( K = 10 \) allows us to obtain a performance that is almost indistinguishable by that of the continuous case. The gap between the ideal case (which has a deterministic knowledge of the next energy inter-arrival time) and the continuous case is of about 27\%, 24\%, 22\%, and 20\% for \( e_j = 5, 10, 20, 30 \) s, respectively. The constant transmission case gives the worst performance. Interestingly, the latter scheme is also outperformed by the step-wise function with \( K = 1 \) and \( \delta = 5\eta \). The comparison between these two schemes shows that it is preferable to transmit less power \((1/5)\) but for a larger duration \((5 \text{ times})\), that is a consequence of the convexity of the log function in (2).

V. Conclusions

In this work we studied the power allocation policies for energy harvesting devices when causal information on the energy amount and statistical information on the energy arrival is available: we assumed that the available energy and the PDF of the energy arrival instants are known, while no information is given about the amount of energy that will be harnessed in the future. Exact solutions have been obtained for the cases of step-wise and time-continuous transmission power profiles. We also showed that the optimal allocation is a non increasing function of time until a new energy arrival occurs, and a non decreasing function of the residual energy. Numerical results, obtained assuming an exponential distribution of the energy arrival intervals, demonstrated a significant performance increase compared to a constant allocation (based on the average energy inter-arrival time), and highlighted the limited loss compared to an ideal non-causal allocation.

References


