FRANCESCA BECCACECE and ALESSANDRA CILLO

APPLYING THE BENCHMARKING PROCEDURE: A DECISION CRITERION OF CHOICE UNDER RISK

ABSTRACT. Modeling risk in a prescriptively plausible way represents a major issue in decision theory. The benchmarking procedure, being based on the satisficing principle and providing a probabilistic interpretation of expected utility theory, is prescriptive. Because it is a target-based language, the benchmarking procedure can be applied naturally to finance. In finance, the centrality of risk is widely recognized, but the risk measures that are commonly used to assess risk are too poor as a decision making tool. In this paper we propose a two-stage decision criterion of choice under risk that provides an application of benchmarking to finance through a risk measure. We will analyze some nonexpected utility theories, in particular lottery dependent utility, as potential frameworks for our criterion.

KEY WORDS. Benchmarking, decision criterion, two stage procedure, risk measure, lottery dependent utility

1. INTRODUCTION

Since the 1940's, the expected utility (EU) paradigm of von Neumann and Morgenstern (vN-M) (1944) has been the standard theory for decision making under risk. However, for the last twenty five years, several psychologists and economists have challenged the descriptive validity of EU. Nonexpected utility models have been proposed to capture EU anomalies (for a review see Starmer, 2000). Some authors have focused more on psychologically intuitive concepts (Kahneman and Tversky, 1979; Bell, 1982, 1985; Loomes and Sugden, 1982, 1986; Becker and Sarin, 1987). Others have given more emphasis to proper axiomatizations by either weakening the independence axiom (Quiggin, 1982; Yaari, 1987), or by demonstrating that the EU analysis does not depend on the independence axiom, but may be derived from the much weaker assumption of smoothness of preferences over alternative probability distributions (Machina, 1982).

A model is prescriptive if it is a "practical aid to choice" (Starmer, 2000, p. 334). A prescriptive model must incorporate the theoretical foundation of normative theory with the observations of descriptive theories. None[1] of the currently accepted nonexpected utility models above are prescriptive because they fail to meet these criteria (Castagnoli and Li Calzi, 1999).

Simon (1955) took a step towards prescriptive theories. Simon (1955) coined the term <u>bounded rationality</u> to refer to rational choices given the cognitive limitations of the individual decision maker in terms of acquiring and processing information. A boundedly rational decision is not irrational or even sub-optimal. Rather, it is rational and optimal given the constraints faced by the decision maker. Gigerenzer and Selten (2001, p. 6) state that "bounded rationality provides an alternative to current norms, not an account that accepts current norms. Bounded rationality means rethinking the norms as well as studying the actual behavior of minds and institutions." A <u>satisficing decision</u> (Simon, 1956) is an example of a boundedly rational decision.

<u>Benchmarking under risk</u> (Castagnoli and Li Calzi, 1996) is based on the satisficing principle and it provides a probabilistic interpretation of EU theory where the notion of vN-M utility function is avoided (Castagnoli and Li Calzi, 1996; Bordley and Li Calzi, 2000). Because it is based on the satisficing theory, and satisficing theory is prescriptive, benchmarking can be considered prescriptive as well. More importantly, the benchmarking approach fits the

definition given above of a prescriptive model. Indeed, it is a practical aid to choice because benchmarking is a natural procedure to construct preferences, and it combines theory and descriptive observation because it represents a probabilistic interpretation of EU theory. However, although benchmarking is appealing because it has prescriptive features, not many applications of it have been proposed so far.

In the finance literature, the recognition of the centrality of risk in financial problems has caused a shift from utility models to more natural tools for managing financial risk (Roy, 1952; Telser, 1955-56; Kataoka, 1963; Arzac, 1976). A risk measure is defined as the cost the individual has to add to the current position for making an unacceptable risk acceptable. Apart from variance, alternative risk measures such as first partial moments (Fishburn, 1977), value-at-risk (VaR) (Jorion, 1997), and coherent[2] risk measures (Artzner et al., 1997, 1999) have been proposed to provide a more realistic representation of business behavior. However, risk measures do not seem to be robust enough to construct risky decisions in a financial setting. As several authors argue (Sarin and Weber, 1993; Schmidt, 2003), an integration of these risk measures with a choice structure is necessary.

The goal of this paper is to propose a decision criterion of choice under risk that provides an application to finance of the prescriptive benchmarking procedure. As a contribution to finance, we suggest a solid example of inserting risk measures within a decision process in order to make financial choices in a more robust way.

The criterion recalls the two stage procedure introduced by Simon (1995), and then developed by Munier (1999). According to this procedure, individual mental processes follow two stages: 1) recognition, and 2) heuristic search. The first stage consists of reducing the set of lotteries by eliminating the ones that are too risky via a generic risk measure. The second stage consists of partitioning the set of lotteries into disjoint subsets according to their riskiness, assessing a different benchmark for each class, and choosing the best lottery according to the probability of outperforming an assigned benchmark.

The criterion is also enriched by a straightforward link with some nonexpected utility models. Lottery dependent utility developed by Becker and Sarin (1987) and Schmidt (2001), is the most natural framework for deriving it. The major assumption of lottery dependent utility consists of the dependence of the utility function on the lottery itself. In our criterion, such dependence relies on the risk measure of the considered lottery. This result has intuitive and mathematical support. From an intuitive viewpoint, it makes sense that the risk measure of a lottery has an impact on the overall utility of the lottery itself. From a mathematical viewpoint, our criterion -when reduced to the second stage- fits the axiomatization of lottery dependent utility provided by Schmidt (2001).

This paper is organized as follows. The next section is devoted to a brief review of the benchmarking approach. Section 3 provides the decision criterion by combining the benchmarking procedure with a risk measure. Section 4 shows that our decision criterion preserves first order stochastic dominance. Section 5 proposes lottery dependent utility and prospect theory as potential frameworks where our criterion fits in a natural way. Section 6 concludes by suggesting potential applications and further research.

2. BENCHMARKING PROCEDURE

Nonexpected utility theories fall in two major categories: rank-based and betweenness-based theories. Both sets of theories account for most of the empirical violations of EU. However, these theories lack prescriptive power: they are not able to help individuals make decisions. These theories presuppose the existence of a preference relation (Castagnoli and Li Calzi, 1999). However, individuals need a systematic procedure to derive reasonable preferences because most of the time they do not have ready-made preferences. As Castagnoli and Li Calzi (1999) point out "benchmarking is the natural procedure to construct preferences over lotteries in a way that conforms to EU."

The benchmarking procedure is a target-based model (Castagnoli and Li Calzi, 1996, 1999; Bordley and Li Calzi, 2000). It is a prescriptive approach for two reasons. First, benchmarking is based on the satisficing principle introduced by Simon (1955). The satisficing approach recognizes the cost or the practical impossibility of searching among all possible actions for the optimal one. Therefore, it becomes rational to adopt rules of thumb as a way to economize on cognitive faculties. One rule of thumb could be that individuals should establish some target (benchmark), and then pick the first action which meets the target. Second, benchmarking combines normative and descriptive features –as a prescriptive model by definition should do– because it satisfies the vN-M's (Castagnoli and Li Calzi, 1996) and the Savage's (Bordley and Li Calzi, 2000) axiomatization through a probabilistic and, therefore, more intuitive interpretation of EU theory.

Considering a compact interval J of outcomes in $\mathbb{R}$, Castagnoli and Li Calzi (1996) define a lottery X any random variable that takes values on J. They write $X \rightsquigarrow F$ to indicate that the cumulative distribution function (c.d.f.) of X is F. They assume exogenous probabilities and reduction of lotteries to c.d.f.s, i.e., they state that the c.d.f. F of a lottery X is known to an individual and that a lottery X can be replaced by its c.d.f. F. According to the authors, the EU of a (monetary) lottery X can be read as the probability that X outperforms another given independent lottery H whose c.d.f. is denoted by U. Thus, the EU of X can be written as:

$$B(X) = \int_J U(x) \mathrm{d}F(x) = \int_J \Pr(H \leq x) \mathrm{d}F(x) = \Pr(X \geq H). \tag{1}$$

Expression (1) provides a natural interpretation for the Bernoulli index $B(X)$: the individual chooses a stochastic benchmark H representing his uncertain target and evaluates a lottery X through its probability of outperforming the benchmark H. Hence, if an individual wants to rank two lotteries X and Y, he needs to first assess a benchmark $H \rightsquigarrow U$ and then compare their probabilities to outperform it. The two lotteries will be ranked according to the rule:

$$X \succeq Y \iff \Pr(X \geq H) \geq \Pr(Y \geq H).$$

Despite the appealing features of benchmarking, few researchers focus on its applications. Bordley and Kirkwood (2004) develop an approach based on performance targets to assess a preference function for a multi-objective decision under uncertainty. The approach has particular applicability for resource allocation decisions, such as new product development or decision making in regulated environment. However, the literature lacks extensive research on applications of benchmarking to finance, a field where it would fit naturally because benchmarks are widely used in finance.

The construction of our decision criterion was motivated by one major need. Decision theory needs prescriptive models. Benchmarking is a prescriptive procedure, but there are few applications of it. Therefore, we provide an application to finance of a procedure which is theoretically solid and intuitively sound. As a contribution to finance, by combining risk measures with a choice structure, we suggest a way of making more sophisticated choices in a risky context. Indeed, in finance risk is an extremely relevant topic, and traditionally risk measures have been used to assess risk. However, these measures are too poor as a decision making tool because in reality choices are much more complex. An integration of risk measures with a choice structure seems to be a viable answer to the requirement of dealing with decision problems.

3. A DECISION CRITERION OF CHOICE UNDER RISK

We propose a decision criterion that comprises two stages. The first stage represents the recognition of the available lotteries via a risk measure. A risk measure can be interpreted as a cost. Therefore, since the higher the risk measure, the riskier the lottery is, individuals will reject those lotteries whose risk measure is higher than the maximum value of risk they are able to bear. The second stage consists of applying a rule of thumb to compare lotteries with different profiles. Lotteries are partitioned into different subsets according to their risk measure. The benchmarking approach makes it possible to compare lotteries belonging to different subsets by attributing a comparative advantage to the lottery that is less risky. More specifically, given two lotteries with different level of risk, benchmarks are assigned to the two lotteries such that the less risky lottery has a benchmark that is easier to outperform. Hence, the individual chooses the best lottery with respect to the probability of outperforming the assigned benchmark.

Assume we are in a continuous setting. Let $\mathcal{X}$ be the set of all lotteries taking values on the compact interval $I = [\alpha, \beta]$ of outcomes in $\mathbb{R}^+$. Given a lottery $X \rightsquigarrow F$, let $\rho(X)$ indicate the risk measure of X, and $\overline{r}$ the maximum level of risk that an individual is able to bear.

In the first stage, a lottery $X \in \mathcal{X}$ will be maintained if $\rho(X) \leq \overline{r}$. The feasible set for individual choice reduces from $\mathcal{X}$ to the class $L = \{X \in \mathcal{X} : \rho(X) \leq \overline{r}\}$.

In the second stage, first the class L is partitioned into subsets that share the same risk measure. We define L_{r_X} as the subset containing the lotteries that display the same risk measure r_X. Then, the individual chooses the best lottery according to the probability of outperforming the benchmark assigned to that subset. If we define by $H_X \rightsquigarrow U_X$ the benchmark assigned to the lotteries belonging to the subset L_{r_X}, H_X can be seen as a lottery with c.d.f. U_X. The problem becomes building a benchmark in such a way that a comparative advantage is attributed to the lotteries that are less risky.

Assume that $X \in L_{r_X}, Y \in L_{r_Y}$, $X \rightsquigarrow F$, $Y \rightsquigarrow G$, and $r_X < r_Y \leq \overline{r}$. Since X displays a better risk structure than Y, the benchmark $H_X \rightsquigarrow U_X$ assigned to X must be easier to outperform than the benchmark $H_Y \rightsquigarrow U_Y$ assigned to Y. A natural implication is that:

$$U_X(t) > U_Y(t) \quad \forall t.$$

Given that individuals are risk adverse, it is reasonable to assume that they are able to bear a maximum and minimum level of risk. In the benchmarking interpretation, this can be represented as follows. Let H be a lottery on I with risk measure $\overline{r}$ and concave c.d.f. U, and V a lottery on I with uniform c.d.f. D. The c.d.f. U represents the most risk adverse attitude, while D the risk neutral attitude. Therefore, one way of obtaining the c.d.f.s of the assigned benchmarks is through a weighted mean of U and D with weights depending on the risk measure of the underlying lotteries. Thus, as Figure 1 shows, the c.d.f.s of H_X and of H_Y are, respectively:

$$U_X(t) = \frac{r_X}{\overline{r}}D(t) + (1 - \frac{r_X}{\overline{r}})U(t)$$

$$U_Y(t) = \frac{r_Y}{\overline{r}}D(t) + (1 - \frac{r_Y}{\overline{r}})U(t).$$

Once the benchmarks have been assigned to each subset, we can rank X and Y according to the rule:

$$X \succeq_H Y \Longleftrightarrow \Pr\{X \geq H_X\} \geq \Pr\{Y \geq H_Y\}$$

where $\succeq_H$ represents the order in L through the benchmarking approach. The probabilities of X and Y of outperforming the assigned benchmarks are represented, respectively, by the indices $B(X)$ and $B(Y)$:

$$B(X) = \int_\alpha^\beta U_X(t, r_X)dF(t) = \int_\alpha^\beta [\frac{r_X}{\overline{r}}D(t) + (1 - \frac{r_X}{\overline{r}})U(t)]dF(t)$$

$$B(Y) = \int_\alpha^\beta U_Y(t, r_Y)dG(t) = \int_\alpha^\beta [\frac{r_Y}{\overline{r}}D(t) + (1 - \frac{r_Y}{\overline{r}})U(t)]dG(t)$$

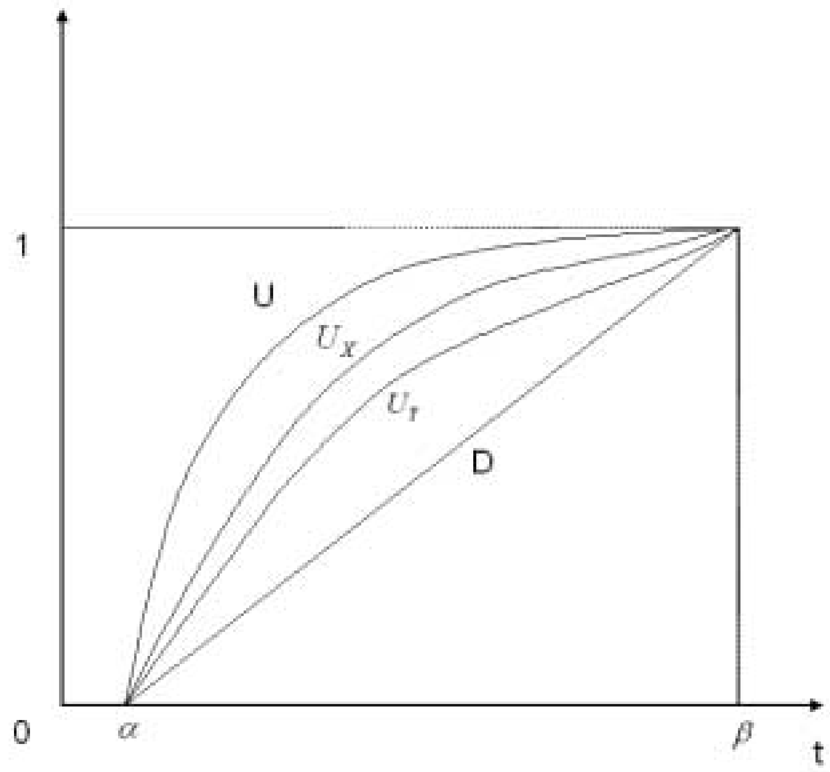

Figure 1: c.d.f.s of V, H, H_X, and H_Y

with $B : L \to \mathbb{R}$. In this case, the preference of one lottery over the other depends on both the risk measures and the c.d.f.s. Instead, if two lotteries $X_1 \rightsquigarrow F_1$ and $X_2 \rightsquigarrow F_2$ belong to the same subset L_{r_X}, we obtain:

$$X_1 \succeq_H X_2 \Longleftrightarrow \Pr\{X_1 \geq H_X\} \geq \Pr\{X_2 \geq H_X\}.$$

Therefore, the functional $B_{r_X} : L_{r_X} \to \mathbb{R}$ will only depend on the c.d.f.s:

$$B_{r_X}(X_1) = \int_\alpha^\beta U_X(t, r_X) dF_1(t) = \int_\alpha^\beta [\frac{r_X}{\overline{r}} D(t) + (1 - \frac{r_X}{\overline{r}}) U(t)] dF_1(t)$$

$$B_{r_X}(X_2) = \int_\alpha^\beta U_X(t, r_X) dF_2(t) = \int_\alpha^\beta [\frac{r_X}{\overline{r}} D(t) + (1 - \frac{r_X}{\overline{r}}) U(t)] dF_2(t).$$

4. SOME FEATURES OF THE DECISION CRITERION

4.1. Premium/Penalty function

If a lottery has a lower risk measure, and therefore a better risk structure, it will be assigned to a benchmark that is easier to outperform. Thus, we attribute a comparative advantage to the less risky lottery. As we will show next, we can quantify such comparative advantage through a premium function.

Let $X \in L_{r_X}$, $Y \in L_{r_Y}$, $X \rightsquigarrow F$, $Y \rightsquigarrow G$, and $r_X < r_Y \leq \overline{r}$. The c.d.f. of the benchmark associated to the lottery X:

$$U_X(t) = \left[\frac{r_X}{\overline{r}} D(t) + (1 - \frac{r_X}{\overline{r}}) U(t)\right]$$

is greater than the c.d.f. of the benchmark assigned to the lottery Y:

$$U_Y(t) = \left[\frac{r_Y}{\overline{r}} D(t) + (1 - \frac{r_Y}{\overline{r}}) U(t)\right].$$

We can represent the c.d.f. of the benchmark assigned to X in terms of the c.d.f. of the benchmark associated to Y:

$$U_X(t) = U_Y(t) + p(r_X, r_Y)\left[U(t) - D(t)\right]$$

where $p(r_X, r_Y) = \dfrac{r_Y - r_X}{\overline{r}}$. Hence, we have:

$$\begin{aligned} B(X) &= \int_\alpha^\beta U_X(t, r_X) dF(t) \\ &= \int_\alpha^\beta \left\{U_Y(t) + p(r_X, r_Y)\left[U(t) - D(t)\right]\right\} dF(t) \\ &= B(X, r_Y) + p(r_X, r_Y) A(X) \end{aligned}$$

where $A(X) = \int_\alpha^\beta \left[U(t) - D(t)\right] dF(t)$. Since $p(r_X, r_Y)$ and $A(X)$ are positive, $B(X) > B(Y)$ and the quantity $\Pi(X, r_X, r_Y) = p(r_X, r_Y) A(X)$ represents the premium given to X since its risk measure is r_X and not r_Y. Vice versa, if $r_X \geq r_Y$, then $p(r_X, r_Y)$ is negative (or zero), and $\Pi(X, r_X, r_Y)$ represents the penalty for the lottery with higher risk structure.

Therefore, we can define a premium/penalty function by:

$$\Pi(X, r_X, r_Y) = \frac{r_Y - r_X}{\overline{r}} \int_{\alpha}^{\beta} [U(t) - D(t)]\, dF(t).$$

The function $\Pi(\cdot,\cdot,\cdot)$ allows us to adjust the evaluation of the lotteries according to the spread of the risk measures. However, it is interesting to note that $\overline{r}$ –the maximum risk the individual can afford– plays a role in the premium/penalty function. For example, if we assume that $r_X < r_Y$, then a premium will be attributed to X, but the magnitude of such premium will also depend on $\overline{r}$. The lower value of $\overline{r}$ (higher risk aversion of the individual) results in a higher premium.

4.2. First order stochastic dominance

The normative validity of a decision making model depends on several factors, including first order stochastic dominance. An important result of the first order stochastic dominance principle is the following. Assume $X \rightsquigarrow F$ and $Y \rightsquigarrow G$. If X first order stochastically dominates Y, and we write $X \succeq_{FSD} Y$, then $\int Z(x) \mathrm{d}F(x) \geq \int Z(x) \mathrm{d}G(x)$ for every nondecreasing function $Z : \mathbb{R} \to \mathbb{R}$.

We want to prove that $\succeq_P$ preserves first order stochastic dominance.

PROPOSITION 1. For any lotteries $X, Y \in L$ with $X \rightsquigarrow F$ and $Y \rightsquigarrow G$, we have that:

$$X \succeq_{FSD} Y \Rightarrow X \succeq_H Y.$$

Proof. By considering U as the c.d.f. of a benchmark W, we can interpret $\int_{\alpha}^{\beta} U(x) \mathrm{d}F(x) = \Pr\{X \geq W)$. Therefore, we can say that if $X \succeq_{FSD} Y$ then $\Pr\{X \geq W) \geq \Pr\{Y \geq W)$ for all the possible benchmarks. Let H_X be the benchmark associated to X. We have that:

$$X \succeq_{FSD} Y \Rightarrow \Pr(X \geq H_X) \geq \Pr(Y \geq H_X). \tag{2}$$

Since $X \succeq_{FSD} Y$ with $r_X \leq r_Y$, this implies that H_X is easier to outperform than H_Y. In particular, we know that:

$$\Pr(Y \geq H_Y) = \Pr(Y \geq H_X) + \Pi(Y, r_X, r_Y)$$

where $\Pi(Y, r_X, r_Y) \leq 0$ is the penalty attributed to Y. Thus, it follows that $\Pr(Y \geq H_X) \geq \Pr(Y \geq H_Y)$ and from (2):

$$X \succeq_{FSD} Y \Rightarrow \Pr(X \geq H_X) \geq \Pr(Y \geq H_Y).$$

Hence, we have proved that:

$$X \succeq_{FSD} Y \Rightarrow X \succeq_H Y.$$

5. POTENTIAL FRAMEWORKS FOR OUR DECISION CRITERION

Our criterion fits in some nonexpected utility models. Lottery dependent utility (LDU)[3], developed by Becker and Sarin (1987) and Schmidt (2001), is the most natural framework for deriving it.

Let $\mathcal{X}$ denote the set of all lotteries over the compact interval $[\alpha, \beta]$ of outcomes in $\mathbb{R}$. The preference functional $V : \mathcal{X} \to \mathbb{R}$ of LDU is given by:

$$V(X) = \int_{\alpha}^{\beta} u(x, h(X)) dF(x) \tag{3}$$

where $u(\cdot, \cdot)$ also depends on the lottery being evaluated, and $h(\cdot)$ is a continuous function on $\mathcal{X}$. For any X, Y, $X \succeq Y$ if and only if $V(X) \geq V(Y)$.

Becker and Sarin (1987) derive the main representation theorem by adding two properties of the utility function itself besides the axioms of weak ordering (O), continuity (C), and stochastic dominance. Moreover, most of the results they obtain are true under the assumption of linearity of $h(\cdot)$. This special case is labeled lottery dependent expected utility (LDEU).

LDU has not received a lot of emphasis in the decision theory literature. This might be because 1) it is difficult to test, and 2) the intuition behind it is not so immediate. To the first point, there is experimental evidence in the work proposed by Abdellaoui and Munier (1998) where preferences towards risk change according to the type of probability distribution individuals face. With respect to the second point, we believe we have taken a step toward clarifying the intuition. Our criterion provides a clear example on how the utility of an outcome might depend on the lottery itself. Let $h(\cdot)$ be the risk measure of the lottery: $h(\cdot)$ has an impact on the overall utility of the lottery itself.

In most financial contexts both stages of our criterion are useful to help people make more complete risky decisions. However, in some situations, for example when there are only few lotteries to be evaluated, it might be more efficient to start from the second stage on. In this particular case, our criterion, reduced to the second stage, naturally fits the axiomatization of lottery dependent utility developed by Schmidt (2001).

Schmidt (2001) makes an assumption on the function $h(\cdot)$ that is more general than the linearity proposed by Becker and Sarin (1987). He imposes that all the subsets with equal value of $h(\cdot)$ are σ-convex. The σ-convexity implies that if for any X_i such that $h(X_i) = c$ then $h(\sum_{i=1}^{\infty} \lambda_i X_i) = c$ for $\lambda_i \geq 0$, $i = 1, 2, ..., \infty$. This formulation is labeled lottery dependent convex utility (LDCU). Schmidt (2001) derives an axiomatization of LDCU by adding lottery dependent independence (LDI) and linearity (L) assumptions to the two usual O and C axioms. LDI imposes independence only for the lotteries that belong to the same subset and whose linear combination belongs to the same subset itself. L imposes that the preference relation between two lotteries belonging to different subsets still holds when we mix each lottery with whatever lottery belonging to the same subset.

The axiomatization of our criterion would follow the same steps proposed by Schmidt (2001). The intuition of the proof goes as follows -for details, see Schmidt (2001). We partition the set of lotteries in σ-convex subsets sharing the same risk measure. It is easy to show that $B_{r.}(\cdot)$ always has the functional form of EU, that is, $B_{r_X}(X) = E[U_X(t, r_X)]$ $\forall X \in L_{r_X}$. Since $B(X) = B_{r_X}(X) \quad \forall X \in L_{r_X}$ is true for every subset $L_{r.}$, we get that $B(X) = E[U_X(t, \rho(X))]$ $\forall X \in L$.

Prospect theory (PT) (Kahneman and Tversky, 1979), one of the most well known nonexpected utility theories, is another possible framework for our criterion. PT distinguishes two phases in the choice process: an early phase of editing and a subsequent phase of evaluation. The editing phase consists of a preliminary analysis of the offered lotteries, which often yields a simpler representation of these lotteries. Coding is one of the most well known operations of the editing phase: outcomes are framed as gains and losses defined relative to some neutral reference point. In the evaluation phase, the edited lotteries are evaluated and the lottery of highest value is chosen. In this phase, two scales are introduced: π that associates with each probability a decision weight $\pi(\cdot)$, and v that measures the value of deviations of the outcomes from the reference point. The overall value V of an edited lottery $(x, p; y, q)$ is expressed as follows:

$$V(x, p; y, q) = \pi(p)v(x) + \pi(q)v(y).$$

The first stage of our criterion and the editing phase of PT display some similarities. The objective of our first stage is "to organize and reformulate the options so as to simplify subsequent evaluation and choice" (Kahneman and Tversky, 1979, p. 274), too. We achieve this by discarding lotteries with a risk measure higher than $\overline{r}$, which can act as a reference point.

Both the evaluation phase of PT and the second stage of our criterion modify outcomes and probabilities. Consider a lottery X with risk measure r_X. In our setting, the value function v is represented by U. We transform U into the c.d.f. U_X of the benchmark H_X such that $U_X = \pi(U, r_X)$: this is exactly our decision weight. In this framework, we can rewrite $B(X)$ as follows:

$$B(X) = \int_{\alpha}^{\beta} \pi(U(t), r_X) dF(t)$$

where the transformation on the c.d.f. of the benchmark H also depends on the risk measure of X.

6. CONCLUSIONS

The design of prescriptive models for choice under risk characterizes a notable part of the current discussion in decision theory. The benchmarking procedure has prescriptive features, but not many applications of it have been proposed so far. We attempt to fill this gap by providing a decision criterion that represents an application of the benchmarking procedure to finance via a risk measure.

This criterion displays several advantages. First, it is prescriptively oriented because it represents an application of a prescriptive procedure. Second, it unifies different streams of research integrating the benchmarking procedure with risk measures and some nonexpected utility models, in particular LDU. Third, it is useful in finance. For example, benchmarks are used to evaluate mutual funds' performance. A portfolio is evaluated by comparing its performance with indices that reflect the fund's investment style. More specifically, the benchmarks constructed to measure the portfolio performance are based on the characteristics of stocks held in the portfolios that are evaluated. Therefore, our idea of a risk measure influencing the choice of the benchmark makes logical sense.

The choice of the benchmark still remains a crucial problem in many financial contexts. For example, in mutual funds it is relevant because it has an impact not only on the performance of the portfolio but also on the evaluations of funds' managers. We have proposed one simple and intuitive path to construct benchmarks. Much work remains to be done in this direction.

ACKNOWLEDGMENTS

We thank Philippe Delquié, Enrico Diecidue, Marco Li Calzi, Fabio Maccheroni and Ayse Önçüler for valuable suggestions. We are extremely grateful to two anonymous referees and the editor for helpful and constructive comments.

NOTES

1. Indeed, Bleichrodt et al. (2001) use the descriptive findings based on prospect theory but to improve prescriptive applications of expected utility.

2. A risk measure is coherent if it satisfies the four axioms of translation invariance, subadditivity, positive homogeneity, and monotonicity.

3. Modesti (2003) adapts the benchmarking procedure to LDU.

REFERENCES

Abdellaoui, M. and Munier, B.: (1998), The risk-structure dependence effect: experimenting with an eye to decision-aid Annals of Operations Research 80: 237-252.

Artzner, P., Delbaen, F., Eber, J. M. and Heath, D.: (1997), Thinking coherently Risk 10: 68-71.

Artzner, P., Delbaen, F., Eber, J. M. and Heath, D.: (1999), Coherent measures of risk Mathematical Finance 9(3): 203-228.

Arzac, E. R.: (1976), Profits and safety in the theory of the firm under price uncertainty International Economic Review 17(1): 163-171.

Becker, J. L. and Sarin, R. K.: (1987), Lottery dependent utility Management Science 33(11): 1367-1382.

Bell, D.: (1982), Regret in decision making under uncertainty Operations Research 30(5): 961-981.

Bell, D.: (1985), Disappointment in decision making under uncertainty Operations Research 33(1): 1-27.

Bleichrodt, H., Pinto, J. L. and Wakker, P.: (2001), Making descriptive use of prospect theory to improve the prescriptive use of expected utility Management Science 47(11): 1498-1514.

Bordley, R. F. and Li Calzi, M.: (2000), Decision analysis using targets instead of utility functions Decisions in Economics and Finance 23(1): 53-74.

Bordley, R. F. and Kirkwood, C. W.: (2004), Multiattribute preference analysis with performance targets Operations Research 52(6): 823-835.

Castagnoli, E. and Li Calzi, M.: (1996), Expected utility without utility Theory and Decision 41: 281-301.

Castagnoli, E. and Li Calzi, M.: (1999), Nonexpected utility theories and benchmarking under risk SZIGMA 29: 199-211.

Fishburn, P. C.: (1977), Mean-risk analysis with risk associated with below-target returns American Economic Review 67: 116-126.

Gigerenzer, G. and Selten, R.: (2001), Bounded rationality: the adaptive toolbox (Cambridge: MA, MIT Press).

Jorion, P.: (1997), Value at risk: the new benchmark for controlling market risk *Irwin*, Chicago.

Kahneman, D. and Tversky, A.: (1979), Prospect theory: an analysis of decision under risk Econometrica 47(2): 263-292.

Kataoka, S.: (1963), A stochastic programming model Econometrica 31(1/2): 181-196.

Loomes, G. and Sugden, R.: (1982), Regret theory: an alternative theory of rational choice under uncertainty Economic Journal 92(368): 805-824.

Loomes, G. and Sugden, R.: (1986), Disappointment and dynamic consistency in choice under uncertainty Review of Economic Studies 53(2): 271-282.

Modesti, P.: (2003), Lottery-dependent utility via stochastic benchmarking Theory and Decision 55(1): 45-57.

Munier, B.: (1999), Two stage rationality under risk: experimental results and perspectives Rivista di Matematica per le Scienze Economiche e Sociali 21(1/2): 3-23.

Quiggin, J.: (1982), A theory of anticipated utility Journal of Economic Behavior and Organization 3(4): 323-343.

Roy, A. D.: (1952), Safety first and the holding of assets Econometrica 20(3): 431-449.

Sarin, R. K. and Weber, M.: (1993), Risk-value models European Journal of Operational Research 70(2): 135-149.

Schmidt, U.: (2001), Lottery dependent utility: a reexamination Theory and Decision 50(1/2): 35-58.

Schmidt, U.: (2003), The axiomatic basis of risk-value models European Journal of Operational Research 145(1): 216-220.

Simon, H. A.: (1955), A behavioral model of rational choice Quarterly Journal of Economics 69(1): 99-118.

Simon, H. A.: (1956), Rational choice and the structure of the environment Psychological Review 63: 129-138.

Simon, H. A.: (1995), The information-processing theory of mind The American Psychologist 50(7): 507-508.

Starmer, C.: (2000), Developments in non-expected utility theory: the hunt for a descriptive theory of choice under risk Journal of Economic Literature 38(2): 332-382.

von Neumann, J. and Morgenstern, O.: (1944), Theory of Games and Economic Behavior. Princeton University Press, Princeton, NJ.

Yaari, M. E.: (1987), The dual theory of choice under risk Econometrica 55(1): 95-115.

Authors' Addresses:

Francesca Beccacece, Bocconi University, Institute of Quantitative Methods, Viale Isonzo 25, 20135 Milan, Italy. Phone: +39 (0)2 58365612; Fax: +39 (0)2 58365634. E-mail address: francesca.beccacece@unibocconi.it

Alessandra Cillo (Contact Author), INSEAD, Boulevard de Constance, 77305 Cedex, Fontainebleau, France. Phone: +33 (0)1 60712536; Fax: +33 (0)1 60745500. E-mail address: alessandra.cillo@insead.edu