

An Evolutionary Approach to Wetlands Design

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Abstract. Wetlands are artificial basins that exploit the capabilities of some species of plants to purify water from pollutants. The design process is currently long and laborious: such vegetated areas are inserted within the basin by trial and error, since there is no automatic system able to maximize the efficiency in terms of filtering. Only at the end of several attempts, experts are able to determine which is the most convenient configuration and choose up a layout. This paper proposes the use of an evolutionary algorithm to automate both the placement and the sizing of vegetated areas within a basin. The process begins from a random population of solutions and, evaluating their efficiency with an state-of-the-art fluid-dynamics simulation framework, the evolutionary algorithm is able to automatically find optimized solution whose performance are comparable with those achieved by human experts.

Keywords: Evolutionary Algorithms, Wetlands Design, Ecological Modelling

1 Introduction

Nowadays, more and more specialists are becoming involved in pollution control, one of the biggest problem of our time. Ecosystems are stressed by pollution. And organic chemicals, while contributing to their destruction, can also make the water not usable by animals and humans. To bring down the quantity of chemical dissolved in water in the latter case, researchers proposed a new approach, based on bio-geochemical processes naturally present in the environment, adopting *free surface constructed wetlands*. A wetland consist of a small artificial basin, partially flooded with water and containing many vegetated areas, in which the water flows and undergoes a natural filtering process from pollutants due to particular plant species, which are able to use these waste products to support its vital functions (e.g., *Phragmites Australis*, *Typha Latifolia*); vegetated areas have to be distributed over the wetland in order to increase the filtering performance. In the last half century a great effort in wastewater treatment has been performed with special plants able to process polluted water. It as been

demonstrated that this approach is more useful with *point sources*, characterized by little quantities of fluid polluted by high concentrations of chemicals, rather than *diffused sources*, characterized by big quantities of fluid polluted by low concentrations of chemicals.

To design a wetland, experts create several configurations which are then processed by a tool to simulate the flow of water and to calculate the efficiency in terms of filtering of the configuration sets. The classic *trial and error* approach is the only viable one, since it is not possible to implement an inverse function able to identify with precision positions and characteristics of each vegetated area to be inserted in the basin, in order to obtain an optimum filtering capability.

The proposed idea is to evolve a population of individuals, each one representing a complete configuration of vegetated area. The evolutionary approach is autonomously able to optimize the performance of the wetland, while an appropriate set of constraints enforces realistic configurations. The preliminary study of a system able to automatically calculate solutions for the problem was verified in [6]. Here, the goal is to tackle a realistic problem by include different constraints.

2 Background

2.1 Wetlands

Cowardin [5] defines a wetland as an ecosystem transitional between aquatic and terrestrial ecosystems, in which the water table is usually at or near the surface or the land is covered by shallow water [4]. Before the extensive land reclamation through the last century, wetlands were common along the coasts, where they functioned as a natural buffer between inner agricultural zones and coastal areas. Today there is a pressing necessity to restore these areas and their role, defining optimal design criteria to obtain, at reasonable costs, the best removal efficiency.

The removal efficiency of natural and constructed free-surface wetlands is controlled by the time spent by contaminants into vegetated zones [18]. The role of vegetation in wetlands is important for two main reasons: water passing through vegetated zones decreases its local velocity, favoring the sedimentation of suspended solids; and biochemical processes determine a transformation of the dissolved substances. In combination with bathymetry, distribution of vegetation can produce preferential pathways of water (hydraulic shortcuts) that can substantially decrease the overall efficiency of a wetland. Removal efficiency is also affected by other hydrodynamic characteristics, as water depth and discharge, both dependent on vegetation distribution and density [1] [14]. Wetlands constructed for waste water treatment are often designed considering an average water residence time [14], even though these methods cannot adequately describe spatial configurations of vegetation in real wetlands [15]. These models, usually called *zero-dimensional*, are often used because they require few data and are easy to manage. Nevertheless, zero-dimensional models produce significant inaccuracies in the prediction of the efficiency of contaminant removal. Other

one-dimensional models with transient storage were recently used [17] to assess the contaminant removal in a constructed wetland, giving in most cases a good approximation of breakthrough curves.

These models, however, fail to describe different flow paths across the vegetation and through main channels. The evidence of different flow pathways results in a clear bimodality of the solute breakthrough curves, that account for the different characteristic time scales of water residence time. Since spatial heterogeneity of the variables assumes a prominent role in determining the removal efficiency, the use of a more detailed *two-dimensional* approach becomes necessary to obtain reliable predictions.

2.2 Evolutionary Algorithms

Natural evolution is not a random process: while it is based upon random variations, their preservation or dismissal is determined by objective evaluations. Darwinian *natural selection* is the process where only changes that are beneficial to the individuals are likely to spread into subsequent generations, and sometimes it strikingly resembles an optimization process. Unlike most optimization processes, however, it does not require the ability to design intelligent modifications, but only the assessment of the effect of random modifications.

Several researchers, independently, tried to replicate such a characteristic to solve difficult problems more efficiently. Evolutionary computation does not have a single recognizable origin, but most scholars agree on identifying four macro areas: genetic algorithms [13], evolution strategies [20], evolutionary programming [9], and genetic programming [16].

The different paradigms share some key concepts, and can be cumulatively called evolutionary algorithms. An EA starts by generating an initial set of usually random candidate solutions for the given problem. These solutions, called *individuals*, are evaluated using problem-dependent metrics. The result of the evaluation, that is, the *goodness* of the solution, is termed *fitness*. The set of candidate solutions, also known as *population*, is then sorted on its fitness values. Subsequently, offspring is produced by altering the existing solutions: often the best solutions have a higher probability of being selected for reproduction. Offspring might be added to the existing population, or replace it entirely; in any case, some of the worst solutions are deleted before iterating the process, starting from reproduction. When a given stop condition is met, the iterations end and the best solutions are returned to the user.

Being based on a population, EAs are more robust than pure hill climbing. Both small and large modifications are possible, but with different probabilities. Sexual recombination makes it possible to merge useful characteristics from different solutions, exploring efficiently the search space. Furthermore, EAs are quite simple to set up, and require no human intervention when running. They are inherently parallel, and a nearly-linear speed-up may be easily achieved on multiple instruction/multiple data (MIMD) architectures. Finally, it's easy to trade-off between computational resources and quality of the results.

3 Proposed Approach

In the proposed approach the design of a wetland is fully automated exploiting an evolutionary algorithm. Each individual of the population represents a complete configuration of the wetland, expressed as a set of patches of vegetation arranged within the area of the basin; each vegetated area is defined by its position and diameter. The evolutionary algorithm handles the creation and evolution of individuals, while the actual evaluation is performed by a tool able to simulate the flow of water within the wetland and calculate the filtering capacity. Differently from the feasibility study, candidate solutions has been provided more stringent constraints in order to evolve towards optimized solutions close to a real ones. This constraint has been applied to the maximum area that can be covered by vegetation patches; the limit was set at 60%, in order to push the evolution towards the realization of optimized individuals describing more closely a configuration similar to those that are actually made.

3.1 Mathematical Models

A wetland is modeled using a two-dimensional depth averaged model that solves hydrodynamics, coupled with a two-dimensional solute transport equation with a first order decay term. Under the assumption of hydrostatic pressure, stationary flow, and negligible wind and Coriolis forces, the depth-averaged velocity field and water depth can be described by the following equations [23]:

$$\frac{\partial(hU)}{\partial x} + \frac{\partial(hV)}{\partial y} = 0 \quad (1)$$

$$\frac{\partial(hU^2)}{\partial x} + \frac{\partial(hUV)}{\partial y} = = -gh \frac{\partial z_s}{\partial x} + \frac{1}{\rho} \frac{\partial(hT_{xx})}{\partial x} + \frac{1}{\rho} \frac{\partial(hT_{xy})}{\partial y} - \frac{\tau_{bx}}{\rho} \quad (2)$$

$$\frac{\partial(hUV)}{\partial x} + \frac{\partial(hV^2)}{\partial y} = = -gh \frac{\partial z_s}{\partial y} + \frac{1}{\rho} \frac{\partial(hT_{yx})}{\partial x} + \frac{1}{\rho} \frac{\partial(hT_{yy})}{\partial y} - \frac{\tau_{by}}{\rho} \quad (3)$$

The quantities U and V represent the depth-averaged velocities [$m s^{-1}$] along the x and y direction, respectively, h is the water depth [m], z_s is the water surface elevation [m], and ρ the water density [$kg m^{-3}$]. The bed shear stresses τ_{bx} and τ_{by} [$N m^{-2}$] in the x and y direction respectively are calculated using the following relationships:

$$\tau_{bx} = \rho c_f m_b U \sqrt{U^2 + V^2} \quad (4)$$

$$\tau_{by} = \rho c_f m_b V \sqrt{U^2 + V^2} \quad (5)$$

In the case modeled here, the bed slope is set to zero and the investigated velocity range makes it possible to consider the friction coefficient as a constant. This assumption generally holds where the velocity is sufficiently fast to assume turbulent flow. For a flat bathymetry, the bed slope coefficient m_b is unitary and the coefficient of friction c_f can be rewritten using Manning equation as

$c_f = gn^2h^{-1/3}$. The effect of different vegetation densities is modeled here using different values of Manning roughness coefficient. This choice is confirmed by many studies that relate vegetation density, stem diameter and flow conditions to an equivalent roughness coefficient [3] [11] [21]. Fluid shear stresses $T_{ij}(i, j = x, y)$ associated to viscous and turbulent effects, are determined using the Boussinesq assumption:

$$T_{xx} = 2\rho(\nu + \nu_t)\frac{\partial U}{\partial x} \quad (6)$$

$$T_{xy} = T_{yx} = \rho(\nu + \nu_t)\left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}\right) \quad (7)$$

$$T_{yy} = 2\rho(\nu + \nu_t)\frac{\partial V}{\partial x} \quad (8)$$

where ν, ν_t , are the kinematic and eddy viscosities [$m^2 s^{-1}$]. Since the kinematic viscosity has a lower value than the eddy viscosity, it can be neglected in most cases. For a turbulent flow regime, as it was assumed in this preliminary study, ν_t can be expressed using Elder depth-averaged parabolic model [7] as $\nu_t = \alpha U_* h$, where the term α is an empirical coefficient [-] and U_* is the shear velocity [$m s^{-1}$]. For longitudinal dispersion Elder proposed a value of the coefficient α of about 5.9 [7], for transverse dispersion, Fischer found that α varies between 0.3-1.0 in irregular waterways with weak meanders [8]. In accordance with [2] [23] a value of 6.0 and 0.6 was chosen for the longitudinal and transversal dispersion coefficients respectively.

Solute transport of a reactive tracer through the wetland is simulated with a depth-averaged solute transport model accounting for the effect of advection, turbulent diffusion, dispersion and decay. In the simulations, the tracer is assumed to interact with vegetation and the chemical breakdown due to the permanence in the vegetated zones is modeled with a first order decay relationship. The equation governing the transport of a reactive tracer in the wetland can be modeled as:

$$\frac{\partial(hUC)}{\partial x} + \frac{\partial(hVC)}{\partial y} = \frac{\partial}{\partial x}(hE_x \frac{\partial C}{\partial x}) + \frac{\partial}{\partial y}(hE_y \frac{\partial C}{\partial y}) - h\lambda C \quad (9)$$

where C is the depth-averaged solute concentration [$kg m^{-3}$], U, V are the vertically integrated velocity components under steady flow conditions [$m s^{-1}$] in the x, y directions respectively. Coefficient E_x, E_y [$m^2 s^{-1}$], account for both turbulent diffusion and dispersion. A constant homogeneous value of E_x, E_y is chosen ($10^{-5} m^2 s^{-1}$) throughout the entire domain.

3.2 Evolutionary Core

The EA used is μ GP[19], is a versatile toolkit developed at Politecnico di Torino in the early 2000s and available under the GNU Public License from Sourceforge⁴. μ GP original use was to assist microprocessors' designers in the generation of programs for test and verification, hence, the Greek letter mu in its

⁴ <http://ugp3.sourceforge.net/>

name. But over the years has been used as optimizer in a much wider spectrum of problems, including numerical optimizations.

The algorithm initially creates a set of random candidate solutions to the given problem, that are then evaluated, and sorted by their fitness value (see Subsection 3.3). Offspring is then created favoring the fittest individuals and also trying to favor diversity among the population. New candidate solutions are then evaluated and added to the initial population. Solutions are again sorted, and the worst ones are removed until the population returns to its original size. The process is then iterated, starting from offspring generation, until a stop condition is reached.

Two categories of genetic operators are used to generate the offspring: *mutations*, or single-parent operators, and *crossovers*, or recombination operators. Mutation operators create new candidate solutions by altering one single parent solution; crossover operators mix the information contained in two or more parents solutions to create offspring. The most common operators are available inside μ GP, but the toolkit also implements *differential evolution*, and other operators specially calibrated for real parameters.

Individuals are internally represented as a multigraph, μ GP relies on an external configuration file constraints the multigraphs to sensible structure, and maps the internal individuals to valid solutions of the problem. In the specific context, each individual encodes a candidate configurations of the wetland, that is, it details the features of the several patches of vegetation, with variable number of occurrences from 20 to 35, that are going to be placed in the water; the order in which the patches are described within the individual is irrelevant. All islands are assumed to be of circular shape. Since they can overlap, however, they can create more complex shapes. An island is characterized by its position (x, y coordinates expressed in real values) in the wetland and its radius; in this simplified approach friction value is always the same. An island's position is constrained by the size of the wetland; its radius is constrained following the minimum and maximum size of actual islands of vegetation used in real wetlands.

3.3 Fitness Function

The definition of an appropriate fitness function is a key aspect in the use of an EA. The process of evolution is based on *differential survival*, that is, different individuals must have a different chance to spread their offspring in future generations. In the artificial environment modeled by an EA, it is essential that different individual get different fitness values. It is a common practice to include in the fitness some heuristic knowledge, in order to help the EA explore the most promising regions of the search space.

In μ GP, the fitness is not a single value but a vector of positive coefficients. The individual A is considered to be fitter than the individual B if the first j elements of the two fitness vectors are equals, and the $(j + 1) - th$ element of the A 's fitness is greater than the $(j + 1) - th$ element of the B 's fitness. In the context of wetland optimization, three values have been used.

In order to evaluate the goodness of a candidate wetland layout, a simulation of the hydrodynamic field is performed extracting computed values of discharge $Q [m^3 s^{-1}]$ and water depth h at the inlet and at the outlet sections of the wetland. During the simulation, a *reactive tracer* with a known concentration is injected at the inlet. Thanks to the presence of vegetation the tracer is gradually degraded and reaches the outlet section. Mass flux $\hat{M} [kg s^{-1}]$ passing through these sections is measured, and the difference between the two values represent the first parameter of the fitness function. In order to obtain the optimal vegetation distribution, this difference must be maximized.

On the other hand, a candidate layout must still let the water flow, avoiding configurations where the vegetation is so dense to make the flow impossible. The energy requested by the water to flow can be represented by the difference between the water depth at the inlet and outlet section. This difference represents the second parameter of the fitness function. This parameter is minimized by the algorithm: solutions that completely block the water flow are then heavily penalized.

The third and last fitness parameter measures the difference of discharge between the inlet and the outlet sections of the wetland. This value assures that the stationary flow conditions are reached and that the mass fluxes are finely computed. This discharge difference is strongly minimized.

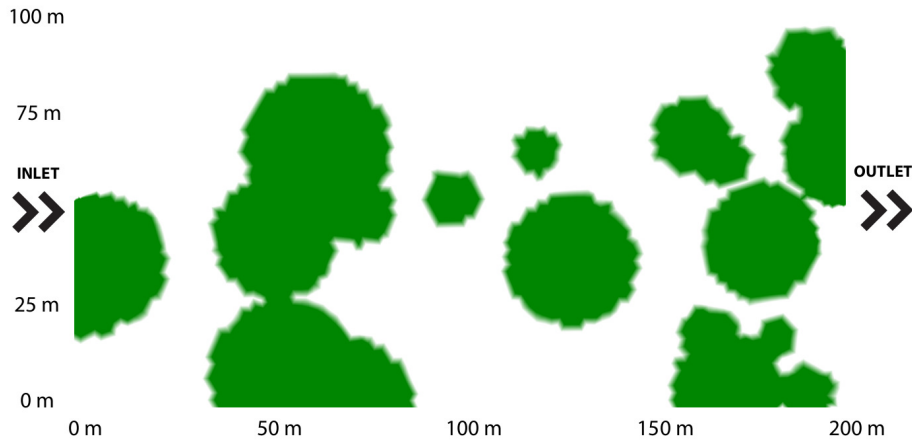


Fig. 1. Individual B: Representation of the phenotype of an individual extracted from the first generation of evolution; dark areas show the distribution of vegetation over the wetland surface.

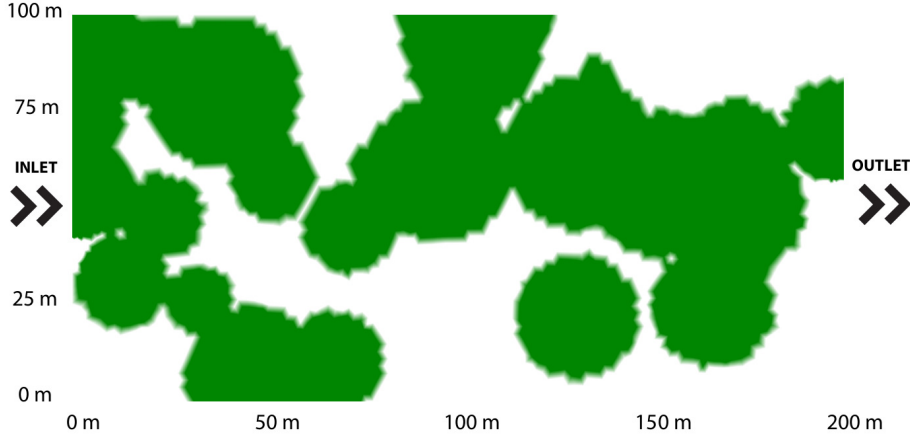


Fig. 2. Individual 7: Individual with percentage of vegetation next to the maximum limit but without good filtering performance, due to the distribution not optimized within the basin.

4 Experimental Evaluation

4.1 Setup

The artificial basin take into consideration in this work has a rectangular shape with dimensions $200m$ -long-by- $100m$ -wide, with a water depth considered constant over the entire surface and equal to $0.5m$. The inlet and outlet sections are located at the centre of the shorter sides of the wetland and have $10 m$ of size amplitude. In this way can be reached two important objectives: the first, related to the proportions of the area, concerns the total spread of the incoming water flow over the entire section of the basin; the second, due to the constant depth, makes this basin more similar to the natural ones and also makes it possible to simplify the system, which will not consider any slopes of the basin's bed [22] [23]. In addition, a constant discharge of $0.2m^3s^{-1}$ is imposed at the inlet section. The rest of the wetland was considered impermeable and laws of friction have not been applied at the side walls. In order to monitor the filtering process of the wetland, within the inlet section is injected a reactive solute with a constant concentration of $1kgm^{-3}$; in this way it is possible to extract the fitness value (which indicates the filtering capability of the basin) by calculating the average value of the concentration of this reagent in outlet area.

In order to simulate the hydrodynamic flow within the basin and the correct values of decay related to pollutants, it has been necessary to set some parameters into the simulation tool. The basin was defined through an adaptive triangular mesh, so as to ensure a sufficient numerical stability and the required resolution in case of steep gradients of the hydrodynamic and solute transport solutions. In addition, was applied to each node a value of the Manning roughness coefficient and a decay value, depending on the structure of each individual.

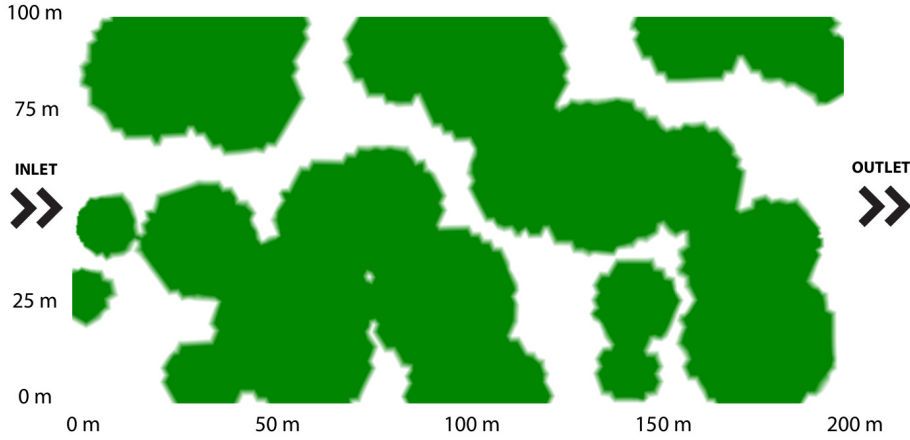


Fig. 3. Individual AAU: Representation of the individual that reached the best optimization level. The percentage of vegetation is close to the imposed limit to 60% but, thanks to the best arrangement of vegetation patches, its filtering performance is optimal.

In the particular configuration of this experiment, in which we impose the constraints that cannot exist individuals with vegetated area greater than 60% related to the total area of wetland, it has been chosen to simplify the decay coefficients, and the the structure of the vegetated patches. In conclusion, it was chosen to apply a single law of decay to a node of the mesh in which there is an island, or a zero coefficient otherwise; it was chosen a decay coefficient equal to $5^{-6} s^{-1}$. In the same way, Manning roughness coefficients are set to $0.20 sm^{-\frac{1}{3}}$ to nodes with vegetation, and $0.02 sm^{-\frac{1}{3}}$ otherwise.

As previously introduced, to achieve this automatic optimization system were used two different tools, both open-source and freely available on internet. The tool used for evolutionary algorithm is μGP version 3.2.0 (revision 198). To simulate and evaluate each individual instead was used a tool called *TELEMAC2D*, part of the wider set of programs *openTELEMAC* [10] [12]. The code of the latter has been specifically modified in order to extract information relating to fitness in the format required by the μGP tool.

Each individual evolved by the evolutionary tool is converted to the *TELEMAC2D* format, that consists of a map of basin's nodes, and each of these nodes can be covered or not by a vegetation patch. For this reason, each individual undergoes a sort of pre-processing that inserts in the nodes of the map values associated with vegetated areas. The process has been elaborated on a single machine, equipped equipped with an *Intel Core i7-950* CPU running at 3.06 GHz, and the whole system was setting in order to process up to 4 individuals simultaneously, with an average computation time of 90 minutes for each individual.

4.2 EA Configuration and Result Discussion

In order to obtain the results described in this paper, the EA has been configured in such a way to create a random initial population of 20 individuals ($\mu = 20$), on which they are applied, at each generation of the evolution, 12 genetic operators ($\lambda = 12$) chosen among the 20 available in μ GP tool. The entire process evolved for 90 generations, for a total of 1070 individuals generated. During the conversion of individuals to the format compatible with *TELEMAC2D*, a certain percentage of them was discarded because it was violating the introduced constraint about maximum area that vegetation patches can cover.

Starting from a random population, the evolution has shown several interesting features, which show the actual goodness of this approach. Among individuals of first generations, it's possible to find some as the individual *B* which are formed by a low number of vegetated areas clearly separated between them, configuration that shows a low filtration capacity; in particular, the configuration shown in Figure 1 ensures a performance of pollutant reduction of 21% respect to the inlet concentration. As evolution proceeds, grows the trend of evolutionary algorithm to generate individuals which respect the constraint of the maximum coverage and, using the maximum available number of islands, the EA is able to combine them to create complex shapes able to modify the water flow and to optimize the filtering performance.

The figure Figure 2 and Figure 3 compares the two individuals 7 and *AUU*, both characterized by a vegetated coverage very close to the imposed limit of 60%, but with different fitness. Individual 7 belongs to the third generation, in which evolution is still very close to the starting stage and, despite the use of maximum coverage allowed, performances in terms of filtering amount to 27%. Individual *AUU* instead represents the best configuration achieved in this experiment, comparable to previous in terms of vegetated area; in this case the filtering capacity has been optimized to achieve performances of 33.2%.

5 Conclusions

Wetlands are artificial ponds, and nowadays are extensively used to filtrate and purify water. Optimizing their design is an extremely complex task, and it is currently carried on by experts using a trial-and-error approach on the basis of fluid-dynamics simulations. In this paper, an evolutionary algorithm is applied to the wetlands design problem. Each candidate solution is evaluated by a state-of-the-art fluid-dynamics simulator, on the basis of several relevant metrics. Experimental results on the best solution provided by the algorithm show a performance comparable with human-devised designs, despite the absence of human intervention during the optimization process.

Future works will include a more complex individual representation, with patches of several different shapes and a more refined management of friction values. Managing larger populations, or different sub-population, might also prove beneficial to the quality of the final solutions: nevertheless, the computational-intensive simulations needed to evaluate a single candidate represent a severe bottleneck. For this reason, further developments will probably exploit the parallelism innate in evolutionary algorithms, using clusters or grids to speed up the process. Finally, the choice of decay coefficients has a predominant role in determination of the final breakdown efficiency: a more detailed analysis on a real case should be used to demonstrate the potential of the proposed approach, that shows promising results in this first experience.

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