Compressed sensing MRI combined with SENSE in partial $k$-space

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Compressed sensing MRI combined with SENSE in partial \( k \)-space

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Abstract
Compressed sensing (CS), parallel imaging and partial Fourier (PF) acquisition are all effective methods to reduce \( k \)-space sampling and therefore accelerate MR acquisition. The combined use of these methods gives us more options to balance the needs for scan speed and image quality. We conducted simulations on full \( k \)-space data to demonstrate the potential use of combining CS-SENSE with PF acquisition in anatomical MRIs of the human brain. To test the accelerated acquisition of high-resolution T1-weighted images of brain, we modified a 3D FSPGR sequence on a GE 3T scanner to implement different undersampling schemes based on CS, including partial Fourier CS-SENSE. Partially sampled \( k \)-space data were acquired and then reconstructed to brain images. CS-SENSE combined with PF sampling is able to provide better reconstructed images than CS only, or than CS-SENSE without PF for the same total acceleration. Combining PF sampling with CS-SENSE enables us to further accelerate image acquisition or improve image quality while holding the acceleration rate constant.

(Some figures may appear in colour only in the online journal)

1. Introduction
In conventional MRI with Fourier encoding, the number of \( k \)-space samplings is determined by the field of view (FOV) and image resolution. The straightforward way to accelerate image acquisition is to reduce the amount of data acquired in \( k \)-space, especially in phase-encoding directions. Usually this means reducing the resolution of the image in these directions. Partial \( k \)-space sampling is one way to reduce the number of \( k \)-space samples without reducing resolution, as it takes advantage of the Hermitian conjugate symmetry of \( k \)-space data for the real image (Noll \textit{et al} 1991, McGibney \textit{et al} 1993). Phase errors should be corrected if Fourier reconstruction is to produce images that are of good quality. Partial \( k \)-space sampling
can be used either to shorten the echo time (partial-k in the readout direction) or reduce the acquisition time (partial-k in the phase-encoding direction). With the development of multi-channel RF coils, parallel imaging (PI) MRI techniques (SENSE, SMASH or GRAPPA) can reconstruct images from multi-channel k-space data, which are sampled at a lower rate than the Nyquist rate required for a specific FOV, with the additional information on the sensitivity maps of all coil channels (Pruessmann et al 1999, Griswold et al 2002, Sodickson and Manning 1997). Although the theoretical maximum acceleration factor is decided by the number of coil channels, noise and imperfection in the coils limits the acceleration factor in practice to a level that is much lower than this maximum (Pruessmann et al 1999, Sodickson and Manning 1997). These PI techniques have been implemented extensively and effectively in both research and clinical settings. Compressed sensing (CS) applied in MRI (CS-MRI) exploits the fact that MRI images are sparse after certain transformations, and that Fourier encoding is incoherent under the sparse transformation (Candes et al 2006, Lustig et al 2007, Donoho 2006). SparseMRI is CS-MRI applied to Cartesian trajectories (Lustig et al 2007), which reduces k-space sampling based on the sparsity of MR images.

Because CS, PI and partial k-space sampling exploit different redundancies in MRI images, CS can be combined with PI or partial k-space sampling to further reduce acquisition time (Lustig et al 2007). The straightforward combination of CS with PI is SparseSENSE and its equivalent (Liu et al 2008, Zhao et al 2008, King 2008), which in the nonlinear convex problem of SparseMRI replaces Fourier encoding with sensitivity encoding. CS-SENSE, yet another way of combining CS with PI, separates CS reconstruction and SENSE reconstruction into two distinct steps (Liang et al 2009). In addition, PF acquisition, which can reduce scan time through asymmetric k-space sampling in the phase-encoding direction, can also be used in combination with either PI (King and Angelos 2000) or CS (Doneva et al 2010). CS, SENSE and PF all have their limitations, however, one of which is that their acceleration factors are highly constrained. Combining CS, SENSE and PF techniques can either enable highly accelerated imaging (Feng et al 2011), or give us more freedom to achieve the balance between scan time and image quality. In this paper, we (a) develop an approach for combining CS, SENSE and PF in MR image acquisition and reconstruction, and (b) implement this combined technique, which we term PF-CS-SENSE, to reduce acquisition time while maintaining image quality. Our simulation and experimental results show that PF-CS-SENSE can achieve a greater acceleration factor while maintaining image quality, and better image quality than CS-SENSE when using the same acceleration factor.

2. Theory

CS and SENSE can be combined one of two ways, either using SparseSENSE (Liu et al 2008, Zhao et al 2008, King 2008) or CS-SENSE (Liang et al 2009). In SparseSENSE, full k-space is sampled randomly using the same sampling scheme as that of SparseMRI, and final image is reconstructed from the undersampled k-space data from an array of coil channels by solving the same nonlinear optimization problem as in SparseMRI, except that sensitivity encoding replaces Fourier encoding. In contrast, CS-SENSE decouples CS and SENSE into two steps: first, k-space is reduced to produce an aliased image with reduced FOV; second, the already reduced k-space is randomly undersampled. Image reconstruction in CS-SENSE applies CS and SENSE steps sequentially. CS-SENSE has been shown to be superior to SparseSENSE in preserving image resolution, which is achieved by decoupling of CS and SENSE (Liang et al 2009). Regularization is used only in the CS component of the procedure, not in the SENSE component. The regularization at a lower acceleration factor for the CS component means that less resolution must be sacrificed to achieve the faster scan times. Nevertheless, noise is
amplified in the SENSE component and can be larger in CS-SENSE than in SparseSENSE. The acceleration factor for the SENSE component should be kept low to minimize noise amplification from ill-conditioned SENSE.

In PF acquisition, slightly more than half of $k$-space is sampled. Only the symmetric center portion of $k$-space data is used to calculate the low frequency phase map. In homodyne detection (Noll et al 1991), the amplitude of the asymmetric portion of $k$-space data is doubled in order to compensate the missing data, and then phase is removed from the reconstructed image and its real part is used to form the final image. PF acquisition and SENSE can be combined to yield further acceleration (King and Angelos 2000). In homodyne detection, phase information is removed from the final image after the last phase correction step. SENSE reconstruction, in contrast, needs to be applied before the phase correction step because SENSE processing cannot correctly unwrap aliased images without proper phase information. Similarly, PF acquisition and CS can be combined in a way in which the low frequency phase map serves as another constraint by performing the reconstruction based on a projection onto convex sets (POCS) (Doneva et al 2010).

We can combine together CS, SENSE and PF, in various ways. Figure 1 shows some examples of differing combinations of PF with CS and SENSE in both 2D and 3D cases, in which PF could be chosen either in a frequency-encoding direction or a phase-encoding direction. We also implement auto-calibration for PI, in which the center of $k$-space is fully sampled to produce sensitivity maps for the multi-channel coil. This step can be replaced with a separate calibration scan to get the sensitivity maps. Image reconstruction can implement the techniques for CS, SENSE and PF in differing combinations. PF reconstruction should be applied after SENSE because PF reconstruction normally removes the phase info of image, which is necessary for unfolding aliased images correctly in SENSE reconstruction (King and Angelos 2000). As already noted, we prefer CS-SENSE to SparseSENSE for the maintenance of image resolution. We then combine homodyne detection with CS-SENSE. Separate steps for CS, SENSE and PF are used to construct the final images from undersampled $k$-space data acquired in the scheme of PF-CS-SENSE (figure 1). Image reconstruction from undersampled $k$-space data includes three basic steps: (1) CS reconstruction for each coil channel using all the undersampled $k$-space data and only the symmetric part (figure 1, the region between the dashed lines) of the undersampled $k$-space data, respectively; (2) SENSE reconstruction.
using sensitivity maps obtained from the fully sampled center \( k \)-space data or a separate calibration scan; (3) correction of the final image using the phase map estimated from the symmetric portion of \( k \)-space data. Before CS reconstruction in step 1, the spectral weighting for homodyne reconstruction is first applied on the \( k \)-space data, e.g. doubling the amplitude of the asymmetric portion of \( k \)-space data (Noll et al 1991). Then CS reconstruction is performed using SparseMRI (Lustig et al 2007). With random undersampling in \( k \)-space, aliased images with reduced FOV from each coil channel can be simply reconstructed by solving

\[
\min \| \Psi m_i \|_{1, s.t.} \| F_0 m_i \|_2 < \varepsilon
\]

(1)

where \( \Psi \) is the sparse transform (e.g. wavelet transform), \( F_0 \) is the undersampled Fourier transform, \( y \) is the \( k \)-space data, and \( m_i \) is the aliased image reconstructed in \( i \)th coil channel.

For each channel, an aliased image with higher resolution is reconstructed from the whole undersampled \( k \)-space data and another aliased image with lower resolution is reconstructed from the symmetric portion of the undersampled \( k \)-space. In the SENSE reconstruction of step 2, the sensitivity map of each channel is calculated from the fully sampled center of \( k \)-space, and then both aliased images from step 1 can be unfolded using the SENSE algorithm (Pruessmann et al 1999) to produce a high-resolution image and a low-resolution image, respectively. The full FOV image, \( m \), can be reconstructed from the set of aliased images in all channels by solving

\[
S m = m_i
\]

(2)

where \( m \) is modulated by sensitivity matrix from all channels, \( S \), to generate a set of aliased images, \( m_i \). To obtain the final image in step 3, the phase map \( \phi \) estimated from the low-resolution image from step 2 is used to correct the high-resolution aliased image and yield the final image,

\[
m' = Re[me^{-i\phi}]\quad (3)
\]

Here the final image \( m' \) is obtained by taking the real part of the phase corrected image \( m \) (Noll et al 1991).

PF-CS-SENSE can be implemented in either 2D or 3D sequences (figure 1). For 2D acquisition, random sampling and SENSE can be easily implemented in the phase-encoding direction, and the PF acquisition can be implemented either in the same phase-encoding direction to reduce scan time or in the readout direction to reduce the minimum echo time (TE). The sampling schemes for 2D PF-CS-SENSE are shown in figure 1(a) to reduce TE (left) and to reduce scan time (right). The advantages of CS are better exploited in 3D acquisition, however, in which the readout direction in \( k \)-space \( (k_0) \) is fully sampled and \( k \)-space random sampling is easily implemented in the two phase-encoding directions \( (k_y \text{ and } k_z) \) (Lustig et al 2007). For simplicity of illustration, we consider only 1D SENSE and 1D PF sampling. In the 3D sampling scheme, SENSE undersampling of \( k \)-space and PF undersampling of \( k \)-space can be in the same or different phase-encoding directions (figure 1(b), left and right, respectively). When reconstructing an image from a 3D acquisition using PF-CS-SENSE, 1D FFT must first be performed along the fully sampled readout direction in \( k \)-space \( (k_0) \). The subsequent steps for reconstruction using PF-CS-SENSE are illustrated in figure 2.

The three mechanisms for \( k \)-space reduction—PF, CS and SENSE—can be combined in different ways to achieve the same overall acceleration factor, \( R \). We can tailor and optimize the end product by selecting the proper acceleration factors for PF, CS and SENSE separately. The acceleration factor for SENSE, \( R_S \), is usually limited by the number of the coil channels. Nevertheless, \( R_S \) should be kept low, e.g. \( R_S = 2 \) for an 8-channel coil, to avoid large noise amplification in the SENSE step (Liang et al 2009). The acceleration factor for PF, \( R_P \), is normally slightly less than 2, e.g. \( R_P \approx 1.78 \) for the case in which extra 16 rows in addition
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Figure 2. Flow chart of image reconstruction in 3D PF CS-SENSE with fully sampled $k$-space center for auto-calibration. The sensitivity maps can also be obtained with a separate calibration scan, instead of using the fully sampled center of $k$-space. The symmetric and asymmetric parts of $k$-space are shown in figure 1.

to half the plane are sampled for a $k$-space plane of 256 rows. The acceleration factor for CS, $R_C$, can be based on the sparsity of image and the overall acceleration factor desired. For example, assuming $R_S = 2$ with an 8-channel coil, for an overall acceleration of $R = 6$ in PF-CS-SENSE, $R_C \approx 1.8$; for the same overall acceleration of $R = 6$ in CS-SENSE, $R_C \approx 3.1$. Consequently, PF-CS-SENSE can yield superior signal-to-noise ratio (SNR) or resolution than CS-SENSE because of the smaller $R_C$ used in PF-CS-SENSE.

3. Materials and methods

We did all simulations on a MRI dataset available online (Lin, website link), which contains one slice extracted from a fully sampled 3D data acquired with an 8-channel coil. In a 3D acquisition, this fully sampled $k$-plane (acquisition matrix $256 \times 256$) is equivalent to the $k_y-k_z$ plane extracted after 1D FFT is performed along the readout direction ($k_x$). We used this dataset to test the undersampling schemes that would be used in 3D acquisition (figure 1(b)). The image reconstructed from the fully sampled data in eight channels using the conventional sum-of-squares (SoS) method was used as the reference image for comparison. We then performed simulations with differing schemes for $k$-space sampling on this dataset in which
the center of the $32 \times 32$ k-space data was used to estimate the sensitivity map of each coil channel. Our sampling schemes employed CS, SENSE and PF hierarchically to assess the contributions of each component to image quality: CS only, CS-SENSE and then PF-CS-SENSE (figure 1(b)). In the CS component, we used the Daubechies-4 wavelet as the sparse transform $\Psi$ in equation (1) and also included a total variation (TV) penalty (Lustig et al 2007). The weights for TV minimization and L1 norm minimization were 0.002 and 0.005, respectively. Inclusion of TV minimization would reduce the background noise, which was used in determining SNR of the reconstructed image (Rudin et al 1992). Image reconstruction was implemented in Matlab 7.7 (MathWorks, MA) using the toolboxes SparseMRI (Lustig et al 2007) and PULSAR (Ji et al 2007). In order to quantitatively measure the performance of differing schemes for k-space sampling, we calculated the correlation coefficients (CC) and the normalized mean square error (NMSE) between the reference image and the reconstructed image (Ying and Sheng 2007):

$$\text{NMSE} = \frac{\sum |I_{\text{estimated}}(x, y) - I_{\text{standard}}(x, y)|^2}{\sum I_{\text{standard}}^2(x, y)}$$

where $I_{\text{standard}}$ is the reference image reconstructed using the SoS method and $I_{\text{estimated}}$ is the image reconstructed from the undersampled k-space data. NMSE is the combined result from image noise, artifacts and resolution, which can measure the performance of differing undersampling schemes.

We acquired in vivo MRI data on a GE Signa 3T HDx scanner (GE Healthcare, WI) using an 8-channel head coil (InVivo, FL). Informed consent was obtained in accordance with the local IRB policy. To partially sample the k-space, we modified the standard pulse sequences to implement differing undersampling schemes. The modified sequences we tested were based on 3D FSPGR. The scanning parameters included: sagittal plane, freq. encoding direction = superior/inferior, FOV = 25.6 cm, acquisition matrix $256 \times 256$, 184 slices, 1.0 mm with 0 mm gap, TI = 500 ms, flip angle = 11°. In 3D acquisition, we could undersample k-space (i.e. CS-SENSE and PF-CS-SENSE) in two phase-encoding directions (within the $k_y$–$k_z$ plane, which in this case was the axial plane), with the center of the $k_y$–$k_z$ plane ($32 \times 32$) being fully sampled for SENSE auto-calibration (figure 1(b)). For the undersampled 3D data in k-space, 1D FFT was first performed on all available fully sampled $k_x$ lines. Then the reconstruction algorithm for CS-SENSE with or without PF was used to produce an image from the undersampled data in the $k_y$–$k_z$ plane. The standard 3D FSPGR with full k-space sampling required approximately 7.5 min of scan time. For 3D FSPGR implemented with CS-SENSE or PF-CS-SENSE, scan time was reduced to approximately 1.9 min for a total acceleration $R = 4$.

4. Results

Using the full k-space data for our simulation studies, we reconstructed images from the k-space data undersampled using differing combinations of CS, SENSE and PF. We did simulations for an overall acceleration $R$ of 5, 6 and 10, with differing undersampling schemes (CS, CS-SENSE and PF-CS-SENSE) (figure 3). The overall acceleration factor $R = R_C$ for CS only, $R = R_C \times R_S$ for CS-SENSE and $R = R_C \times R_S \times R_P$ for PF-CS-SENSE, where $R_C$, $R_S$ and $R_P$ were the individual acceleration factor for CS, SENSE and PF, respectively. With $R_S = 2$ and $R_P = 1.78$ in these cases, $R_C$ was adjusted to achieve the desired $R$. To evaluate the performance of differing undersampling schemes, we calculated CC and NMSE between the reference image from full k-space and the reconstructed images from undersampled k-space (figure 3), with the corresponding error images shown in figure 4. For the same undersampling scheme, the
Figure 3. Simulation results for total reduction $R = 5$ (row 1), $R = 6$ (row 2) and $R = 10$ (row 3) using differing $k$-space sampling schemes, with the reference image (left) from full $256 \times 256$ $k$-space. $R = R_C$ for CS only, $R = R_C \times R_S$ for CS-SENSE and $R = R_C \times R_S \times R_P$ for PF-CS-SENSE, where $R_S = 2$, $R_P = 1.78$, and $R_C$ is adjusted to get target $R$. ROI is selected in the reference image (white square box). The corresponding ROIs in the reconstructed images using differing sampling schemes are shown in the right panel: (1a–3a) CS only; (1b–3b) CS-SENSE; (1c–3c) CS-SENSE with PF using pattern in figure 1(b), left; (1d–3d) CS-SENSE with PF using pattern in figure 1(b), right. The numbers listed below each ROI image are the correlation coefficient (CC) and NMSE in this ROI between the reconstructed image and the reference image.

Figure 4. Error images corresponding to the entire images reconstructed in figure 3. Compared to CS, both CS-SENSE and PF-CS-SENSE show reduced noise in the background. Error images from PF-CS-SENSE show a little more structures in the brain than those from CS-SENSE.
CC decreased while the NMSE increased as $R_C$ increased, the consequence of more noise and artifact as $k$-space sampling grew increasingly sparse. For $R = 5, 6$ and 10, CS had the least CC and the biggest NMSE compared to CS-SENSE and PF-CS-SENSE with the same $R$, which meant that CS-SENSE and PF-CS-SENSE had improved quality for the reconstructed image. With the addition of SENSE to CS (CS-SENSE), image resolution was better preserved because less undersampling was needed for CS, whereas the SENSE component would amplify noise and possible artifact due to undersampling. Nevertheless, PF provided another degree of freedom for undersampling so that less CS undersampling was needed when applied in combination with PF sampling in order to achieve the same acceleration factor. For $R = 5$ and 6, PF-CS-SENSE had better CC and NMSE than CS-SENSE, while for $R = 10$, PF-CS-SENSE had slightly worse CC and NMSE. For PF-CS-SENSE, both CC and NMSE were close for differing combinations of PF and SENSE in figure 1(b) (figures 3(1c–3c) and 1d–3d). For $R = 5$ and 6, PF-CS-SENSE with PF and SENSE in the same direction (figure 1(b), left) gave slightly better CC and NMSE than that with PF and SENSE in the different directions (figure 1(b), right).

We also tested differing combinations of $R_S$ and $R_C$ for the same overall acceleration factor ($R = 6$ and 10) while $R_P$ was kept the same (figure 5). To achieve the same overall $R$, we needed a smaller $R_C$ for $R_S = 3$ than for $R_S = 2$. For both CS-SENSE and PF-CS-SENSE, the reconstructed images for $R_S = 3$ were of poorer quality than those for $R_S = 2$, according to the images and the calculated CC and NMSE values. The image improvement due to smaller $R_C$ in the CS component was not enough to compensate the degradation caused by larger $R_S$ in the SENSE component. For the 8-channel coil we used, $R_S$ should be kept as low as $R_S = 2$ to avoid large noise amplification. Table 1 summarizes the comparison among the differing combinations of CS, SENSE and PF in this simulation example.

We acquired in vivo human data with the modified 3D sequences implementing CS-SENSE and PF-CS-SENSE. Axial images were reconstructed from the undersampled 3D $k$-space data with a total acceleration factor $R = 4$ (figure 6). To estimate SNR, average signal was measured in selected ROIs (labeled as ‘1’ and ‘2’ in figure 6(a)) within the brain, and noise was measured in two large ROIs (labeled as ‘BK’ in figure 6(a)) outside of the brain. In the select ROIs of figure 6, SNR for CS-SENSE in ROI 1 was 9.6 and in ROI 2 was 7.2, whereas the average SNR for PF-CS-SENSE was 12.8 and 9.9, respectively. PF-CS-SENSE was superior to CS-SENSE in preserving resolution and improving SNR. Nevertheless, the sampling scheme employed in PF-CS-SENSE did influence image quality in this case, as the image using one sampling scheme (figure 1(b), left; figure 6(b)) produced less noise than that using the other scheme (figure 1(b), right; figure 6(c)), though both had the same total acquisition time.

The total time for reconstruction varies for these different undersampling schemes and reconstruction algorithms. Table 2 lists the reconstruction times of a single slice in our simulations in figures 3 and 5. All image reconstruction was carried out on a computer with Intel Core 2 Quad Q8200 CPU at 2.33 GHz and 4 GB RAM. Because our Matlab programs were written without using Parallel Computing Toolbox (MathWorks, MA), only single core of CPU was used in computation, which limited the reconstruction speed.

5. Discussion

Combining PF sampling and CS-SENSE enabled us either to further accelerate image acquisition or to improve image quality for a given acceleration factor. Inclusion of PF sampling accelerates image acquisition by a factor slightly less than 2 because it requires sampling slightly more than half of $k$-space. Another option when combining PF with
Table 1. CCs and NMSEs corresponding to the reconstructed images in figures 3 and 5.

<table>
<thead>
<tr>
<th></th>
<th>$R_{total} = 5$</th>
<th>$R_{total} = 6$</th>
<th>$R_{total} = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CS</strong></td>
<td>$R_C = 5$</td>
<td>$R_C = 6$</td>
<td>$R_C = 10$</td>
</tr>
<tr>
<td></td>
<td>CC = 0.9782</td>
<td>CC = 0.9758</td>
<td>CC = 0.9596</td>
</tr>
<tr>
<td></td>
<td>NMSE = 0.0041</td>
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<td>NMSE = 0.0079</td>
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<td>$R_C = 3.1$</td>
</tr>
<tr>
<td></td>
<td>CC = 0.9843</td>
<td>CC = 0.9793</td>
<td>CC = 0.9726</td>
</tr>
<tr>
<td></td>
<td>NMSE = 0.0029</td>
<td>NMSE = 0.0037</td>
<td>NMSE = 0.0049</td>
</tr>
<tr>
<td></td>
<td>$R_C = 2.1$</td>
<td>CC = 0.9506</td>
<td>$R_C = 3.7$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NMSE = 0.0094</td>
<td>CC = 0.9313</td>
</tr>
<tr>
<td><strong>PF-CS-SENSE</strong></td>
<td>$R_S = 3$</td>
<td>$R_C = 1.5^b$</td>
<td>$R_C = 1.8^b$</td>
</tr>
<tr>
<td></td>
<td>CC = 0.9857</td>
<td>CC = 0.9823</td>
<td>CC = 3.2$^b$</td>
</tr>
<tr>
<td></td>
<td>NMSE = 0.0026</td>
<td>NMSE = 0.0032</td>
<td>CC = 0.9693</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>NMSE = 0.0055</td>
</tr>
<tr>
<td></td>
<td>$R_C = 1.5^c$</td>
<td>CC = 0.9849</td>
<td>$R_C = 1.8^c$</td>
</tr>
<tr>
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<td>CC = 0.9814</td>
<td>CC = 0.9814</td>
<td>CC = 0.9694</td>
</tr>
<tr>
<td></td>
<td>NMSE = 0.0027</td>
<td>NMSE = 0.0033</td>
<td>NMSE = 0.0055</td>
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<td></td>
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<tr>
<td></td>
<td>$R_S = 3$</td>
<td>$R_C = 1.2^b$</td>
<td>$R_C = 2.1^b$</td>
</tr>
<tr>
<td></td>
<td>CC = 0.9577</td>
<td>CC = 0.9577</td>
<td>CC = 0.9407</td>
</tr>
<tr>
<td></td>
<td>NMSE = 0.0076</td>
<td>NMSE = 0.0076</td>
<td>NMSE = 0.0107</td>
</tr>
</tbody>
</table>

^a For PF-CS-SENSE, $R_p = 1.78$ in all cases.

^b PF and SENSE are in the same phase-encoding direction (figure 1(b), left).

^c PF and SENSE are in different phase-encoding directions (figure 1(b), right).
Figure 5. Reconstruction of image in figure 3 with differing combination of reduction factors $R_C$ and $R_S$. (a) CS-SENSE with $R_{total} = 6$; (b) PF-CS-SENSE with $R_{total} = 6$; (c) CS-SENSE with $R_{total} = 10$; (d) PF-CS-SENSE with $R_{total} = 10$. $R_P = 1.78$ in all cases. PF-CS-SENSE uses undersampling pattern in figure 1(b), left. In each panel (a–d), the upper row shows the ROI images with differing combination of $R_S$ and $R_C$; the lower row shows the entire error images with calculated CC and NMSE (within the selected ROI) listed below.

Table 2. Total reconstruction time\(a\) for a single slice in the simulations done in figures 3 and 5.

<table>
<thead>
<tr>
<th></th>
<th>CS-SENSE</th>
<th>PF-CS-SENSE</th>
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<tr>
<td>$R_S = 2$</td>
<td>$R_S = 3$</td>
<td>$R_S = 2$</td>
</tr>
<tr>
<td>$R_S = 3$</td>
<td>$R_S = 3$</td>
<td>$R_S = 3$</td>
</tr>
</tbody>
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Recon time per slice $\sim 170$ s $\sim 80$ s $\sim 60$ s $\sim 160$ s $\sim 120$ s

\(a\) Total time includes the reconstruction time for each channel and the time used to combine all channels for the final image.

CS-SENSE is to include PF sampling to achieve the same overall acceleration factor. When implementing PF sampling, a smaller acceleration in CS can be used to achieve the same total acceleration as in CS-SENSE to yield better resolution during the CS component of image reconstruction and less noise propagation during the SENSE component. Nevertheless, every undersampling scheme will necessarily suffer more or less loss of SNR compared to the schemes using full $k$-space data.

With PF-CS-SENSE, we can choose different combinations of $R_C$, $R_S$ and $R_P$ for a given total acceleration $R$. Proper choice of these individual acceleration factors depends on various considerations in a special case and can decide the final quality of image reconstruction. For
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Figure 6. Images from real data acquired with CS-SENSE (a) without and (b, c) with PF, all with total acceleration $R = 4$ ($R_S = 2$, $R_P = 1.78$ and $R_C \approx 2$ for CS-SENSE or $R_C \approx 1.3$ for PF-CS-SENSE). A 3D FSPGR was modified to implement different undersampling patterns in the phase-encoding plane ($k_y$,$k_z$), which in this case was the axial plane. To estimate SNR, average signal was measured in selected ROIs (labeled as ‘1’ and ‘2’) within the brain, and noise was measured in two large ROIs (labeled as ‘BK’) outside of the brain. Plots (b) and (c) show images produced using differing PF-SENSE combinations, i.e. PF and SENSE are in the same phase-encoding direction (figure 1(b), left) and in different phase-encoding directions (figure 1(b), right), respectively. The region indicated by the arrow shows better resolution and less noise in (b) than in (c).

PF sampling, the acceleration factor, $R_P$, is normally kept to a value slightly less than 2, and we do not have much choice to increase it. The acceleration factor of CS, $R_C$, is limited by the sparsity of the image (Lustig et al 2007). When the image is sparse, we can choose a larger $R_C$. For brain images, $R_C$ smaller than 3 is normally chosen for good-quality reconstruction. The smaller the $R_C$, the less the noise propagation to the SENSE component. The acceleration factor of SENSE, $R_S$, is limited by the number of channels of the coil and the g-factor (Pruessmann et al 1999). We can choose larger $R_S$ when the number of channels is larger and the g-factor is small. Figure 5 shows the reconstruction results from differing combination of $R_C$, $R_S$ and $R_P$ for a given total acceleration ($R = 6$ or 10). The images with $R_S = 2$ are better than those with $R_S = 3$ for the same total acceleration. In our case, we maintain it at $R_S = 2$ for our 8-channel head coil to avoid the excessive noise amplification that comes with ill-conditioned SENSE (Liang et al 2009). If we have a coil with more channels, we may increase $R_S$ without sacrificing much of image quality. In a summary, we can always optimize these acceleration factors ($R_C$, $R_S$ and $R_P$) to balance the requirements for acquisition speed and image quality. The choice of these acceleration factors will be based on multi-channel coil, characteristics of scan object and the need for scan speed.

PF-CS-SENSE will especially benefit 3D image acquisition, which normally requires long acquisition times. In 3D image acquisition, undersampling (using CS, SENSE or PF) can be implemented easily in either of the two phase-encoding directions. From our demonstration, PF-CS-SENSE normally has better reconstructed image than CS-SENSE for a given total acceleration (figures 3 and 6) because of the smaller $R_C$ used in the CS component. However, as the total acceleration factor increases (e.g., $R = 10$), both CS-SENSE and PF-CS-SENSE require larger $R_C$ ($R_C > 5$ for CS-SENSE and $R_C > 3$ for PF-CS-SENSE in our case). Because the brain image is not sparse enough, the larger $R_C$ produces more noise and artifact which propagate to the following reconstruction steps and greatly degrade the final image. The improvement caused by reduced $R_C$ may not be enough to compensate the degradation due to
PF. Therefore, the advantages of PF-CS-SENSE over CS-SENSE will diminish as the overall acceleration factor $R$ is increased too much (figures 3(a)–(d)). In addition, the precise ways in which the combination of PF, SENSE and CS is implemented may also influence the quality of the reconstructed images. Our simulation (figures 3(1c–3c) and 1d–3d) showed that 1D PF and 1D SENSE could be combined either in the same phase-encoding direction or in two different phase-encoding directions (figure 1(b)). In the in vivo scan we did (figure 6), the combination of 1D PF and 1D SENSE in the same phase-encoding direction yielded a better image with less noise in one region of the brain (indicated by the arrow in figure 6).

PF-CS-SENSE, which is a direct extension of CS-SENSE, is not the only way to combine CS, PI and PF. SparseSENSE can also be combined with PF (Doneva et al 2010, Feng et al 2011). Those problems inherent to CS-SENSE and SparseSENSE (Liang et al 2009) still exist when PF is included. The loss of resolution related to regularization with larger reduction factor is still a problem in SparseSENSE with PF (SparseSENSE-PF), as in SparseSENSE compared to CS-SENSE. Nevertheless, in PF-CS-SENSE, noise and artifacts due to undersampling from CS can be amplified in SENSE. In either PF-CS-SENSE or SparseSENSE-PF, the undersampling pattern in $k$-space and the algorithm for nonlinear image reconstruction can be optimized in different applications. Inclusion of PF further complicates the comparison between PF-CS-SENSE and SparseSENSE-PF. A careful and detailed comparison between them is worth further investigation in different applications.

Partial Fourier CS-SENSE will benefit from future developments in either the CS or SENSE algorithms, as the CS, SENSE and PF components in the PF-CS-SENSE image reconstruction are applied separately. Both PI and CS are fast-developing research fields in which new algorithms for CS or SENSE can be implemented easily. One major limitation of PF-CS-SENSE is its long reconstruction time. Nevertheless, reconstruction time will be reduced greatly with the rapidly improving computation power afforded by multi-core CPUs and GPUs (Chang and Ji 2010, Stone et al 2008).

In conclusion, we can combine CS, PI and PF acquisition to balance the needs for scan speed and image quality in MR image acquisition. Compared to CS or CS-SENSE only, combining PF sampling with CS-SENSE enables us to further accelerate image acquisition or improve image quality while keeping the acceleration rate.

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