Sports Eng (2010) 12:163–169
DOI 10.1007/s12283-010-0041-4

ORIGINAL ARTICLE

A determination of the dynamic response of softballs

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Published online: 1 May 2010
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Abstract An apparatus is described for measuring the stiffness and coefficient of restitution (COR) of balls with application to softballs. While standardized test methods currently exist to measure these properties, they do not represent the displacement rate and magnitude that occur in play. The apparatus described herein involves impacting a fixed, solid cylindrical surface (matched to the diameter of the bat) with a ball and measuring the impact force during impact and speed of the ball before and after impact. The ratio of the ball speeds determines the COR, while the impact force is used to derive a ball stiffness. For an example of the contribution of the new ball test, the performance of hollow bats, which is sensitive to ball stiffness, was compared. Bat performance showed a much stronger dependence on the proposed ball stiffness than the traditional measure. Finally, it was shown that to achieve similar conditions between impacts with fixed and recoiling objects, the impact speed should be chosen so that the centre of mass energy was the same in the two cases. The method has application to associations wishing an improved method to regulate ball and bat performance.

Keywords Coefficient of restitution · Ball compression · Ball stiffness · Ball performance

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1 Introduction

It has been well established experimentally that hollow aluminium or composite bats generally outperform solid wood bats of similar length and mass [1]. To preserve the traditional balance between the pitcher and batter, governing organisations seek to regulate the performance of non-wood bats. One of the key factors affecting bat performance is the ball–bat coefficient of restitution (BBCOR), denoted by e, which is defined as the ratio of the relative ball–bat velocity after the collision to that before the collision. It is related to energy dissipation in the collision since the fraction of the original energy in the centre of mass (CM) frame that is dissipated in the collision is $1 - e^2$ [2]. For a perfectly elastic collision, no energy is dissipated, the two bodies recede with the initial relative velocity, and $e = 1$. For a perfectly inelastic collision, the two bodies stick together, all the initial CM energy is dissipated, and $e = 0$.

To understand the BBCOR, it is necessary to describe briefly the dynamics of the ball–bat collision. The collision involves large forces acting over short times of order 1 ms, during which the ball compresses to a fraction of its undistorted radius, comes to a momentary halt, and then recovers to its original shape. This process is inherently inefficient, with a large fraction of the original kinetic energy dissipated in the internal structure of the ball. For baseballs and softballs e is on the order of 0.5, so that about 75% of the initial CM energy is dissipated during the bat–ball collision [2]. When the ball collides with a massive rigid object, such as a brick wall, all of the energy loss comes from dissipation in the ball and the corresponding COR is commonly referred to as the “ball COR,” denoted by e_0. For solid wood bats impacted in the barrel near the nodes of the lowest few bending vibrational modes (the so-called "sweet spot zone"), the bat is much less compressible than the ball, so that $e \approx e_0$ [3]. Hollow bats, on the other hand, are considerably more compressible than wood bats due to the compliance of the thin shell. As a result, the ball and the bat mutually compress each other during the collision, so that some of the CM energy that might otherwise have gone into compressing the ball instead goes into compressing the bat. Therefore, less energy is stored and dissipated in the ball and e is enhanced relative to e_0, resulting in a BBCOR which can be larger than e_0, sometimes considerably larger. This phenomenon is popularly referred to as the "trampoline effect" [4] and is one of the reasons that hollow aluminium or composite bats outperform wood bats.

If e is to be a meaningful metric of bat performance, it is necessary to control the properties of the balls used to measure it. One such property is e_0, where $1 - e_0^2$ is the fraction of the compressive strain energy stored in the ball that is dissipated. However, the forgoing description of the ball–bat collision makes it clear that there is a second property, the "stiffness" of the ball, a term that is independent of e_0 and that will be defined more precisely below. For a given bat, the ball stiffness controls how the initial CM energy is partitioned between strain energy stored in the ball and that stored in the bat. The larger the ball stiffness, the more strain energy is stored in the bat and the less strain energy is stored in the ball, leading to less overall energy dissipation and a larger value of e. Both e_0 and ball stiffness can affect e, and therefore both quantities need to be measured and controlled for balls used to test bats.

The following is primarily concerned with the impact response of 300 mm (12 in.) circumference softballs. The balls are made from a dense polyurethane foam core with a thin (1 mm) leather or synthetic cover. The formulation of the polyurethane core is proprietary, but allows e_0 and the ball stiffness to be independent. In a recent study [5], it was found that a 10% change in either quantity produced a similar effect on e for commonly used non-wood softball bats. Balls may be purchased commercially to desired values of e_0 and stiffness (identified on the ball as COR and compression, respectively). For commercially available softballs ball stiffness can differ by >100% between models, while e_0 differs by only 10%. The effect of ball stiffness, therefore, can be a significant factor in determining e.

The standard method for measuring e_0 is to impact the ball fired at 26.8 m/s (60 mph) onto a rigid flat surface [6], and e_0 is just the ratio of final to initial speed. The standard method for determining the stiffness of baseballs and softballs is to measure the peak force from displacing a ball 6.3 mm (0.25 in.) over 15 s between flat platens [7]. Dividing the peak force by the centre of mass displacement (3.2 mm or 0.12 in.) produces the quantity k_s, which is referred to herein as the quasi-static stiffness. There are potential problems with both techniques. For the e_0 measurement, the prescribed impact speed of 26.8 m/s (60 mph) is considerably lower than speeds typical of the game. For example, in upper level recreational slow-pitch softball play, the relative bat–ball speed can approach 49.2 m/s (110 mph) [8]. Given that e_0 is known to depend on the impact speed and that balls with nearly identical e_0 at low speed do not necessarily have identical e_0 at high speed [9], it is important to know e_0 at a higher speed for balls used to test bats. One focus of the present paper is to relate impact speeds with fixed and recoiling objects. For the stiffness measurement, the magnitude and rate of the displacement in the quasi-static test are 5 and 10,000 times lower, respectively, than under typical game conditions [5]. Given the known rate dependence of the modulus of the common polyurethane softball [10], the relevance of the quasi-static stiffness k_s to e is far from obvious. The second focus of this paper is to report on an apparatus to measure a dynamic stiffness of the ball k_d and to determine experimentally the relevance of k_d and k_s to determining the BBCOR.

2 Measuring ball stiffness

Balls were projected onto a fixed solid cylindrical surface, as depicted in Fig. 1, from which the impact force was measured. The half cylinder was intended to represent the shape of a softball bat, having a diameter of 57 mm (2.25 in.). The impact force was measured from an array of three load cells (PCB model 208C05), placed between the half cylinder and a rigid wall. Data from the load cells were summed and collected at 200 kHz, from which an impulse curve, as shown in Fig. 2, was obtained.

To ensure uniform temperature and moisture content the balls were stored at 22 ± 1°C (72 ± 2°F) and between 45 and 55% relative humidity for at least 14 days prior to testing. The balls rested for at least 2 min between impacts to minimize the effect of internal frictional heating. Each ball was impacted no more than 40 times so that ball degradation effects would be small [11].

Balls were projected at speeds ranging from 26.8 to 49.2 m/s (60–110 mph) using an air cannon. The balls travelled in a sabot, which separated from the ball prior to impacting the half cylinder. The sabot helped control ball speed and ball orientation. As shown in Fig. 1, the balls were impacted on the four regions of the ball with the largest spacing between the stitches. Ball speed before and after impact was measured using infrared light screens, placed as shown in Fig. 1.

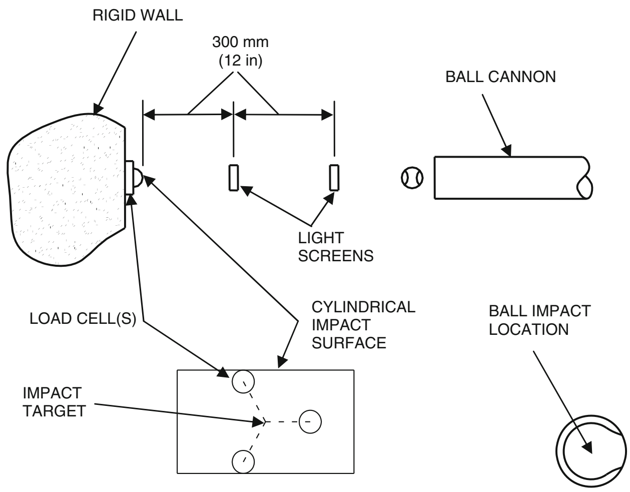

Fig. 1 Schematic of apparatus used to measure the softball impact force, *left*; diagram showing ball impact location, *lower right*

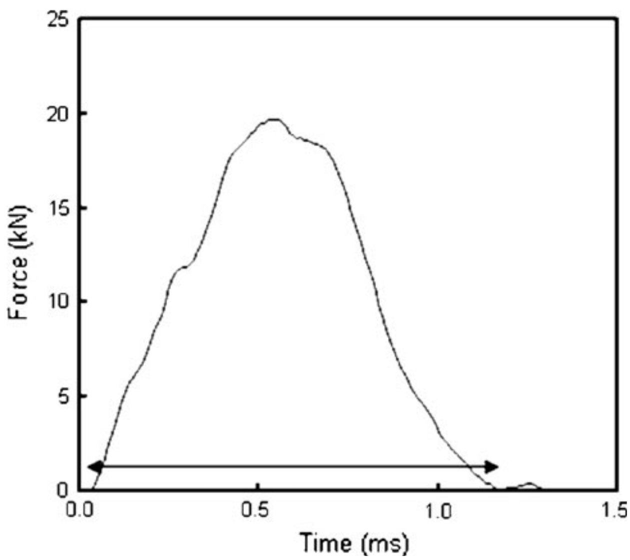

Fig. 2 Representative impulse curve of a softball impacting a fixed half cylinder at 42.5 m/s (95 mph). *Arrow* indicates range of integration to find impulse

The centre of mass displacement of the ball may be found by dividing the impact force by the ball mass and integrating over time twice. A representative ball force–displacement curve of a fixed cylinder impact at three speeds is shown in Fig. 3. The oscillations during the loading phase were observed for both fixed and free-cylinder impacts. The oscillations, therefore, are likely to be related to vibrations in the ball.

To derive the ball stiffness, it was assumed that the ball behaved as a non-linear spring during the loading phase according to $F = kx^n$, where F and x were the force and displacement of the spring, respectively, and k was the spring constant. The exponent n expressed the non-linearity of the spring. The dynamic stiffness was defined to be the

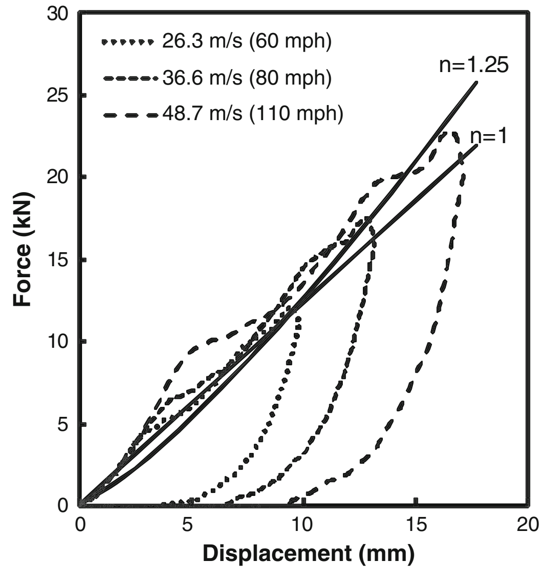

Fig. 3 Representative force–displacement curves (*dotted* and *dashed lines*) of a ball impacting a fixed cylinder. The *solid lines* are $F = kx^n$ for $n = 1$ and $n = 1.25$

value of the spring constant k_d corresponding to a linear spring, $n = 1$. A classical Hertzian contact results in a spring with $n = 1.5$. An expression for the spring constant may be obtained by equating the kinetic energy before impact with the potential energy at maximum deflection during impact as

$$k = \left[\frac{2}{m_b(n+1)}\right]^n \frac{F_p^{n+1}}{v_i^{2n}} \quad (1)$$

where m_b is the ball mass, F_p is the peak impact force, and v_i is the incoming ball speed. The force–displacement curve corresponding to $n = 1$ and $n = 1.25$ are shown in Fig. 3. The dependence of the peak impact force on the initial speed appears to be slightly non-linear, and will be discussed in more detail below.

The ball speed and impact force each provide an independent measure of the impact impulse. Ideally these measures would be equal, according to

$$m_b(v_i + v_r) = \int F(t)\mathrm{d}t \quad (2)$$

where v_r is the rebound ball speed and $F(t)$ is the measured impact force as a function of time. While experimental error prevents the equality, the redundant measures can be used to improve the accuracy of the result, especially when comparing results for two different test fixtures. If, for instance, the speed measurements are more accurate than the force measurements,[1] the latter may be scaled according to

[1] This scenario is plausible given the 1% calibration error typical of most load cells in comparison to the 0.1% accuracy of speed measurements attainable from modern light screens and high speed timers.

$$F'(t) = F(t)\frac{m_b(v_i + v_r)}{\int F(t)dt} \tag{3}$$

where $F'(t)$ is the scaled impact force. To investigate the effect of this procedure on k_d ($n = 1$), 12 balls were tested on two separate fixtures. The procedure for measuring k_d was similar to that used for ball COR [6]. Each ball was impacted six times, from which the average k_d was determined for each of the fixtures. These average values differed by 3% using the measured force, while the difference decreased to 1.5% when the scaled force was used. Since the dynamic stiffness results reported below were obtained on the same fixture, the load scaling procedure described above was not used.

3 Ball stiffness results

The peak force and spring constant k are plotted as a function of incoming speed in Fig. 4. To compare these quantities on a common axis, they were normalised by dividing by their respective values at the maximum speed. The values of k calculated from Eq. 1 are included in the figure for $n = 1$ and $n = 1.25$ to illustrate the degree of non-linearity of the ball. It was observed that the spring constant was independent of speed when $n = 1.25$ (less than the $n = 1.5$ classical Hertzian contact). This result is surprising as large deformation effects tend to increase the exponent for Hertzian contact [12].

The impacts considered here have three components of non-linearity. First, classical small deformation Hertzian contact is non-linear. Second, the geometric effects of large deformation Hertzian contact are non-linear. Third, the core of the softball is made from polyurethane, which undergoes non-linear softening with large deformation [10]. While the small and large geometric components of non-linearity tend to increase n, the material softening tends to lower n. The present results indicate that the effect of material softening is more dominant than the geometric non-linearities.

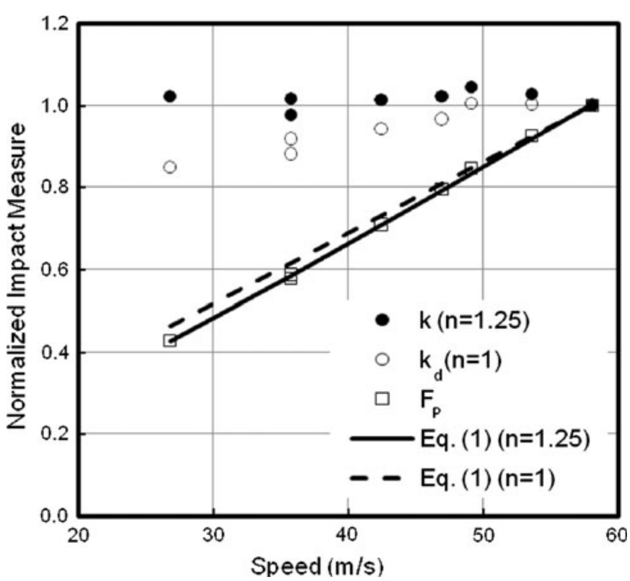

Fig. 4 Peak impact force and spring constants as a function of incoming ball speed (values were normalized with respect to the 58 m/s or 130 mph case). The lines were obtained by solving Eq. 1 for F_p

One aim in the ball dynamic stiffness study was to develop an improved metric for the purpose of regulating ball stiffness. It is desirable, therefore, to simplify its evaluation to encourage acceptance without impeding its utility as a ball stiffness metric. Accordingly, Eq. 1 is utilized with $n = 1$, so that it simplifies to

$$k_d = \frac{1}{m_b}\left(\frac{F_p}{v_i}\right)^2. \tag{4}$$

When k_d is used to compare ball stiffness, the test speed is constant, so that the non-linear effects on the relative stiffness between balls are negligible. The following comparisons of ball stiffness were accordingly performed using k_d ($n = 1$).

The k_s and k_d (with $v_i = 95$ mph or 42.5 m/s) values of 150 softballs comprising 18 models from five manufacturers are compared in Fig. 5. The results show a measurable difference between the two quantities. The dynamic stiffness, k_d, is consistently larger than k_s by more than a factor of two, a feature consistent with polymer rate effects, where modulus increases with the displacement rate. While there is substantial scatter in the comparison, the data generally fall in three groups, identified as "a", "b", and "c" in the figure. The balls in groups "a" and "b" are of a

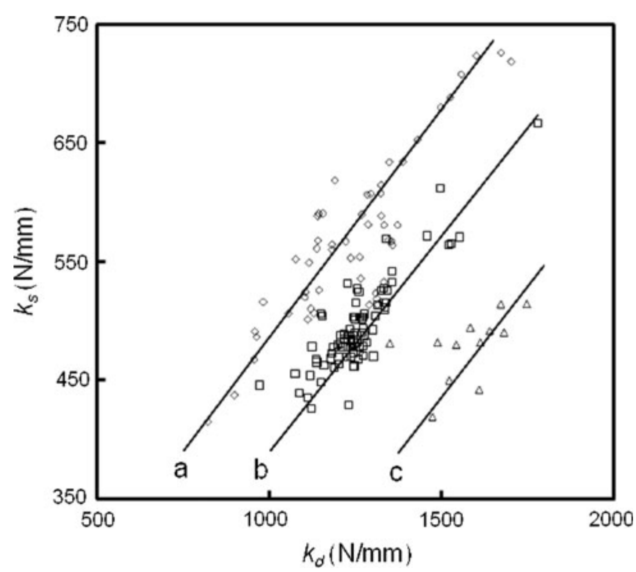

Fig. 5 Comparison of k_s and k_d of 150 softballs. The three plotting symbols correspond to balls from different manufacturers

similar one-piece core construction and are likely to be produced from different urethane formulations. (A relatively small number of overseas facilities produce balls for a large number of domestic companies.) The balls in group "c" have a unique construction having a core that is made of a relatively soft 3-mm thick outer shell and a harder interior. The figure highlights potential shortcomings of using quasi-static measures to imply impact response.

4 Effect of ball stiffness on the BBCOR

The effect of k_s and k_d on the BBCOR, e, was investigated next. To this end, four bats were tested according to ASTM F2219 [13]. A ball was fired from a high-speed air cannon onto the barrel of an initially stationary bat, which was free to recoil after impact by pivoting about a point on the handle. The inbound and rebound ball speeds, v_i and v_r, respectively, were measured, from which e can be derived using [2]:

$$e = \frac{v_r}{v_i}(1 + r) + r, \tag{5}$$

where the so-called recoil factor r is given by

$$r = \frac{mb^2}{I}, \tag{6}$$

where m is the ball mass, b is the distance from impact point to pivot, and I is the moment of inertia of the bat about the pivot. The nominal value of v_i was 49.2 m/s (110 mph). The bats included a solid wood bat, an aluminium bat, and two composite bats. Softballs were selected from the groups a and b of Fig. 5 to provide similar COR and varying k_d. The results of the study are presented in Figs. 6, 7, in which e is shown as a function of k_s in Fig. 6 and as a function of k_d in Fig. 7.

The results in Figs. 6, 7 show that e is insensitive to k_s or k_d for the wood bat. Indeed, no trampoline effect is expected for a solid wood bat due to the near incompressibility of the barrel, so no dependence of e on either the quasi-static or dynamic ball stiffness is expected. If the trampoline effect is active, then a monotonic increase in e with the ball stiffness for a hollow bat is expected. When plotted versus the quasi-static stiffness, no clear monotonic effect is observed (Fig. 6). However, when plotted versus the dynamic stiffness, the expected effect is found (Fig. 7). The comparison illustrates that it is the dynamic stiffness and not the quasi-static stiffness that governs the size of the trampoline effect in hollow bats.

5 Establishing an appropriate test speed

A desired outcome of this work was to measure ball properties that may subsequently be used to normalize e to

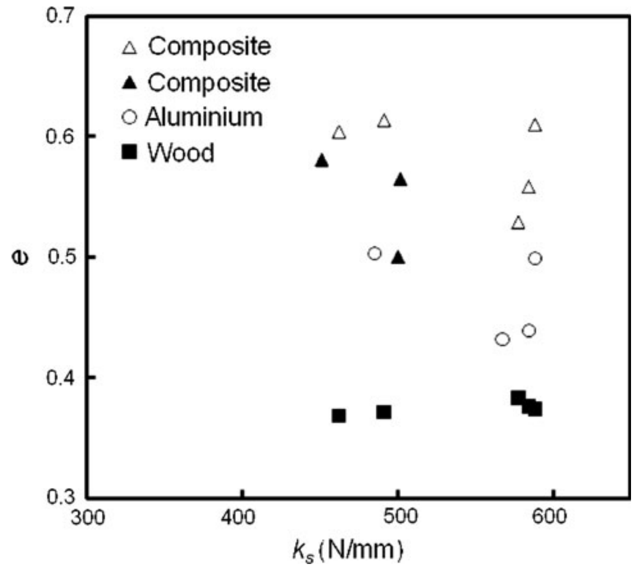

Fig. 6 BBCOR, e, as a function of as a function of k_s

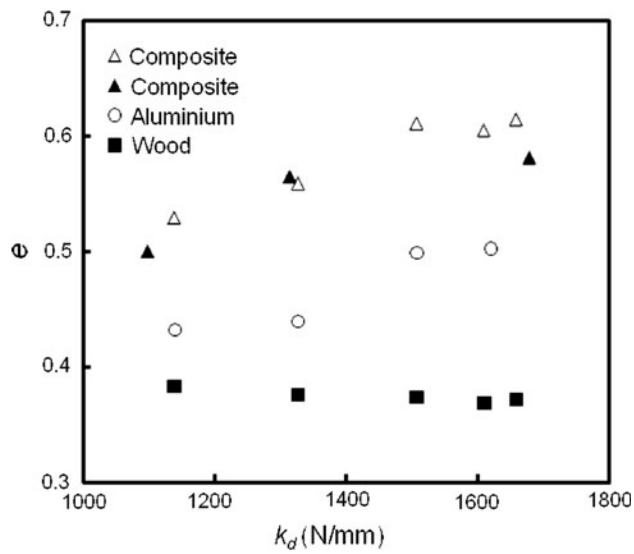

Fig. 7 BBCOR, e, as a function of k_d

variations in e_0 and k_d. The data in Fig. 4 show that k_d increases with increasing ball speed. Moreover, it is well known that e_0 decreases with increasing speed, as is typical of many sport ball impacts [14–16]. Since both e_0 and k_d change with speed, the question arises as to what test speed in the impact of the ball with a rigid wall corresponds to the speed of the ball when impacting a bat that is free to recoil. The rigid-wall test is distinguished from the bat test condition by the effective mass that the ball impacts in each case. While the effective mass of the rigid wall is infinite, the effective mass of the recoiling bat is finite. To state the issue more precisely, suppose v is the initial speed when the ball impacts a rigid wall. What is the corresponding speed v^* when the ball impacts a bat with recoil factor r

(defined in Eq. 6) that would give the same values of e_0 and F_p? One possible answer is that v^* is the same initial velocity in the two cases. Another is that v^* is such that the CM energy is the same in the two cases. A third is that v^* is the velocity such that the impulse to the ball is the same in the two cases. These three possibilities can be written in the following compact form

$$v_p^* = v(1 + r)^p, \quad (7)$$

where $p = 0$, ½, and 1 correspond to constant initial speed, constant CM energy, and constant impulse, respectively. For the case of F_p, when impacting an otherwise rigid object that is free to recoil (such as a solid wooden bat in the sweet spot zone), all of the CM energy must be converted to strain energy in the ball. Therefore, it is the CM energy that determines the maximum displacement, F_p, and k_d. The answer is not as obvious for the case of e_0. When compared to the recoiling case, constant CM energy for the rigid wall necessarily means the same maximum displacement and peak force but a smaller impulse. Constant impulse means a smaller maximum displacement and peak force and the same impulse. The question then reduces to which effect is more important for determining e_0: the maximum displacement or the impulse? To the author's knowledge, there has been no investigation of this question previously in the literature. However, in an experiment done long ago [17], it was claimed without justification that constant impulse is the correct answer, a result adopted more recently by Adair [14].[2]

To investigate these issues, values of e_0 obtained from impacting a fixed cylinder to those obtained by impacting free solid cylinders of differing mass (and therefore differing values of r) were compared. The impact force, F_p, was also compared in the free-cylinder impacts using an accelerometer mounted to the back side, as shown in Fig. 8. To reduce error associated with the different means of force measurement (load cell for the fixed case and accelerometer for the free-cylinder case), the loads were scaled with the speed measurements as described by Eq. 3. Solid-free cylinders were made from aluminium (716 g or 25.2 oz) and steel (2,043 g or 71.9 oz). The cylinders measured 57 mm (2.25 in.) in diameter and were 102 mm (4 in.) long, while the softballs had a nominal mass of 200 g (7 oz). Impacting at the centre of relatively short cylinders minimized energy loss due to bending vibrations and resulted in a recoil factor that was the ratio of ball to cylinder mass [2].

[2] Briggs carried out the "inverse experiment," in which a block of wood was projected onto a baseball initially at rest. To determine the equivalent "ball on wall" speed, Briggs used Eq. 7, with $p = 1$ (constant impulse).

Softballs were impacted against the aluminium-free cylinder at speeds ranging from 30 to 55 m/s (67–124 mph). The values for e_0 and F_p were taken from the average of six balls, where each ball was impacted six times. The steel cylinder was impacted similarly at speeds ranging between 28 and 51 m/s (63 and 115 mph). The resulting values of e_0 and F_p are shown in Figs. 9, 10, respectively. The values are plotted against the three possibilities of v_0*, $v_{1/2}$* and v_1*. It is clear from the plots that the values of both e_0 and F_p fall on a smooth curve only when plotted versus $v_{1/2}$*, demonstrating conclusively that the appropriate value of v^* is the one corresponding to constant CM energy (i.e., $p = 1/2$ in Eq. 7) for both e_0 and F_p.

6 Conclusions

The foregoing described an apparatus to measure the stiffness, k_d, of a ball under conditions representative of play. The ball stiffness was derived from the peak impact force, ball mass, and incoming ball speed. For the softball impacts considered here, the ball stiffness was shown to increase with incident speed, but at a lower rate than the peak impact force. The speed dependence was attributed to geometric and material non-linearities. Taking advantage of redundant measures of impulse, a method was presented that may improve the reproducibility of the ball stiffness measure beyond that typically attainable from load cells alone. The proposed dynamic test distinguished differences in ball response that were not apparent using standard quasi-static measures (k_s). It was shown that it is k_d and not k_s that governs the size of the trampoline effect in hollow bats. Finally, it was shown that to achieve similar conditions between impacts with fixed and recoiling objects, the

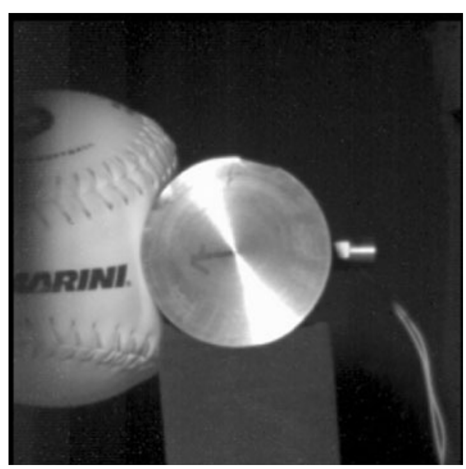

Fig. 8 Softball impacting a free cylinder, supported on a compliant foam pedestal. Accelerometer is shown opposite the impact surface

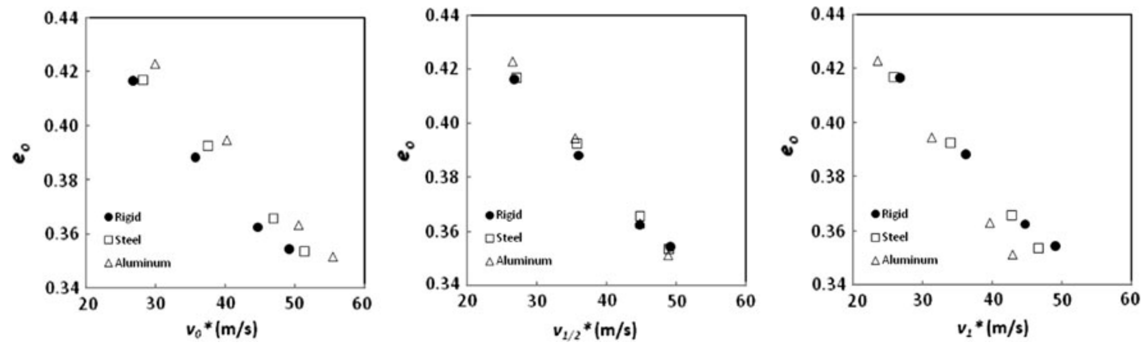

Fig. 9 COR as a function of impact speed (v_p*), where v_p* equals the incident velocity ($p = 0$), the constant-CM velocity ($p = 1/2$), and the constant-impulse velocity ($p = 1$)

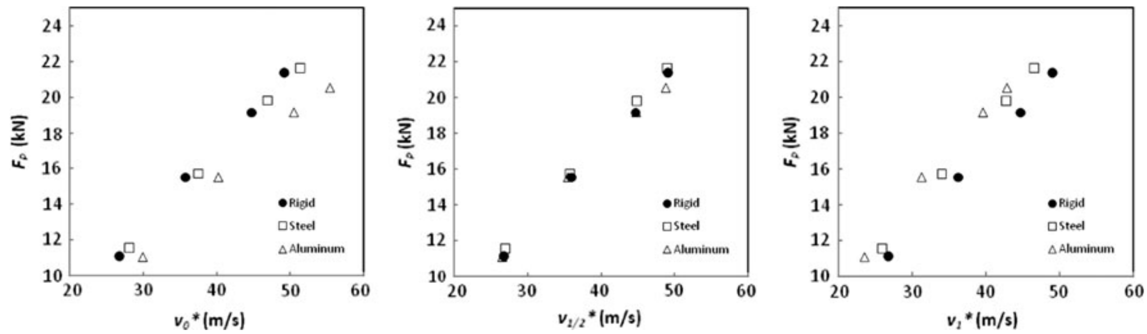

Fig. 10 Peak impact force as a function of impact speed (v_p*), where v_p* equals the incident velocity ($p = 0$), the constant-CM velocity ($p = 1/2$), and the constant-impulse velocity ($p = 1$)

impact speed should be chosen so that CM energy is the same in the two cases.

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