Robust observer-based fault tolerant control for vehicle lateral dynamics

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Abstract — This paper presents on line Sensor Fault Detection, Isolation (FDI) and the associated fault tolerant control (FTC) algorithm for a vehicle lateral dynamics represented by the uncertain Takagi-Sugeno (TS) fuzzy model. Fault detection scheme based on a bank of observers based method is considered. Using the LMI formulation, the T-S fuzzy model of the vehicle nonlinear dynamics is used to design an observer based output feedback controller. To demonstrate the effectiveness of the proposed strategy, simulation results are given.

Keywords: Fuzzy model, observer, controller, LMI, Bank of observes, Fault-tolerant control, FDI, vehicle lateral dynamics.

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1 Introduction

To satisfy increasing safety, efficiency and comfort requirement, many new vehicles are equipped with different driver assisted systems such as: Traction Control System (TCS), Dynamic Stability Control (DSC-BMW, Jaguar); Electronic Stabilization Program (ESP-Audi, Mercedes-Benz, SAAB ESP Electronic Stability Program. Robert Bosch GmGH, Stuttgart) (Bauer, 1999) These systems use a combination of wheel speed, ABS information, yaw rate, lateral acceleration and steer angle to improve the active safety by stabilizing the vehicle in extreme driving situations (Kienck and Nielsen, 2000). Therefore the lateral control system must have fault tolerant ability such that the system maintains stability and acceptable performance even when some sensors have failed.

The most common approach in dealing with such a problem is to split the overall design in two distinct phases. The first phase addresses the so-called “fault detection and isolation” (FDI) problem, which consists in designing a dynamical system (filter) which, by processing input/output data, is able to detect the presence of an incipient fault and to isolate it from other faults and/or disturbances (Isermann, 2001; Ding, Schneider, Ding and Rehm, 2005; Blanke, Kinnaert, Lunze and Staroswiecki, 2003; Gertler, 1998; Oudghiri, Chadli and A. El Hajjaji, 2007; Oudgiri, Chadli and A. El Hajaji, 2008). Once the FDI filter has been designed, the second phase usually consists in designing a supervisory unit which, on the basis of the information provided by the FDI filter, reconfigures the control so as to compensate for the effect of the fault and to fulfill performance constraints. In general, the latter phase is carried out by means of a parameterized controller which is suitably updated by the supervisory unit.
Our objective is to develop model-based FTC-scheme for vehicle lateral dynamics. This study is motivated by the practical demands for such monitoring systems that i) automatically and reliably detect and isolate faults from sensors ii) deliver reliable and fault tolerant estimates of the vehicle lateral dynamics and iii) are practically realizable. In this paper, we propose an observer-based fault tolerant control to detect, identify and accommodate sensor failures. The given method is based on the single failure assumption which states that at most one sensor can fail at any time.

To know the vehicle response, the proposed controller needs to know the yaw rate and the lateral velocity in order to generate the suitable output. If the yaw rate can be directly measurable by a yaw rate sensor (gyroscope), the lateral velocity will have to be estimated using an observer because it is not measurable easily. In this paper, a fuzzy controller is designed by considering the lateral velocity estimated using an observer. In the analysis and design, the vehicle lateral will be represented by a Takagi-Sugeno (T-S) fuzzy model (Takagi and Sugeno, 1985), largely used these last years (Xioodong and Qingling, 2003; Chadli, Maquin and Ragot, 2005; Kirakidis, 2001; Tanaka and Wang, 1998; Chadli and El Hajjaji, 2006). It is usually referred to as the bicycle model. Moreover, we consider the uncertain Takagi-Sugeno (T-S) fuzzy model to describe the vehicle dynamics in large domains and by the same way to improve the stability of vehicle lateral dynamics (Oudghiri, Chadli and A. El Hajjaji, 2007b; Chadli, El hajjaji and Oudghiri, 2008). The proposed algorithm is formulated in terms of linear matrix inequalities (LMI) (Boyd and al, 1994) which are easy to implement using classical numerical tools (such as LMI Toolbox for Matlab).

The structure of the paper is organized as follows. Section 1 describes the vehicle lateral and its representation by uncertain T-S fuzzy model. Sections 2 and 3 present the observer-based fault tolerant control strategy. The fourth Section is dedicated to simulations of sensor faults and result analysis. Conclusions are given in Section 5.

Notation: symmetric definite positive matrix $P$ is defined by $P > 0$, the set $\{1, 2, ..., n\}$ is defined by $I_n$ and symbol $*$ denotes the transpose elements in the symmetric positions.
2 Vehicle Model

Vehicle lateral dynamics have been studied since the late 1950’s. Segel (Segel, 1956) developed a three-degree-of-freedom vehicle model to describe the vehicle directional responses, which includes the yaw, lateral and roll motions. Most of the previous research works on vehicle lateral control have relied on the bicycle model (figure 1) that considers only lateral and yaw motions. It is based on the following assumptions:

- There is no roll, pitch or bounce
- The relative yaw between the vehicle and the road is small
- The steering angle is small
- The tire lateral force varies linearly with the slip angle

The following simplified model is obtained:

\[ m(\dot{v} + ur) = 2(F_f + F_r) \]

\[ J\dot{\delta} = 2(a_f F_f - a_r F_r) + M_z \]  

(1)

Figure 1. Bicycle model
where \( u \) and \( v \) (\( v = \beta \times u \)) are components of the vehicle velocity along longitudinal and lateral principle axes of the vehicle body, \( r \) is yaw rate, \( \beta \) denotes the side slip angle, \( m \) and \( J \) are the mass and the yaw moment of inertia respectively, \( a_f \) and \( a_r \) are respectively distances of the front and rear axle from the center of gravity, while yaw moment \( M_z \) is the control input, which must be determined from the control law, \( F_r \) and \( F_f \) are rear and front lateral forces respectively. They are described by magic formula (Lin, Popov and Mcwilliam, 2004) as

\[
F_r = D_f(\mu)\sin\left[ C_f(\mu)\tan^{-1}\left\{ B_f(\mu)(1 - E_f(\mu))\alpha_f + E_f(\mu)\tan^{-1}(B_f(\mu)\alpha_f)\right\}\right],
\]

\[
F_f = D_f(\mu)\sin\left[ C_f(\mu)\tan^{-1}\left\{ B_f(\mu)(1 - E_f(\mu))\alpha_f + E_f(\mu)\tan^{-1}(B_f(\mu)\alpha_f)\right\}\right]
\]

(2)

Coefficients \( D, C, B \) and \( E \) depend on the tire characteristics, road adhesion coefficient \( \mu \) and the vehicle operational conditions, \( \alpha_f \) and \( \alpha_r \) represent tyre slip-angles at the front and rear of the vehicle respectively. Given that

\[
\begin{align*}
\alpha_f &= -\frac{v}{u} - \tan^{-1}\left(\frac{a_f}{u} r \cos\left(\frac{v}{u}\right)\right) + \delta_f \\
\alpha_r &= -\frac{v}{u} + \tan^{-1}\left(\frac{a_r}{u} r \cos\left(\frac{v}{u}\right)\right)
\end{align*}
\]

(3)

where \( \delta_f \) is the front steer angle.

To obtain the TS fuzzy model, we have represented the front and rear lateral forces (2) by the following rules:

If \( |\alpha_f| \) is \( M_1 \) then

\[
\begin{align*}
F_f &= C_{f1}(\mu)\alpha_f \\
F_r &= C_{r1}(\mu)\alpha_r
\end{align*}
\]

(4)

If \( |\alpha_f| \) is \( M_2 \) then

\[
\begin{align*}
F_f &= C_{f2}(\mu)\alpha_f \\
F_r &= C_{r2}(\mu)\alpha_r
\end{align*}
\]

(5)
where $C_f$, $C_r$ represent front and rear lateral tire stiffness, which depend on road adherence $\mu$.

The overall forces are obtained by:

$$
\begin{cases}
F_f = h_1(\alpha_f) C_{f1}(\mu) \alpha_f + h_2(\alpha_f) C_{f2}(\mu) \alpha_f \\
F_r = h_1(\alpha_f) C_{r1}(\mu) \alpha_r + h_2(\alpha_f) C_{r2}(\mu) \alpha_r
\end{cases}
$$

(6)

where $h_j$ $(j=1,2)$ is the $j^{th}$ bell curve membership function of fuzzy set $M_j$.

They satisfy the following constraints

$$
\begin{cases}
\sum_{i=1}^{2} h_i(\alpha_f) = 1 \\
0 \leq h_i(\alpha_f) \leq 1 \ \forall i = 1, 2
\end{cases}
$$

(7)

The expressions of membership functions $h_j$ $(j=1,2)$ used are as follows

$$
h_i(\alpha_f(t)) = \frac{\beta_i(\alpha_f(t))}{\sum_{i=1}^{2} \beta_i(\alpha_f(t))}, i = 1, 2
$$

(8a)

with

$$
\beta_i(\alpha_f) = \frac{1}{\left(1 + \left(\frac{\alpha_f(t) - c_i}{a_i}\right)^{2b}\right)}
$$

(8b)

The membership function parameters and consequence of rules are obtained using an identification method based on the Levenberg-Marquadt algorithm (Lee, Lai and Lin, 2003) combined with the least square method, allow to determine parameters of membership functions $(a_1, b_1, c_1)$ and stiffness coefficient values

$$
av_1 = 0.5077, \quad b_1 = 3.1893, \quad c_1 = -0.4356, \quad a_2 = 0.4748, \quad b_2 = 5.3907, \quad c_2 = 0.5622
$$

(9a)

$$
C_{f1} = 60712.7, \quad C_{f2} = 4814, \quad C_{r1} = 60088, \quad C_{r2} = 3425
$$

(9b)

Using the above approximation idea of nonlinear lateral forces by TS rules and by considering that

$$
\alpha_f \approx \frac{-v - a_f r}{u} + \delta_f, \quad \alpha_r \approx \frac{-v + a_r r}{u}
$$

(10)
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nonlinear model (1) can be represented by the following TS fuzzy model:

If $\alpha_f$ is $M_1$ then
\[
\begin{align*}
\dot{x} &= A_1 x + B_1 M_z + B_{f_1} \delta_f \\
y &= C_1 x + D_1 \delta_f
\end{align*}
\]
(11a)

If $\alpha_f$ is $M_2$ then
\[
\begin{align*}
\dot{x} &= A_2 x + B_2 M_z + B_{f_2} \delta_f \\
y &= C_2 x + D_2 \delta_f
\end{align*}
\]
(11b)

where $x = (v, r)^T$, $y = (y_1, y_2)^T = (a_y, r)^T$ and

\[
\begin{align*}
A_i &= \begin{pmatrix}
-2 \frac{C_{fi} + C_{ni}}{mu} & -2 \frac{C_{fi} a_f - C_{ai} a_r}{J} \\
-2 \frac{C_{fi} a_f - C_{ni} a_r}{J} & -2 \frac{C_{fi} a_f^2 + C_{ni} a_r^2}{J u}
\end{pmatrix} \\
B_i &= \begin{pmatrix}
\frac{2C_{fi}}{mu} \\
\frac{2 a_f C_{fi}}{J}
\end{pmatrix}, \quad B_i = B = \begin{pmatrix}
0 \\
\frac{1}{J}
\end{pmatrix}, \quad D_i = \begin{pmatrix}
\frac{C_{fi}}{m} \\
0
\end{pmatrix}
\end{align*}
\]
(12a)

The output vector of system $y$ consist of measurements of lateral acceleration $a_y$ and the yaw rate about center of gravity $r$

The defuzzified output of this T–S fuzzy system is a weighted sum of individual linear models
\[
\begin{align*}
\dot{x} &= \sum_{i=1}^{2} h_i(\alpha_f) (A_i x + B_i M_z + B_{f_i} \delta_f) \\
y &= \sum_{i=1}^{2} h_i(\alpha_f) (C_i x + D_i \delta_f)
\end{align*}
\]
(13)

From the expressions of front and rear forces (4), (5), we note that stiffness coefficients $C_{fi}$ and $C_{ni}$ are not constant and vary depending on the road adhesion. To take into account these variations, we assume that these coefficients vary as follows:

\[
\begin{align*}
C_{fi} &= C_{fi0} (1 + d_f, f_i) \\
C_{ni} &= C_{ni0} (1 + d_f, f_i)
\end{align*}
\]
\[
\|f_i\| \leq 1
\]
(14)
where $d_i$ indicates the deviation magnitude of the stiffness coefficient from its nominal value.

After some manipulations, the TS fuzzy model can be written as:

$$
\begin{align*}
\dot{x}(t) &= \sum_{i=1}^{2} h_i(\|x\|) \left[ \left( (A_i + \Delta A_i)x(t) + BM_z + \left( B_{ji} + \Delta B_{ji} \right) \delta_f \right) \right] \\
y(t) &= \sum_{i=1}^{2} h_i(\|x\|) \left[ \left( C_i x + D_i \delta_f \right) \right]
\end{align*}
$$

where $\Delta A_i$ and $\Delta B_{ji}$ represent parametric uncertainties represented as follows:

$$
\Delta A_i = H_i \Sigma_i(t) E_{Ai}
$$

with $\Sigma_i(t)$ $(i = 1, 2)$ are matrices uncertain parameters such that $\Sigma_i(t) \Sigma_i(t)^T \leq I$, $E_i$ is known real matrix of appropriate dimension that characterizes the structures of uncertainties.

# Output Feedback Design

## 3.1 TS Fuzzy observer structure

Consider the general case of uncertain T-S fuzzy model (Takagi and Sugeno, 1985):

$$
\begin{align*}
\dot{x}(t) &= \sum_{i=1}^{q} h_i(z(t)) \left( (A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t) \right) \\
y(t) &= \sum_{i=1}^{q} h_i(z(t)) C_i x(t)
\end{align*}
$$

with properties:

$$
\sum_{i=1}^{q} h_i(z(t)) = 1, h_i(z(t)) \geq 0 \quad \forall i \in I_q
$$

where $q$ is the number of sub-models, $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input vector, $y(t) \in \mathbb{R}^l$ is the output vector, $A_i \in \mathbb{R}^{n \times n}, B_i \in \mathbb{R}^{n \times m}, C_i \in \mathbb{R}^{l \times n}$ are the $i^{th}$ state matrix, the $i^{th}$ input matrix and the $i^{th}$ output matrix respectively. Vector $z(t)$ is the premise variable.
depending on measurable variables. \( \Delta A_i \) and \( \Delta B_i \), \( i \in I_n \) are time-varying matrices representing parametric uncertainties in the plant model. These uncertainties are admissibly norm-bounded and structured, defined as

\[
\Delta A_i = H_i \Sigma_i(t) E_{A_i}, \quad \Delta B_i = H_i \Sigma_i(t) E_{B_i}
\]  

(19)

The overall fuzzy observer has the same structure as the TS fuzzy model. It is represented as follows:

\[
\begin{align*}
\dot{x}(t) &= \sum_{i=1}^{q} h_i(z(t)) \left( A_i \dot{x}(t) + B_i u(t) + G_i \left( y(t) - \hat{y}(t) \right) \right) \\
\dot{\hat{x}}(t) &= \sum_{i=1}^{q} h_i(z(t)) C_i \hat{x}(t)
\end{align*}
\]  

(20)

where \( G_i \), \( i \in I_n \) are the constant observer gains to be determined.

### 3.2 TS Fuzzy controller

Like the fuzzy observer, the TS fuzzy controller is represented as follows

\[
u(t) = -\sum_{i=1}^{q} h_i(z(t)) K_i \hat{x}(t)
\]  

(21)

where \( K_i \), \( i \in I_n \) are the constant feedback gains to be determined. We define the error of estimation as

\[
e(t) = x(t) - \hat{x}(t)
\]  

(22)

From systems (20), (21) and (22), we have

\[
\begin{align*}
\dot{x}(t) &= \sum_{i=1}^{q} \sum_{j=1}^{q} h_i(z) h_j(z) \left( A_i \dot{x}(t) - (B_i + \Delta B_i) x(t) + (B_j + \Delta B_j) \right) \\
\dot{\hat{x}}(t) &= \sum_{i=1}^{q} \sum_{j=1}^{q} h_i(z) h_j(z) \left( A_i \dot{\hat{x}}(t) - (B_i + \Delta B_i) \right) + (\Delta A_i + \Delta B_i) x(t)
\end{align*}
\]  

(23)

The augmented system can be expressed as:

\[
\dot{\tilde{x}}(t) = \sum_{i=1}^{q} \sum_{j=1}^{q} h_i(z) h_j(z) \left( \tilde{A}_i + \Delta \tilde{A}_i \right) \tilde{x}(t)
\]  

(25)

where

\[
\tilde{x} = \begin{pmatrix} x \\ e \end{pmatrix}, \quad \tilde{A}_i = \begin{pmatrix} A_i - B_j K_j & B_j K_j \\ 0 & A_i - G_i C_j \end{pmatrix}, \quad \Delta \tilde{A}_i = \begin{pmatrix} \Delta A_i - \Delta B_j K_j & \Delta B_j K_j \\ \Delta A_i + \Delta B_j K_j & -\Delta B_j K_j \end{pmatrix}
\]  

(26)
The global asymptotic stability of the TS fuzzy model (25) is summarized in the following theorem:

**Theorem 1:** If there exist symmetric and positive definite matrices $Q$ and $P$, some matrices $K_i$ and $G_i$ such that the following LMIs are satisfied $\forall (i, j) \in I_q^2, i < j$, then TS fuzzy system (25) is globally asymptotically stable via TS fuzzy controller (21) based on fuzzy observers (20):

\[
\begin{align*}
\begin{bmatrix}
\Phi_{ii} & * & * & \cdots \\
E_{ii}Q - E_{ii}M_i & - (\varepsilon_{ii}^{-1} + I) & * & \cdots \\
H_i^T & 0 & - (\varepsilon_{ii} + I) & \cdots \\
\end{bmatrix} < 0 & \quad (27a) \\
\begin{bmatrix}
\Psi_{ii} & * & * & \cdots \\
E_{ii}Q - E_{ii}M_j & - (\varepsilon_{ii}^{-1} + I) & * & \cdots \\
E_{ii}Q - E_{jj}M_i & 0 & - (\varepsilon_{jj}^{-1} + I) & * \\
H_i^T & 0 & 0 & - (\varepsilon_{jj}^{-1}) \\
H_j^T & 0 & 0 & 0 & - (\varepsilon_{ii}^{-1}) \\
\end{bmatrix} < 0 & \quad (27b) \\
\begin{bmatrix}
T_{ii} & * & * & \cdots \\
E_{ii}K_i & - (\varepsilon_{ii}^{-1} + I) & * & \cdots \\
H_i^T P & 0 & - (\varepsilon_{ii} + I) & \cdots \\
\end{bmatrix} < 0 & \quad (28a) \\
\begin{bmatrix}
\Theta_{ii} & * & * & \cdots \\
E_{ii}K_j & - (\varepsilon_{ii}^{-1} + I) & * & \cdots \\
E_{jj}K_i & 0 & - (\varepsilon_{jj}^{-1} + I) & * \\
(P_{ji})^T & 0 & 0 & - (\varepsilon_{jj}^{-1}) \\
(P_{ij})^T & 0 & 0 & 0 & - (\varepsilon_{ii}^{-1}) \\
\end{bmatrix} < 0 & \quad (28b)
\end{align*}
\]

with

\[\Phi_{ii} = QA_i^T + A_iQ - M_i^TB_i^T - B_iM_i + I\]
\[ \Psi_{ij} = QA^T_i + A_iQ + QA^T_j + A_jQ - MT^T_j B^T_i - B_jM_i - MT^T_i B^T_j - B_jM_i + D_iD^T_i + D_jD^T_j + 2I \]

\[ T_i = A^T_i P + PA_i - C^T_i N^T_i - N_iC_i + K^T_i B_i K_i \]

\[ \Theta_{ij} = A^T_i P + PA_i + A^T_j P + PA_j - C^T_i N^T_j - N_iC_j - C^T_j N^T_i - N_jC_i + K^T_i B^T_j B_j K_i + K^T_j B^T_i B_j K_j \]

The controller and the observer are defined as follows

\[ K_i = M_i Q^{-1} \]  
(29a)

\[ G_i = P^{-1} N_i \]  
(29b)

**Proof:** The proof can be inspired directly from (El Hajjaji, Chadli, Oudghiri and Pagès, 2006).

### 3.3 Case of common input matrix

In the case of common input matrix \( B \) \( (B_i = B \ \forall i \in I_q) \), the above result is simplified. The new stability conditions are given in the following corollary

**Corollary 1:** If there exist symmetric and positive definite matrices \( Q \) and \( P \), some matrices \( K_i \) and \( G_i \) such that the following LMI are satisfied \( \forall i \in I_q \), then TS fuzzy system (25) is globally asymptotically stable via TS fuzzy controller (21) based on fuzzy observers (20):

\[
\begin{bmatrix}
    \Phi_{ii} & * \\
    E_{ii}Q & - (\epsilon^{-1}_{ii} + 1)^{-1} I & * \\
    H^T_i & 0 & - \epsilon^{-1}_{ii} I
\end{bmatrix} < 0
\]  
(30a)

\[
\begin{bmatrix}
    T_{ii} & * \\
    H^T_i P & -I
\end{bmatrix} < 0
\]  
(30b)

with

\[ \Phi_{ii} = QA^T_i + A_iQ - MT^T_i B^T - BM_i + I \]

\[ T_{ii} = A^T_i P + PA_i - C^T_i N^T_i - N_iC_i + K^T_i B^T_i BK_i \]

The controller and the observer gains are as defined in (29).
**Proof**: The result is obtained directly from theorem 1.

### 3.4 Remarks and discussion

1. The derived stability conditions are LMI on synthesis variables $P > 0$, $Q > 0$, $M_i$, $N_i$, and scalars $\varepsilon_i > 0$. However, the problem to resolve becomes nonlinear in $K_i$, $i \in I_q$ (inequalities (27)-(28)/(30)-(31)). A method allowing the use of numerical tools to solve these constraints is given in the following.

2. To resolve the obtained BMI (bilinear matrix inequality) conditions using LMI tools (LMI toolbox of Matlab software for example), we propose to solve synthesis conditions (27) (or (30)) sequentially:
   - First, we solve LMI (25) and (26) in the variables $Q$, $M_i$, and $\varepsilon_i$.
   - Once gains $K_i$ have been calculated from (29a), conditions (28) become linear in $P$ and $N_i$ can be easily resolved using the LMI tool to determine gains $L_i$ from (29b).

3. Result of corollary 1 derive directly from the TS fuzzy model (15) (with common input matrix $B_i = B, i \in I_2$, and $\delta_f = 0$). This case leads to four constraints to resolve, whereas the result of theorem 1 leads to six constraints, which means less conservatism.

### 4 FTC STRATEGY

It is important to be able to carry out fault detection and isolation before faults have a drastic effect on the system performance. Even in case of system changes, faults should be detected and isolated. Observer based
estimator schemes are used to generate residual signals corresponding to the difference between measured and estimated variables (Chen and Patton, 1999). The residual signals are processed using either deterministic (e.g. using fixed or variable thresholds) (Ding, Schneider, Ding and Rehm, 2005) or stochastic techniques (based upon decision theory) (Chen and Liu, 2000). Here, the first one is used.

The method that we propose is illustrated in figure 2, where it can be seen that the FDI functional block uses two observers, each one is driven by a single sensor output. The failure is detected first, and then the faulty sensor is identified. After that, the state variables are reconstructed from the output of the healthy sensor. The lateral control system enters the degraded mode that guaranteed stability and an acceptable level of performance.

Figure 2 shows the block diagram of the proposed closed system, $[a_y, r]^T$ is the output vector of the system, where $a_y$ denotes the lateral acceleration and $r$ is the yaw rate about the center of gravity. Two observer based controllers are designed, one based on the observer that uses the measurement of lateral acceleration $a_y$ and the other one based on the observer that uses the measurement of yaw rate $r$.

![Figure 2. Block diagram of the observer-based FTC](image-url)
Assumptions
Let \( C_i^l (i, l = 1, 2) \) denote the \( l \)th row of matrix \( C_i \) (12c.). We assume that \( \{A_i, C_i^l\} \) are observable, which implies that it is possible to estimate the state through either the first output \((a_y)\) or the second one \((r)\) for the vehicle model (15).

Sensor failures are modeled as additive signals to sensor outputs
\[
y = \begin{pmatrix} a_y \\ r \end{pmatrix} = C_i x + D_i \delta_f + Ff
\]
where
For failure of sensor 1
\[
F = \begin{pmatrix} 1 \\ 0 \end{pmatrix}
\]
For failure of sensor 2
\[
F = \begin{pmatrix} 0 \\ 1 \end{pmatrix}
\]
We also assume that at any time one sensor only fails at the most. This assumption has been implied by the two possible values of \( F \).

Observer-based FDI design
If each \( \{A_i, C_i^l\} \) \( i, l = 1, 2 \) is observable, then it is possible to construct a TS fuzzy observer for the TS fuzzy model of the vehicle as described in section III.

For observer 1, the state is estimated from the output of the first sensor \( (a_y) \). It is given as:
\[
\hat{x}_i(t) = \sum_{j=1}^{2} \mu_i(a_j) \left( A_i \hat{x}_i + B_i a_j + B_i M \right)
\]
\[
\hat{a}_j = \sum_{i=1}^{2} \mu_i(a_j) \left( C_i \hat{x}_i + D_i \delta_f \right)
\]
\[
\dot{x}_2 = \sum_{i=1}^{2} \mu_i(|\alpha_f|) \left( A_i \dot{x}_2 + B_i \delta_f + B_i M \dot{z} + G_i^2 (r - \dot{r}_i) \right)
\]
\[
\dot{\hat{r}}_2 = \sum_{i=1}^{2} \mu_i(|\alpha_f|) \left( C_i^2 \dot{x}_2 + D_i \delta_f \right)
\]

where \( C_i^l \) and \( D_i^l \) are the \( l \)-th rows of matrices \( C_i \) and \( D_i \) (equations 10) respectively and \( G_i^l(i,l=1,2) \) are the constant observer gains to be determined. \( \dot{x}_i, \dot{a}_y \) and \( \dot{r}_i \) are respectively the state estimation, the lateral acceleration estimation and yaw rate estimation with observer \( i \).

The TS fuzzy controller is represented as follows

\[
M_z(t) = -\sum_{i=1}^{2} \mu_i(|\alpha_f|) K_i \dot{x}_i(t)
\]

with

\( l = 1 \) If sensor 2 fails
\( l = 2 \) If sensor 1 fails

We define the residual signals as

\[
R_{1,ay} = \hat{\dot{a}}_{y1} - a_y \quad R_{2,ay} = \hat{\dot{a}}_{y2} - a_y \\
R_{1,r} = \hat{\dot{r}}_1 - r \quad R_{2,r} = \hat{\dot{r}}_2 - r
\]

Note that \( R_{1,ay} \) and \( R_{1,r} \) are related to observer 1 and \( R_{2,ay} \) and \( R_{2,r} \) are related to observer 2.

with

\[
\begin{pmatrix}
\dot{\hat{a}}_{y1} \\
\dot{\hat{r}}_1
\end{pmatrix} = \sum_{i=1}^{2} h_i(|\alpha_f|) \left( C_i \dot{x}_i + D_i \delta_f \right)
\]

\[
\begin{pmatrix}
\dot{\hat{a}}_{y2} \\
\dot{\hat{r}}_2
\end{pmatrix} = \sum_{i=1}^{2} h_i(|\alpha_f|) \left( C_i \dot{x}_2 + D_i \delta_f \right)
\]

The FDI scheme developed in this study follows a classical strategy such as the well-established observer based FDI methods (Isermann, 2001; Huang and Tomizuka, 2005; Oudghiri, Chadli and El Hajjaji, 2007). The residual signals \( R_{1,ay}, R_{1,r}, R_{2,ay}, R_{2,r} \) are used for the estimation of the model uncertainties and then, for the construction of model uncertainty indicators. The decision bloc is based on the analysis of these residual signals.
signals. Indeed faults are detected and then switching operates according to the following scheme:

**Detection:** if 
\[
\max\left(\|R_{1,ay}, R_{1,r}\|, \|R_{2,ay}, R_{2,r}\|\right) > T_h
\]
then the fault has occurred where \(T_h\) the prescribed threshold is and \(\|\cdot\|\) denotes the Euclidian norm at each time instant.

**Switching:** if 
\[
\|R_{1,ay}, R_{1,r}\| \succ \|R_{2,ay}, R_{2,r}\|
\]
then switch to observer 2. If not switch to observer 1.

Since model uncertainties and sensor noise also contribute to nonzero residual signals under the normal operation, threshold \(T_h\) must be large enough to avoid false alarms while small enough to avoid missed alarms. In this paper, we do not further discuss the selection of the thresholds.

## 5 SIMULATION RESULTS

To show the effectiveness of the proposed FTC based on bank of observer algorithm, we have carried out some simulations using the vehicle model (1) and MATLAB software. In the design, the vehicle parameters considered are given in table 1. To take account of uncertainties, stiffness coefficients \(C_{fi}\) and \(C_{ri}\) are supposed to be varying depending on road adhesion.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>(I_z) Kg.m(^2)</th>
<th>m</th>
<th>(a_f) m</th>
<th>(a_r) m</th>
<th>U m/s</th>
<th>Nominal stiffness Coefficients (N/rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>3214</td>
<td>1740</td>
<td>1.04</td>
<td>1.76</td>
<td>20</td>
<td>(C_{f10}) (C_{f20}) (C_{r10}) (C_{r20})</td>
</tr>
<tr>
<td></td>
<td>60712</td>
<td>4812</td>
<td>60088</td>
<td>3455</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1

with the following uncertainties
\[
D_1 = D_2 = \begin{pmatrix} 0.4 & 0 \\ 0 & 0.4 \end{pmatrix}
\]

We point out that only the yaw rate is directly measurable by a yaw rate sensor (gyroscope), the lateral velocity is unavailable and is estimated using the proposed observer.
By solving the derived stability conditions of theorem 1, the designed controller and observer gains are:

\[
K_1 = 10^4 \begin{pmatrix} -1.1914 & 1.1616 \\ -1.2623 & 1.3102 \end{pmatrix}, \quad K_2 = 10^4 \begin{pmatrix} -35.9102 & 6.2245 \\ -223.2973 & 43.8026 \end{pmatrix}
\]

(41)

\[
G_1 = \begin{pmatrix} -50.7356 & 5.7456 \\ -28.2271 & 3.0782 \end{pmatrix}, \quad G_2 = \begin{pmatrix} -28.2271 & 3.0782 \end{pmatrix}
\]

(42a)

(42b)

Figure 3 shows the additive signals that represent sensor failures. The first one has been added to sensor 1 output between 2s and 8s, and the second one has been added to sensor 2 output between 10s and 16s.

![Additive signal to sensor 1 output](image1)

![Additive signal to sensor 2 output](image2)

**Figure 3. Failure of sensors**

All the simulations are realized on the nonlinear model given in (1) with vehicle speed 20 m/s. The simulation results are given in figures 4 and 5 with and without the FTC strategy. In figure 4 the law control is based on one observer (observer 2) without using the switching bloc. We can see between 10s and 16s that the vehicle lost its performance just after the yaw rate sensor became faulty.

Figure 5 shows vehicle state variables and the estimated signals, when the law control is based on the bank of two observers with the switch bloc. We can note that the vehicle remains stable despite the presence of faults, which shows the effectiveness of the proposed FTC strategy.
Figure 4. Vehicle states without FTC strategy

Figure 5. Vehicle states with FTC strategy
The switching from observer 1 to observer 2 is visualized clearly at $t \approx 8s$ (figure 6). We notice that switching observers is carried out without loss of control of the system state.

The second simulations are realized to show the importance of the proposed FTC method based on an output fuzzy controller, on the stability of the vehicle dynamics. Simulations propose to show the difference between the vehicle dynamics behaviour with TS fuzzy yaw control based on a fuzzy observer (figure 5) and its behaviour with the linear yaw control based on a linear observer (figure 7). Figure 7 clearly shows that the linear control fails to maintain the stability of the vehicle in presence of sensor faults despite a short magnitude of the additive signal ($f = 0.1$) and also a very low front steering angle $\delta_f = 0.001$. Indeed, we can see that by using the proposed fuzzy yaw control based on a fuzzy observer and the algorithm proposed for detection sensors faults, the results are better than these with linear control.
Figure 7a. Additive signals to sensors output

Figure 7b. Vehicle states without sensor faults using linear control with road friction coefficient fixed at 0.5

6 CONCLUSION
Using an algorithm based on a bank of two observers, a fault tolerant control has been presented. The vehicle nonlinear model is first represented by an uncertain Takagi-Sugeno fuzzy model. Then, a robust output feedback controller is designed using LMI terms. Based on the designed robust observer-based controller, a fault tolerant control method
is utilized. This method uses a technique based on the switching principle, allowing not only to detect sensor failures but also to adapt the control law in order to compensate the effect of the faults by maintaining the stability of the vehicle and the nominal performances. Simulation results show that the proposed FTC strategy based on robust output TS fuzzy controller are better than these with linear control in spite of a short magnitude of the additive signal and very low front steering angle.

References