Model-Driven System Identification of Transcritical Vapor Compression Systems

Bryan P. Rasmussen, Andrew G. Alleyne, and Andrew Musser

Abstract—This paper uses an air conditioning system to illustrate the benefits of iteratively combining first principles and system identification techniques to develop control-oriented models of complex systems. A transcritical vapor compression system is initially modeled with first principles and then verified with experimental data. Both SISO and MIMO system identification techniques are then used to construct locally linear models. Motivated by the ability to capture the salient dynamic characteristics with low order identified models, the physical model is evaluated for essentially nonminimal dynamics. A singular perturbation model reduction approach is then applied to obtain a minimal representation of the dynamics more suitable for control design, and yielding insight to the underlying system dynamics previously unavailable in the literature. The results demonstrate that iteratively modeling a complex system with first principles and system identification techniques gives greater confidence in the first principles model, and better understanding of the underlying physical dynamics. Although this iterative process requires more time and effort, significant insight and model improvements can be realized.

Index Terms—air conditioning, control engineering, modeling, identification, reduced order systems.

I. INTRODUCTION

Controls engineers are often required to develop dynamic models for a class of complex physical systems with little prior knowledge of the system dynamics. Adding to the challenge of this task is the desire to gain physical insight into the underlying dynamics. This task can sometimes span years or decades before the dynamics of the given class of systems are well understood and characterized with simple control-oriented models.

The foremost difficulty in this process is balancing accuracy with simplicity. Although many processes can be accurately modeled with high order ordinary or partial differential equations, the practicing control engineer often needs practical low order controllers [1]. This means that significant time must be spent either 1) developing simple, accurate, control-oriented models prior to the control design phase, or 2) reducing high order controllers after the design phase, but prior to implementation. The distinct advantage of developing simple models is the physical insight gained. This insight is invaluable as a tool in developing general control strategies (i.e. selecting sensors, actuators) and in system design (e.g. sizing components).

Because of these advantages, engineers expend considerable effort to develop low order physical models. For example, efforts to model the two-phase fluid dynamics associated with drum boilers spanned several decades before finding an appropriate balance of simplicity and accuracy [2]. However, once an accurate low order model has been identified, the development of control strategies follows quickly. As an example, this was evident with the publications that followed the Moore-Greitzer compressor model [3-4].

The objective of obtaining a low order model containing physical insight seemingly would preclude the use of system identification techniques, and restrict the engineer to first principles based modeling. However, this work uses an air conditioning system as an example to demonstrate that iteratively using both first principles modeling and system identification techniques can be mutually beneficial. Previously, subcritical air conditioning systems have been modeled using a lumped-parameter, moving-boundary formulation [5-8]. When this methodology is applied to a transcritical cycle, the resulting model is of comparable order to the subcritical cycle models. However, as will be shown, the resulting model is essentially a nonminimal representation of the system dynamics.

Although the specific system discussed in this paper is a vapor compression cycle, many of the benefits will be similar for other complex physical systems. Specifically, the identified models can guide the engineer in selecting the appropriate balance between model fidelity and simplicity. Likewise, insight from the physical model can guide the choice of appropriate inputs/outputs, model order, and structure for system identification.

This iterative procedure is best explained by the diagram shown in Fig. 1. First principles are used to derive the governing dynamic equations for a transcritical air conditioning system, resulting in an 11th order model [9]. This model is validated using data obtained from an experimental system. The data from this system are also used to construct SISO and MIMO empirical models using system identification techniques. While the SISO identification results in 3rd order input/output models, the MIMO identification necessitates a 5th order model for adequate prediction, demonstrating the need for MIMO identification of such systems. Moreover, the simple identified behavior suggests the presence of low order dominant dynamics, and prompts investigation into possibilities for model reduction of the first principles model.
Subsequent analysis reveals that the 11th order model is singularly perturbed [10], which leads to physically-based model reduction. Analysis also reveals that the system is highly coupled, further demonstrating the inadequacy of SISO models in describing the system dynamics. The final reduced order model is of similar order to the MIMO identified model, giving greater confidence in the physical model and, more importantly, a low order, physically-based control-oriented model previously unavailable.

The remainder of this paper is organized as follows. Necessary background information is presented in Section 2, and details regarding the experimental setup are given in Section 3. Section 4 presents the first principles model for a transcritical vapor compression system, and Section 5 presents the system identification results. Section 6 discusses the physically-based model reduction using singular perturbation approximation. Concluding remarks are given in Section 7.

II. BACKGROUND

A majority of air conditioning and refrigeration systems operate using a vapor compression cycle. Modeling the complex thermofluid dynamics of these systems with distributed parameters can result in high order models that are of limited use for control design [11-14]. However, the dominant dynamics of these systems are observed to be low order, nonlinear, and nonminimum phase. Vapor compression systems typically have been controlled using simple single-input single-output (SISO) mechanical controllers. The extensively coupled dynamics cause these control strategies to perform poorly, with occasional instability [15],[5], which suggests that the use of multivariable control strategies could be beneficial [7].

This paper considers a transcritical vapor compression cycle as pictured in Fig. 2. This is an atypical vapor compression cycle that uses carbon dioxide (CO₂) as the working fluid. This type of system has attracted attention as a possible replacement of traditional subcritical refrigerants because CO₂ is a natural fluid, and does not have the negative environmental impacts of traditional refrigerants. From a controls perspective, this system has the distinct advantage of having an extra degree of freedom to trade efficiency for capacity [16].

Beginning at the top of the diagram (Fig. 2), the high-pressure, supercritical fluid flows through a microchannel gas cooler, where heat is rejected. From the gas cooler the refrigerant flows to the “hot” side of a microchannel counterflow heat exchanger. This internal heat exchanger uses the cold fluid leaving the evaporator to cool the hot refrigerant from the gas cooler. This increases the capacity of the air conditioning system, as well as ensuring that only refrigerant vapor enters the compressor. After the internal heat exchanger the refrigerant flows through an expansion valve. Through this valve the fluid expands and transitions from a supercritical fluid to a two-phase mixture. The refrigerant then enters the microchannel evaporator where heat is absorbed as the fluid evaporates. From the evaporator the refrigerant flows through the “cold” side of the internal heat exchanger, and then to the compressor where the fluid is compressed to a higher pressure.

III. EXPERIMENTAL SYSTEM

The test facility was located on the campus of the University of Illinois at Urbana-Champaign. The principal facility was a second-generation prototype CO₂ automotive air conditioning system (MAC2R744). This system was used previously to study steady state efficiency in [17], where a detailed description of the system components is available. A schematic of the system is given in Fig. 3. The experimental system has two insulated environmental chambers that are controlled to simulate desired indoor and outdoor conditions. An electronic heater and glycol chiller are used to heat and cool the air flow after leaving the evaporator and gas cooler respectively, while a motor and clutch system is used to drive the compressor. Extensive steady state calibration of the sensors has been conducted to ensure the validity of all measurements [17]. For transient tests, the sampling frequency was approximately 1 Hz. This frequency maximized the sensor dwell time, thus increasing the signal to noise ratio, for the desired number of sensors to be monitored.

The standard system actuators included an expansion valve, compressor, and air fans. For the transient tests an electronic expansion valve and variable displacement compressor were used. The compressor was fixed at full displacement, and mass flow rate was varied by altering the rotational speed. Air
flow rates were varied by changing the fan speeds, while inlet air temperature was maintained at a desired value.

Each of the component models is given as nonlinear state space models, \( Z(x,u) \cdot x = f(x,u) \), where the elements of \( Z(x,u) \) and \( f(x,u) \) are nontrivial and presented in detail in [9]. The state and input vectors for each of the three component models are defined in (1)-(6). The state variables are defined in terms of pressures, enthalpies, etc. and are the result of the derivation procedure. However, they are not the only possible choice of physical states as discussed in Section 6. For example, the states of the evaporator model \( (x_e) \) are length of two-phase flow \( L_{e,1} \), evaporation pressure \( P_e \), outlet enthalpy \( h_{e,2e} \), and the two lumped wall temperatures \( T_{ew,1} \) and \( T_{ew,2} \).

The inputs to each of the component models are generally outputs of other component models. For example, the inputs to the evaporator model, \( u_e \), are the inlet and outlet refrigerant mass flow rates \( m_{e,in} \) and \( m_{e,out} \) (outputs of the valve and compressor models), the inlet enthalpy \( h_{e,in} \) (output of the valve model), and the temperature and mass flow rate of air \( T_{air,in} \) and \( m_{air,in} \) (inputs to the overall system).

The actuating components such as the compressor and expansion valve have dynamics that are considerably faster than the dominant dynamics of the system. These components are modeled with nonlinear algebraic equations that calculate mass flow as a function of the operating pressures, and outlet enthalpy as a function of the inlet enthalpy. The gas cooler, evaporator, and internal heat exchanger are modeled with nonlinear ordinary differential equations, resulting in 3rd, 5th and 3rd order models respectively. Thus the overall system is an 11th order model.

All fluid in the gas cooler is assumed to be in a supercritical state. Therefore, a lumped parameter model of the gas cooler assumes one single-phase region. The fluid in the evaporator is assumed to enter the evaporator as a two-phase fluid, and exit as a superheated vapor, as depicted in Fig. 4. Therefore, the lumped parameter model assumes two separate fluid regions. The internal heat exchanger is modeled with the three ordinary differential equations based on lumped capacitance assumptions and a counterflow configuration is assumed.

This first principles model is the result of using an established modeling approach [5]. Although the resulting model is sufficiently accurate, the approach results in several
modes that are nonessential for dynamic prediction. This will be seen in the system identification results in the following sections.

Figure 5: Evaporator Pressure for Step Changes in Compressor Speed (Data, Nonlinear and Linear Models)

V. SYSTEM IDENTIFICATION

A common approach for modeling air conditioning systems is using system identification techniques (e.g. [18] and [19]). System identification can yield very accurate models, but has the distinct disadvantage of being dependent on the system considered, and only valid around the operating point considered. However, the resulting models can be used to verify the level of complexity of the first principles model.

For identification purposes, it was necessary to excite the system by varying each of the inputs. Because SISO model identification is concerned only with individual input-output behavior, the data for this identification approach was generated by varying each of the system inputs separately using a Pseudo-Random Binary Sequence (PRBS). Although not ideal for nonlinear identification, this signal is persistently exciting of adequate order, and is sufficient for identifying approximate linear models. The choice of input amplitudes was approximately 10% of the actuator range, generally resulting in a 5-10% change in the outputs, which was sufficient to clearly discern the dynamic behavior of the system. For identifying MIMO models, all of the system inputs were varied simultaneously.

The individual input-output models were constructed offline assuming an ARMAX model structure, \( A(q)y(t) = B(q)u(t) + C(q)e(t) \). The coefficients of the \( A(q) \), \( B(q) \), and \( C(q) \) polynomials are found using a standard nonlinear least squares iterative search method (Gauss-Newton) [20], that minimizes a quadratic prediction error criterion, \( V(\theta, Z) \) as defined in (7), while imposing constraints to ensure that only models with stable predictors are used [21].

\[
V(\theta, Z) = \frac{1}{N} \sum_{t=1}^{N} e^2(t, \theta)
\]  

The minimum model order necessary to adequately model the dynamics, while ensuring whiteness and independence of the model residuals, was 3rd order or lower. The data sets were divided into estimation and validation sets and the models were cross-validated to ensure that the models were not over-fitted to a specific data set. This process was repeated for three common operating conditions (idle, city, and highway driving). For a few of the input/output combinations the resulting models were 2nd order, while others were observed to be unaffected by changes in the inputs. For example gas cooler exit air temperature, \( T_{\text{cco}} \), remained virtually unchanged for changes in expansion valve (Fig. 6) and evaporator air flow rate. The individual transfer functions for each of the input-output pairs for all three conditions can be found in [22]. Comparison between one of the SISO models and data are shown in Fig. 6. As discussed in [21], the initial input/output values are assumed to be close to the equilibrium values and are unnecessary for identification of a linear model. Therefore, the sample means are removed prior to the offline identification and the results are plotted with a zero mean value.

Given the results of the SISO identification, it would be tempting for someone unfamiliar with system identification to assume that only a 3rd order physical model is needed. However, the states of the various SISO models need not be the same, and therefore a MIMO model would be a more appropriate description of the system. Although the individual SISO models could be combined, scaled, and then reduced using a balanced realization/truncation approach, a more direct approach for constructing a MIMO model (subspace algorithms) is preferred.

A state-space model structure is selected of the form of (8), where \( T \) is the sampling time. \( \{A,B,C,D,K\} \) are constant matrices, and \( x(t), u(t), y(t), \) and \( e(t) \) are the time sequences of states, inputs, outputs, and model residuals respectively. This model structure differs notably from those in previous studies that identified air conditioning systems using multivariable ARX models [19]. The particular advantage of the model structure in (8) is its close relationship to physically-based continuous time state space models. Other multivariable methods (i.e multivariable ARX, vector difference equation, etc.) are less easily related to their first principles counterparts [21].

\[
x(t+T) = Ax(t) + Bu(t) + Ke(t) \\
y(t) = Cx(t) + Du(t) + e(t) 
\]  

The algorithm used to identify the \( \{A,B,C,D,K\} \) matrices was a combined prediction error method and subspace algorithm. The initial guess values of these matrices are determined by an N4SID algorithm [23]. Contrary to the classical identification methods which determined the system matrices first and then the system states, the N4SID subspace method identifies the state vector first, and then determines the system matrices using a linear least squares approach. As discussed in [21],[24] the general steps of subspace algorithms are: 1) construct the extended observability matrix from input-output data, 2) select appropriate weighting matrices (possibilities are listed in [24]) and perform a singular value decomposition (SVD), 3) determine the state vector from the SVD, 4) determine the \( \{A,B,C,D\} \) matrices using a linear least squares approach, and 5) determine the \( \{K\} \) matrix from \( \{A,B,C,D\} \) and the covariance of the residuals. For this application, an initial guess for the system matrices is obtained using an N4SID algorithm, and the model is then adjusted by improving the prediction error fit using an approach similar to the SISO algorithm. Models of order 1 through 10 were estimated and cross-validated. The 5th order model adequately
predicted all outputs and had the lowest correlation errors of any of the models generated (Fig. 7). Numerical representations of the identified state space matrices for one possible operating condition (highway driving) are given in (9)-(12).

As a demonstration of the model fit for both approaches, three basic statistics about the model residuals are shown in Table I. The model residuals, $\varepsilon$, are defined as $\varepsilon(t) = \hat{y}(t) - y(t)$ with $\hat{y}$ as the predicted output and $y$ as the measured output. The maximum residual and the average residual are calculated as given in (13) and (14) with $N$ being the number of measurements. Additionally, a relative error measure is calculated as shown in (15), where $\bar{y}$ is the mean value of $y$. The percentage of model fit can then be calculated as: $\%Fit = 100(1 - S_r)$.

\begin{align*}
D_y &= \left[ \begin{array}{c}
1.0482 \\
0.1060 \\
4.0175 \\
0.0163 \\
0.0116 \\
0.0059
\end{array} \right] \\
B_d &= \left[ \begin{array}{c}
-0.001866 \\
0.005277 \\
0.00010445 \\
0.064922 \\
0.003185 \\
0.001658 \\
-0.08877
\end{array} \right] \\
C_d &= \left[ \begin{array}{c}
0.0063097 \\
5.1565e-005 \\
0.035205 \\
-0.053953 \\
-2.8848 \\
-0.1270 \\
0.1218
\end{array} \right]
\end{align*}

\begin{align*}
A_d &= \left[ \begin{array}{cccc}
0.94295 & -0.0091181 & -0.014164 & -0.0065713 \\
-0.064485 & 0.90741 & 0.0640222 & -0.0065357 \\
-0.082379 & -0.10869 & 0.91539 & -0.0049082 \\
-0.062383 & 0.03533 & -0.011021 & 0.80366 \\
-0.011651 & 0.016309 & 0.039517 & -0.10604
\end{array} \right]
\end{align*}

\begin{align*}
B_d &= \left[ \begin{array}{cccc}
8.8717e-005 \\
-0.19545 \\
0.027169 \\
0.004922 \\
0.068274 \\
0.001815 \\
0.021678 \\
0.16158 \\
-0.88477
\end{array} \right] \\
C_d &= \left[ \begin{array}{cccc}
0.53953 \\
-0.35252 \\
0.054908 \\
-0.0063097 \\
0.88477 \\
-0.16138 \\
0.008131 \\
-0.00030371 \\
0.0091829
\end{array} \right] \\
D_y &= \left[ \begin{array}{c}
1.1379 \\
-1.92772 \\
2.6032 \\
2.3568 \\
-0.7687 \\
2.3020 \\
0.9757 \\
-2.036 \\
-8.90199
\end{array} \right]
\end{align*}

\begin{align*}
S_d &= \frac{1}{N} \sum_{t=1}^{N} \varepsilon^2(t) \\
S_r &= \sum_{t=1}^{N} (\hat{y}(t) - y(t))^2 \\
S_y &= \left( \frac{\sum_{t=1}^{N} \varepsilon^2(t)}{\sum_{t=1}^{N} (\hat{y}(t) - y(t))^2} \right)
\end{align*}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline
& & & & & & & & & & \\
\hline
\textbf{TABLE I: MODEL FIT MEASURES FOR IDENTIFICATION} & & & & & & & & & & \\
\hline
\textbf{SDO Identification} & & & & & & & & & & \\
\textbf{MIMO Identification} & & & & & & & & & & \\
\hline
& & & & & & & & & & \\
\hline
& & & & & & & & & & \\
\hline
\textbf{Exp. Valve Opening} & & & & & & & & & & \\
\textbf{Compressor Speed} & & & & & & & & & & \\
\textbf{Exhaust Air Flow Rate} & & & & & & & & & & \\
\textbf{Cone Cooler Air Flow Rate} & & & & & & & & & & \\
\textbf{All Inputs} & & & & & & & & & & \\
\hline
& & & & & & & & & & \\
\hline
\textbf{Exp. Valve Opening} & & & & & & & & & & \\
\textbf{Compressor Speed} & & & & & & & & & & \\
\textbf{Exhaust Air Flow Rate} & & & & & & & & & & \\
\textbf{Cone Cooler Air Flow Rate} & & & & & & & & & & \\
\textbf{All Inputs} & & & & & & & & & & \\
\hline
\end{tabular}
\end{table}

VI. REDUCED ORDER FIRST PRINCIPLES MODEL

There is a clear discrepancy in model complexity between the 11th order first principles model and the lower order identified models. The large difference in model order suggests that the first principles model is more complex than is necessary to capture the low order input-output behavior. Although numerical model reduction techniques could be applied to the linearized physical model to eliminate nonessential modes, the states of the resulting model would lose physical significance, and little insight to the dynamics would be gained. This prompts a close examination of the physical model and the derivation procedure.

Examining the eigenvalues of the linearized component models, they are found to differ by several orders of magnitude, indicating that the various dynamic modes evolve on extremely different time scales. However, the results of Section 5 do not indicate which dynamic modes dominate the system response, and what are the physical significance of these modes.

A unique aspect of thermofluid systems is the redundancy of information of the thermodynamic state variables. For example, given pressure and temperature all the remaining fluid properties can be found (i.e. density, enthalpy, etc.). This results in freedom in selecting variables as dynamic states. In this case, we wish to select the combination of states that most directly decouples the system dynamics into fast and slow modes. In [9] an alternative choice of states is found that effectively separates the fast and slow dynamic modes. This choice of states has intuitive appeal as well. The slow states are the total wall energy of each heat exchanger region, and
total refrigerant mass in each heat exchanger. The fast states are the total refrigerant energy in each heat exchanger region. Additionally, [9] reveals a dynamic mode that is completely redundant. Because the total amount of refrigerant is fixed, there is no need to have separate states for the refrigerant mass in each of the heat exchangers.

Since the eigenvalues of the different modes differ by more than an order of magnitude, the component models are said to be singularly perturbed [10]. A reduced order linearized model can be found by “residualizing” the fast dynamic modes [25]. This process of model reduction replaces the fast dynamic modes with their algebraic equivalents, thus reducing the model order while preserving the physical nature of the state variables. [26] shows that for singularly perturbed systems, there is a connection between the fast dynamic modes and the so-called weakly controllable/observable modes. Thus by eliminating the redundant dynamic mode and the fast dynamic modes, we retain the dominant dynamic behavior of the system and achieve a minimal representation of the system dynamics.

The final form of this reduced order model can be represented in the standard state space form, shown in (16), where the \(A, B, C, D\) matrices are found in [22]. The system inputs/disturbances are given in (17) (valve opening, compressor speed, inlet evaporator air temperature, evaporator air mass flow rate, inlet gas cooler air temperature, and gas cooler air flow rate). The system outputs are given in (18) (evaporator superheat, evaporator pressure, gas cooler pressure, evaporator exit air temperature, gas cooler exit air temperature). The reduced order system model closely approximates the eigenvalues of the full order model, as seen in Table II.

A comparison of the full order and reduced order linear models is shown in Fig. 8, where the full order linear and the reduced order linear models are indistinguishable. Thus, the model reduction prompted by the system identification efforts resulted in a model that was of lower order, but with negligible loss in model accuracy.

\[
\dot{x} = A_x x + B_x u
\]
\[
y = C_x x + D_x u
\]
\[u = [u_v, \omega_t, T_{c,at}, m_{c,at}, T_{c,at}, m_{c,at}]^T\]
\[y = [T_{c,sh}, P_{s}, P_{v}, T_{c,at}, T_{c,sh}]^T\]

Table II: System Eigenvalue Approximation

![Figure 8: Evaporator Pressure for Step Changes in Compressor Speed (Full order and Reduced Order Models)](image)

VII. CONCLUSIONS

The principal contribution of this paper is a demonstration of how control-oriented modeling of complex systems can benefit from iteratively using first principles and system identification. A transcritical air conditioning system is modeled according to the process shown in Fig. 1. The model is initially developed with first principles using an approach established in the thermal systems field. When compared to the results of system identification, the discrepancy in model complexity prompts a detailed evaluation of the physical modeling process resulting in insights into the system dynamics previously unavailable in the literature. Furthermore, modes that are nonessential are identified and physically-based model reduction techniques are subsequently employed to achieve a minimal representation of the dynamics more suitable for control design. The final reduced order physical model sufficiently captures the salient dynamic behavior of the system and results in negligible loss of accuracy compared to the higher order model previously derived.

The development of minimal control-oriented models can benefit from iteration between both first principals modeling and system identification. Although in hindsight the choice of model order is clear, engineers are often required to model complex systems with little a priori knowledge. For many complex nonlinear systems formed by the interconnection of many subsystems, the search for nonessential physical dynamic modes could benefit from the identification results to provide an estimate of the appropriate order. The iterative process described in this paper requires more time and effort, but results in models with the appropriate balance between fidelity and simplicity, and greater confidence in the final model.

<table>
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REFERENCES


