A Pragmatic Bit and Power Allocation Algorithm for NOFDM Signalling

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Abstract— In this paper, a novel pragmatic bit and power allocation algorithm for Non Orthogonal Frequency Division Multiplexing (NOFDM) systems is presented. It is based on the well-known Campello approach and takes into account the lack of orthogonality between adjacent waveforms. Simulation results in a typical NOFDM wireless applicative scenario show the effectiveness of the proposed algorithms.

Index Terms-- Resource Allocation, Bit and Power Loading, Non-Orthogonal Frequency Division Multiplexing.

I. INTRODUCTION

In the last recent years, multicarrier (MC) techniques have received an ever-increasing attention in the context of high data rate transmission over wireless frequency selective channels. In the form of orthogonal frequency division multiplexing (OFDM), MC schemes have been embedded in various standards, such as Wi-Fi WLAN IEEE 802.11 a/g/n and HyperLAN, Wi-Max broadband wireless access IEEE 802.16, and terrestrial digital and video broadcasting (DAB and DVB-T/H) [1]-[3]. This is mainly due to extremely appealing features such as high robustness against fading channels, simple frequency-domain equalization and adaptive bit and power loading capability to attain optimal power-throughput trade-off, that make also OFDM a strong candidate to be considered for the physical layer of next-generation wireless systems. Recently, orthogonal MC systems have been generalized with the introduction of non-orthogonal OFDM techniques, or NOFDM for short, wherein the information-bearing symbols are conveyed using a set non-orthogonal time- and frequency-shifted versions of a given prototype pulse, which can be properly adapted to the available a-priori knowledge of the transmission channel status [4]. Compared with conventional OFDM signaling which employs orthogonal pulses, the NOFDM innovative approach brings forth a number of advantages: i) pulse shaping and the spacing between adjacent atoms in time and frequency domain can be selected so that performance over dispersive fading channels can be optimized; ii) the choice of pulse shaping can be done without any restrictions with the result of a much more flexible design; iii) time and frequency overlapping among waveforms leads to better bandwidth efficiency; iv) reduced sensitivity to narrowband interferers and mitigation of inter-carrier interference (ICI) on severe doubly-selective wireless channels can be achieved without requiring any cyclic extension. What emphasized above fully explains why a specific class of NOFDM systems, namely filter-bank MC (FBMC) schemes, have been adopted for two standards, the return channel of terrestrial DVB (DVB-RCT) [5] and the Release 2 of the terrestrial trunked radio (TETRA) air interface [6]-[7].

The interest surrounding NOFDM and related formats is demonstrated by a number of significant contributions on both channel equalization and signal synchronization; see [8]-[9] and references therein. However, as far as the topic of bit and power allocation (BPA) is concerned, the literature about NOFDM (to the best of authors’ knowledge) seems to not cover this task, that indeed has to be optimally performed at the transmitter improve link reliability and make efficient use of the available resources. On the other side, the well-know algorithms developed for conventional OFDM systems, such as [10] and [11] are valuable approaches to BPA, but cannot be directly employed in the context of NOFDM due to the different structure of the signal format. Having the above as reference, this paper aims at giving a pragmatic solution to the problem of BPA for NOFDM transmissions by properly revisiting the bit-loading algorithm proposed by Campello in [10] for OFDM transmission. Specifically, after a brief discussion of the NOFDM signal model in Section II, the BPA problem is addressed in Section III where the proposed algorithm is formalized and discussed. Simulation results are given in Section IV for the specific class of NOFDM signaling adopted in the TETRA2 standard [6]-[7], while some conclusions are finally drawn in Section V.

II. NOFDM SIGNAL MODEL

The basic concepts NOFDM modulation is based on consist in i) choosing a pulse shaping (the prototype filter) different from the rectangular one adopted in conventional OFDM, ii) avoiding usage of cyclic prefixing thus obtaining higher spectral efficiency, and iii) allowing the time- and frequency-shifted versions of the prototype pulse, the so-called Gabor atoms, to be positioned in time and frequency (TF) domains without any restrictions [4]. Figure 1 gives a schematic representation of the NOFDM TF frame. Each circle stands for a specific atom $g_{t,m}(t)$ localized at the TF point $(T_t,M_f)$, with $0 \leq t \leq L-1$ and $0 \leq m \leq M-1$, where $T$ and $F$ denote the distance along the time and frequency directions between adjacent atoms, and $L$ and $M$ the number of symbols within a
frame and subcarriers, respectively. From the above, the following remarks are of interest: i) the atoms overlap each other in both direction on the TF plane, and accordingly, the orthogonality principle (required in OFDM systems) no longer exists in that both intersymbol interference (ISI) and ICI occur, ii) the number of subchannels within a given spectral interval can be greater than that OFDM allows, and iii) the relationship between symbol duration $T$ and the frequency spacing $F$ is not fixed, but can be chosen in a very flexible way to fulfill different design requirements.

![Figure 1. NOFDM frame on time-frequency plane.](image)

Defining the elementary atom as $g_{l,m}(t) = g(t - lT)e^{j2\pi mt}$, i.e., as obtained by shifting in time and frequency the prototype pulse $g(t)$, and denoting the information-carrying data symbol conveyed by $g_{l,m}(t)$ as $c_{l,m}$, the NOFDM transmitted signal can be written as

$$s(t) = \sum_{l,m} c_{l,m} g_{l,m}(t). \quad (1)$$

The above equation can be equivalently viewed as the Gabor expansion of the transmitted signal $s(t)$ on the TF plane using the set of non-orthogonal basis functions $g_{l,m}(t)$ through the coefficients $c_{l,m}$ [12]. Hence, ideal recovery of the coefficients $c_{l,m}$ from the received data in the case of non-dispersive channel is achieved whenever the receiver set functions $P_{l,m}(t)$ generated by the dual prototype pulse $p(t)$ is such that the biorthogonality condition holds

$$\sum_{l,m} \sum_{n,n'} g_{l,m}[n] g_{l,m}^*[n'] = \delta(n - n'), \quad (2)$$

where a discrete representation is assumed with sampling interval $T_s$, with $T = NT_s$, and $\delta(n)$ is the Kronecker delta which is unitary for $n = 0$ and zero otherwise. Note that any signal can be described via the NOFDM representation by appropriately choosing the prototype pulse $g(t)$ and the parameters of TF grid as time and frequency spacing $T$ and $F$. This includes also for instance single carrier signals, in the case $M = 1$, or conventional OFDM signal whenever the prototype shape is selected to be rectangular and $M = N$.

Further, the practical realization NOFDM transceiver is only by resorting to digital signal processing based on FFT block followed by the polyphase filter banks [13].

### III. ADAPTIVE BIT AND POWER ALLOCATION STRATEGY FOR NOFDM

#### A. Power allocation procedure

The aim of any BPA procedure is to assign the total power $P$ and the required number of total transmitted bits $B$ over all the $M$ subcarriers within one transmitted frame. Assuming that perfect knowledge of the channel gains is available at the transmitter or, if these values change within the frame, that an appropriate channel prediction can be applied, it is well-known that the optimal power allocation strategy can be obtained as application of the water-filling principle [14]. In the case of NOFDM, however, the problem of adaptive BPA turns out to be two-dimensional due to the TF representation of the transmitted signal (1). Thus, the fractional power $P_{l,m}$ allocated to an atom at the time and frequency location $(lT, mF)$ satisfies the two dimensional condition

$$P_{l,m} = K \frac{G_{l,m}}{|H_{l,m}|}, \quad (3)$$

where $G_{l,m}$ the power spectral density of the AWGN component, $H_{l,m}$ the TF channel transfer function, and $K$ is the water surface level to be calculated so that the amount of total transmitted power does not exceed $P$. Hence, taking into account that the atoms $g_{l,m}(t)$ overlap, it is possible to show that

$$P_{l,m} = K \frac{G_{l,m}}{|H_{l,m}|} \frac{\sum_{i,j,l,m} P_{ipg}^{|l-j|=0}}{|H_{l,m}|^2}, \quad (4)$$

where $P_{ipg}^{|l-j|=0}$ denotes the power of the pulse $g_p(t)$ at the TF location $(lT, mF)$, $P_i$ is defined as $P_i^{(00)}$ (i.e. $i=l$ and $j=m$) and $D = \{(i,j) \in Z^2 : 0 \leq i \leq L-1 \wedge 0 \leq j \leq M-1 \wedge i \neq l \wedge j \neq m\}$. Clearly, the assigned power values cannot be negative, so the constraint

$$K \left( G_{l,m} + \sum_{i,j,l,m} P_{ipg}^{|l-j|=0} \right) |H_{l,m}|^{-2} \quad (5)$$

has to be considered. It can be observed from (4) that the power assigned to any atom depends on the values of power allocated to all others TF waveforms. Hence, solving the set of equations...
defined by (4) subject to (5) reduces to an optimization problem with inequality constraints, whose possible solutions are out of the scope of this paper. For the ease of simplicity, we will resort to a simplified and pragmatic yet sub-optimal approach. First, (4) is solved numerically without constraints, then each negative solution \( P_{l,m} \) are made zero and the remainder power values are rescaled to fulfill the total transmit power constraint.

### B. Bit allocation procedure

In order to formalize a bit allocation procedure for NOFDM systems, let us consider the well-known Campello algorithm (CA) proposed in [10]. The main idea consists in a primary bit allocation depending on the channel gains and the noise power across the subcarriers in use. Then, the number of allocated bits found is iteratively modified so as to find a global optimal allocation. It can be shown that this procedure works quite well in OFDM systems, but when applied in the context of NOFDM some consideration arise. First, it has to recalled that the whole processing in NOFDM has to be performed in the TF plane, and accordingly, the allocation algorithm has to necessarily be of two-dimensional type. Second, the effect of overlapping between atoms, i.e., ISI and ICI, has to be considered. Now, although the extension to two-dimensional processing is somewhat straight-forward, the impact of the latter is rather significant and impose several modifications as described in the sequel.

First, the gain-to-noise [10] ratios \( \tilde{g}_{l,m} \) can be rewritten including interference between atoms as

\[
\tilde{g}_{l,m} = \frac{\Gamma \left( \sigma_{l,m}^2 + |H_{l,m}|^2 \sum_{(i,j) \in D} P_{l,m} P_{l,m}^{(i-j-m)} \right)}{|H_{l,m}|^2} ,
\]

where \( \sigma_{l,m}^2 \) stands for the noise power and \( \Gamma_{l,m} = \Gamma \) corresponds to the coding scheme and the target probability of error (and is usually denoted as “SNR gap”). Since the gain-to-noise definition is changed [10], the way to compute the amounts of power \( \Delta P_{l,m} \) (required to sent one additional bit on one atom localized at \((l,m)\) instant on discrete TF grid) has to be modified.

One can conclude from (6), that to calculate the primary bit allocation the final power distribution over TF plane is needed due to the necessity of incorporation of the interference between neighboring atoms. However, these power values should be calculated after finding the final bit allocation (e.g due to the Campello approach in CBR case). In fact we cannot neglect the power of interferences (in other words, we cannot assume, that the initial power allocation is uniform), because the final power allocation will be completely different and – in consequence – the interferences between atoms will be different. To solve this problem, one could propose, to i) compute incremental power values \( \Delta P_{l,m} \), ii) increment the number of bits carried by the atom with the smallest \( \Delta P_{l,m} \), and iii) recalculate the values of \( \Delta P_{l,m} \). However, as it will be shown later, the calculation of power increments is rather complex, and such approach leads to Hughes-Hartogs-like [11] algorithm and is not a subject of this paper. To solve this problem, we propose to find the primary power allocation according to (4) and set to zero all (if any) negative values of power. This power allocation (found from the water-filling principle) is strongly correlated with the final power allocation obtained form the bit-loading algorithm. In such an instant, the interferences between atoms can be calculated and the primary bit allocation can be determined. It is worth to mention, that (4) can be also expressed in the matrix form.

The next modification (due to the overlapping) occurs on the stage of computing of incremental power values \( \Delta P_{l,m} \). Let us recall the relation for the number of bits assigned to one atom in TF scenario:

\[
b_{l,m} = \log_2 \left( 1 + \frac{P_{l,m} P_{l,m} \tilde{g}_{l,m}}{\sigma} \right),
\]

where \( b_{l,m} \) denotes the number of bits assigned to the atom localized at \((l,m)\) instant of TF plane and \( P_{l,m} P_{l,m} \tilde{g}_{l,m} \) is the power assigned to this atom. From (7) and (6) one can obtain,

\[
P_{l,m} = \frac{\left( 2^{b_{l,m}} - 1 \right)}{P_g |H_{l,m}|} \Delta \sigma_{l,m} + |H_{l,m}|^2 \sum_{(i,j) \in D} P_{l,m} P_{l,m}^{(i-j-m)} .
\]

First, let us compute the value of \( \Delta P_{l,m} \), when only one bit is added for only one. Thus:

\[
\Delta P_{l,m} = P_{l,m}^* - P_{l,m}^\prime =
\]

\[
= \frac{\left( 2^{b_{l,m}^*} - 1 \right)}{P_g |H_{l,m}|} \Delta \sigma_{l,m} + |H_{l,m}|^2 \sum_{(i,j) \in D} P_{l,m} P_{l,m}^{(i-j-m)} -
\]

\[
= \frac{\left( 2^{b_{l,m}^\prime} - 1 \right)}{P_g |H_{l,m}|} \Delta \sigma_{l,m} + |H_{l,m}|^2 \sum_{(i,j) \in D} P_{l,m} P_{l,m}^{(i-j-m)} =
\]

\[
= \frac{2^{b_{l,m}^\prime} - 1}{P_g |H_{l,m}|} \Delta \sigma_{l,m} + \sum_{(i,j) \in D} P_{l,m} P_{l,m}^{(i-j-m)}
\]

In the above equations \( P_{l,m}^* \) denotes the power assigned to the atom localized at TF instant \((l,m)\) after the increment operation of number of bits carried by this atom, whereas \( P_{l,m}^\prime \) corresponds to the power assigned to this atoms before above mentioned operation. Although the above equation can be represented in the matrix form, the computation complexity for the increment operation of one bit is significant. Thus, such a way of calculation of the incremental power values \( \Delta P_{l,m} \) cannot be used in practical implementation, especially when many iterations (many increment operations) have to be performed. Instead one could propose to calculate the incremental power values jointly for all time-frequency instants. We can rewrite (9) as follows:
\[ P_{\text{in}} - P_{\text{out}} = \frac{2^{b_{\text{in}} - 1} - 1}{P_s g H_{\text{in}}} \Gamma \left( \sigma_{\text{in}}^2 + \sum_{(i,j) \in D} P_{\text{in}} \sum_{(i,j) \in D} P_{\text{in}} \right) - \frac{2^{b_{\text{in}} - 1} - 1}{P_s g H_{\text{in}}} \Gamma \left( \sigma_{\text{in}}^2 + \sum_{(i,j) \in D} P_{\text{in}} \sum_{(i,j) \in D} P_{\text{in}} \right). \]  

After some derivations one can obtain

\[ P_{\text{in}} - P_{\text{out}} = \frac{2^{b_{\text{in}} - 1} - 1}{P_s g H_{\text{in}}} \Gamma \left( \sigma_{\text{in}}^2 + \sum_{(i,j) \in D} P_{\text{in}} \sum_{(i,j) \in D} P_{\text{in}} \right) - \frac{2^{b_{\text{in}} - 1} - 1}{P_s g H_{\text{in}}} \Gamma \left( \sigma_{\text{in}}^2 + \sum_{(i,j) \in D} P_{\text{in}} \sum_{(i,j) \in D} P_{\text{in}} \right). \]  

which corresponds to the already found bit arrangement. For this purpose let us rewrite (12) in the following manner:

\[ \frac{P_{\text{in}} - P_{\text{out}}}{2^{b_{\text{in}} - 1} - 1} = \Gamma \left( \sigma_{\text{in}}^2 + \sum_{(i,j) \in D} P_{\text{in}} \sum_{(i,j) \in D} P_{\text{in}} \right). \]  

or equivalently

\[ \frac{L - 1}{2^{b_{\text{in}} - 1}} \sum_{i=0}^{L-1} \sum_{j=0}^{L-1} \delta_{i,j} P_{\text{in}} \sum_{(i,j) \in D} P_{\text{in}} \sum_{(i,j) \in D} P_{\text{in}} = \Gamma \sigma_{\text{in}}^2 |H_{\text{in}}|^{-2}. \]

Algorithmically, the procedure can be summarized as follows:

1. Let: \( M, L, H_{\text{in}}, \sigma_{\text{in}}^2, B \) are the known variables

2. Calculate the modified values of gain-to-noise ratios (according to (6)).

3. Calculate the primary power allocation (as in (4)).

4. Based on the Campello approach, find the incremental power values \( \Delta P_{\text{in}} \) using (13) and assign the remaining bits to these atoms, which have the lowest incremental power values.

5. Recalculate the power allocation for the found bit arrangement (using (17)).

IV. SIMULATION RESULTS

In this section, the proposed BPA algorithm is verified by numerical simulations adopting two different prototype waveform: i) Gaussian pulse (GP) which is well localized both in time and in frequency domain so that small overlapping between atoms occurs, ii) and root-raised cosine RRC pulse (RRCP) for which the overlapping in time is more significant than for GP. The number of subcarriers and number of consecutive symbols in one OFDM frame, or equivalently, the number of atoms in time and in frequency domain, are equal to 48 and 34, respectively, as defined in the TETRA2 standard [6]-[7]. The total number of bits to be transmitted in each OFDM frame is set to \( B = M \cdot L \cdot \frac{b}{2} \), where the maximal number of bits per one constellation point is \( b = 10 \). In both cases, the multipath channel (a snapshot of the TF channel gains is given in Figure 2) is modeled with 6 paths with exponentially decaying power delay profile corresponding to a delay spread of 1.76ms, and the SNR is set as high as 35 dB in order to make evident the effects of the interference between neighboring atoms on the total system capacity.
Figure 2. Normalized TF channel gain.

Figure 3. Bit allocation for the proposed algorithm for GP (a) and RRCP (b).

Figure 4. Bit allocation for 2D Campello algorithm for GP (a) and RRCP (b).

Figure 5. Differences in bit allocation for GP.
Figures 3 and 4 show the bit allocation obtained for GP (a) and RRCP (b) when the proposed modified CA (MCA) algorithm (Fig. 3) and the original two-dimensional CA algorithm (2DCA) (Fig. 4) are applied. The differences in bit allocation, coming from the application of MCA and 2DCA, are marked by solid circles for GP and dotted circles for RRCP. One can observe from Figs. 5 and 6 that for the GP (which enables less interference among atoms) the number of atoms that differ in the number of assigned bits is smaller. This can be explained by considering that when overlapping is stronger as when adopting the RRCP (this means that the power leakage between neighboring atoms is greater), the MCA provides a bit arrangement that better copes with the effects of the channel dispersion on the NOFDM transmission. Conversely, when the atoms do not overlap each other significantly as in the case of GP, the bit allocation obtained from MCA and 2DCA are similar.

V. CONCLUSIONS

A pragmatic bit and power allocation procedure for NOFDM systems has been derived from a modified version of the well-known Campello algorithm proposed in the context of OFDM transmission. The rationale of our scheme comes from taking into account the significant interference in term of ISI and ICI that can exist between adjacent atoms (waveforms) in the time and frequency domains wherein the NOFDM transmitted signal is represented. The results obtained for both Gaussian and root-rise-cosine shapes show that, unlike the conventional approach, the proposed algorithm can cope with the interference induced by the dispersive channel thus enabling a more efficient usage of the link resources.

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