Assessment of Time-Domain Models of Wave Energy Conversion Systems

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Abstract—Time-domain models are necessary for the analysis of wave energy conversion systems, due to the presence of nonlinearities which may not be neglected for accurate prediction of their performance and behaviour. Such nonlinearities are contributed in varying degrees by drag, Coulomb friction, fluid compressibility, and also control mechanism. In time-domain, the equations of motion for the system will contain hydrodynamic radiation terms expressed as convolution integrals due to the frequency dependence of the radiation coefficients. The evaluation of the convolution integral is time-consuming and is difficult to carry out by standard adaptive solvers. Hence, various approximations to the convolution integral have been proposed to avoid these problems. The purpose of this study is to systematically assess the quality of some selected time-domain models. Generic models of wave energy conversion systems will be developed, with the possibility of varying the relative importance of the nonlinear terms. The time-domain models are categorised according to the convolution approximation and the numerical integration method used. Selected assessment criteria include computation time as well as the statistics of device motions and converted power. A model is always a trade-off between efficiency and accuracy. It is hoped that this study will provide some guidelines in the choice of time-domain models suitable for simulation of wave energy conversion systems.

Index Terms—wave energy, time-domain simulation, bond graph

I. INTRODUCTION

The need for time-domain models for the simulation of wave energy conversion systems (WECSs) has been recognised as early as late 1970s [1]. Time-domain approach is necessary because WECSs contain substantial degree of nonlinearities, arising mainly from the power take-off system and control mechanism. The common way to formulate a time-domain model is to use integro-differential equations of motion which contain convolution integrals representing the wave radiation force [2], [3]. The convolution integral account for the system memory, signifying the fact that waves radiated by the body in the past continue to affect the body force for all subsequent times [4], [5]. The kernel of the integral is an impulse response function (IRF) also known as the retardation function, which is related to the frequency-dependent hydrodynamic radiation coefficients by Fourier transforms. When the system is nonlinear, this integral is necessary even when the incident waves are monochromatic, as shown in [4].

In time-domain simulations, evaluation of the convolution integral is known to be time-consuming and difficult to carry out with standard adaptive time-stepping solvers. The reason is that one usually has to store discrete values of the IRF sampled at every simulation time step for the whole simulation time length, and one has to re-evaluate the convolution integral at every time step. It is obvious that for a multiple-degree-of-freedom system, which contains a number of these integrals, the computational effort can be tremendous. To avoid this difficulty, a set of coupled linear ordinary differential equations has been proposed as an approximate replacement for the convolution integral, first probably by Jefferys [1] in wave energy context. This so-called state-space representation is more efficient due to its Markovian property: at any instant, the value of the state summarises all the past system information [6]. The need to store a large amount of data and re-evaluate the integral at every time step is therefore eliminated.

Different approaches have been proposed in the literature to identify this state-space radiation force model, and have been summarized recently in [6]. Each of these approaches belongs to either time-domain or frequency-domain identification. In time-domain identification the state-space model is obtained from the corresponding radiation IRF, whereas in frequency-domain identification the state-space model is obtained from the corresponding radiation data in frequency domain.

To assess the quality of these state-space models, one may compare the steady-state responses of a linear system obtained from such models against those from a frequency-domain model. This was done by Jefferys [7], who compared the velocity and power obtained from a state-space model by frequency-domain identification with those from a frequency-domain model, over a range of wave periods. Taghipour et al. [8] compared the displacements from two state-space models, one by time-domain identification and the other by frequency-domain identification, with those from a frequency-domain model. Perez and Fossen [6] later made similar comparisons of the force-displacement frequency response functions (FRFs) obtained from the three models. One may also compare the transient responses obtained from the state-space models with those from a direct convolution integration (without convolution replacement). Considering a linear system, Taghipour et al. [8] compared the displacements in calm water after an initial displacement obtained from the two state-space models and from direct convolution integration. Also, considering a nonlinear system, Jefferys [7] compared the mean power and mean square velocity in a given sea state obtained from a state-space model by frequency-domain
identification, with those obtained from direct convolution integration. Common to all these studies was the considerable saving of computation time resulting from the use of state-space model as opposed to direct convolution integration.

The purpose of this study is to complement those previous works by supplying a more systematic comparison of some selected time-domain models of WECSs. To achieve this purpose, generic models of WECSs are developed, with the possibility of varying the relative importance of their nonlinear terms. Hydrodynamic data are computed from some simple geometries. The time-domain models to be compared are categorised according to the convolution approximation and the numerical integration method used. For the state-space approximations, we will focus on state-space models obtained by frequency-domain identification, with different orders of approximation. Comparisons will be made of response in monochromatic waves and response in polynomial waves.

The assessment criteria include computation time, statistics of device motions, and statistics of converted power. A similar study with relevant discussions has been presented by Ricci et al. [9]. The main difference between their and the authors is that we consider more generic models with variable nonlinear terms, to allow for a more systematic study of the various effects. Moreover, they considered time-domain identification whereas we consider frequency-domain identification for the state-space models.

II. GENERIC MODELS OF WAVE ENERGY CONVERSION SYSTEMS

We first develop generic models of a selection of WECSs in the bond graph formalism [10]. These are thought to represent most wave energy conversion concepts available to date. For all these models, we assume a linear load resistance. Each model contains nonlinear terms. The effects of varying the degrees of nonlinearity of these nonlinear terms will be examined for each model.

A. Floating oscillating water column

We consider firstly a floating oscillating water column (OWC) wave energy device, restricted to move only in one degree of freedom. A bond graph model of such system is shown in Fig. 1, with the following main characteristics. The force balance on the OWC body is represented by the bonds connected to the 1-junction on the upper left. The volume flow balance in the OWC chamber is represented by the bonds connected to the 0-junction on the bottom left. The coupling between the body velocity and the chamber pressure is represented by the TF element connecting the 0-junction to the 1-junction. The TF element to the right of the first 1-junction carries out the transformation between force-velocity and pressure-volume flow. The 0-junction on the right connects 1- and 0-junctions on the left, signifying that the volume flow relative to the body is utilised for power absorption. The circle labelled P connected to the load resistance is a power sensor. The nonlinear terms in the model are the Coulomb and quadratic damping forces on the body, the volume flow due to air compressibility in the chamber, and the volume flow through the relief valve. All other terms are assumed to be linear. The coupled equations of motion for this device in time domain can be written as

$$F_e(t) = \left[ m_m + m(\infty) \right] \dot{u}(t) + k(t) \ast u(t) - C(\infty) \dot{p}(t) - h(t) \ast \dot{p}(t) + (S_1 + S_2) \dot{s}(t) + R_C \text{sgn} \ u(t) + R_q u(t) \| u(t) \| - r \dot{p}(t)$$

$$Q_e(t) = y(t) \ast p(t) + C(\infty) u(t) + h(t) \ast u(t) + Q_v(t) + Q_c(t) + r u(t) + (R_c + 1/R_u) p(t),$$

where $F_e(t)$ is the wave excitation force, $m_m$ is the inertia of the body, $m(\infty)$ is the infinite-frequency added mass, $u(t)$ is the body velocity, $k(t)$ is the radiation IRF, $C(\infty)$ is the real part of the radiation coupling coefficient $H(\omega)$ at infinite frequency (see [11]), $h(t)$ is the radiation coupling IRF, $p(t)$ is the chamber pressure, $S_1$ is the hydrostatic stiffness, $S_2$ is the external stiffness, $\dot{s}(t)$ is the body displacement, $R_C$ is the Coulomb damping coefficient, $R_q$ is the quadratic damping coefficient, $r$ is some transformation factor, $Q_v(t)$ is the excitation volume flow, $\dot{p}(t)$ is the radiation admittance IRF, $Q_c(t)$ is the volume flow through the relief valve, $Q_e(t)$ is the volume flow due to air compressibility, $R_c$ is the external damping coefficient, and $R_u$ is the load resistance.

The flow $Q_v(t)$ through the relief valve depends on the pressure difference across the valve:

$$Q_v(t) = \begin{cases} \frac{p(t)}{p_{cl}} Q_{cl} & \text{if } |p(t)| < p_{cl} \\ \text{sgn}(p(t)) \left[ Q_{cl} + \frac{|p(t)| - p_{cl}}{p_{op} - p_{cl}} (Q_{op} - Q_{cl}) \right] & \text{if } p_{cl} \leq |p(t)| \leq p_{op} \\ C_d A_{max} \sqrt{\frac{2}{p_a} |p(t)| \text{sgn}(p(t))} & \text{if } |p(t)| > p_{op} \end{cases}$$

Fig. 1. Bond graph model of a floating oscillating water column wave energy device restricted to move only in one degree of freedom.
Fig. 2. Bond graph model of a fixed oscillating water column wave energy device.

where

\[ Q_{cl} = C_d A_{\text{min}} \sqrt{\frac{2}{\rho_a}} p_{cl} \]  \hspace{1cm} (4)\]

\[ Q_{op} = C_d A_{\text{max}} \sqrt{\frac{2}{\rho_a}} p_{op}. \]  \hspace{1cm} (5)\]

In the above, \( C_d \) is the discharge coefficient, \( \rho_a \) is the air density, \( A_{\text{min}} \) is the leakage area, while \( A_{\text{max}} \) is the fully open flow area. The pressures \( p_{cl} \) and \( p_{op} \) are reference pressures for the closing and opening of the valve. The valve is closed if \(|p| < p_{cl}\), and is fully open if \(|p| > p_{op}\). To be realistic, the leakage area \( A_{\text{min}} \) is introduced to allow possible leakage when the valve is closed.

The air compressibility in the chamber can be modelled according to this nonlinear relationship:

\[ p_0 + p = p_0 \left( \frac{V_0}{V_0 - \Delta V} \right)^\gamma, \]  \hspace{1cm} (6)\]

where \( p_0 \) is the atmospheric pressure and \( V_0 \) is the average air volume in the chamber. The volume change due to compressibility is denoted by \( \Delta V = \int_0^t Q_c(t) dt \). The specific heat ratio \( \gamma \) depends on whether the expansion and compression occur rapidly or slowly. The value \( \gamma = 1.4 \) is usually adopted.

B. Fixed oscillating water column

The second device we consider is a fixed OWC. A bond graph of this type of device is shown in Fig. 2. We have intentionally maintained the layout of the bond graph for the floating OWC in Fig. 1 to show that the bond graph for a fixed OWC is a subset of the former. Since there is no coupling with the body motions, the bond graph structure representing the body dynamics is removed. The nonlinear terms in the model are the volume flow due to air compressibility and the volume flow through the relief valve. The equation of motion for this device can be written as

\[ Q_c(t) = y(t) \ast p(t) + Q_c(t) + Q_c(t) + (R_c + 1/R_a) p(t), \]  \hspace{1cm} (7)\]

using the same notations as for the floating OWC (Section II-A).

C. Oscillating single body

Lastly, we consider an oscillating single-body wave energy device, restricted to move only in one degree of freedom. A bond graph of this type of device is shown in Fig. 3. Again, we have shown that the bond graph for an oscillating single-body wave energy device is a subset of that for the floating OWC. The \( TF \) element in the model is some transformation from one energy domain to another. The nonlinear terms in the model are the Coulomb and quadratic damping forces. The equation of motion for this device can be written as

\[ F_c(t) = [m_m + m(\infty)] \ddot{u}(t) + k(t) \ast u(t) + (S_b + S)s(t) + R_C \text{sgn} u(t) + R_d u(t) |u(t)| + r R_a u(t). \]  \hspace{1cm} (8)\]

III. COMPUTATION OF HYDRODYNAMIC PARAMETERS

A. Body geometries

For the fixed and floating OWC device we consider a vertical square cylinder with a square moonpool in the centre. The cylinder is 10 m by 10 m, the moonpool is 5 m by 5 m, and the draft is 5 m. For the floating OWC, the cylinder is restricted to move only in heave. The incident wave is assumed to propagate in the direction normal to any of the cylinder side walls.

For the oscillating single-body device we consider a geometry similar to the Edinburgh duck [12]. It is a horizontal cylindrical body with a cross section formed by a combination of a semicircle and a right triangle meeting at 30 degree angle. The body is free to move only in rotation about the axis passing through the centre of the arc. The arc centre is located 1.5 m below the water line and the arc radius is 3 m, making a draft of 4.5 m, while its length (measured along the rotation axis) is 8 m. The incident wave is assumed to propagate in the direction perpendicular to the rotation axis.

Three-dimensional views of the submerged body geometries are shown in Fig. 4.

B. Computation in frequency domain

The computation of the frequency-domain hydrodynamic parameters is carried out using a three-dimensional higher-order panel method [13]. For all computations, infinite water
depth is assumed. Computations are performed for wave frequencies from 0 to 6 rad/s, in interval of 0.05 rad/s, and also for the infinite-frequency limit. As the wave frequency increases, greater number of panels is required for convergence, and the computational burden at the same time increases. A panel subdivision is chosen which gives converged results up to a certain cut-off frequency. Beyond the cut-off frequency, the computed hydrodynamic parameters start to fluctuate around the correct values. For accurate evaluation of the radiation impulse response function (to be discussed in the next section), there is a need to have hydrodynamic data for high frequencies. For this purpose, the values for wave frequencies above the cut-off frequency are approximated by extrapolation using a fitting function fitted to the tail of the data. Given the fact that fitting is done in the least-squares sense, although values computed beyond the cut-off frequency for the chosen panel subdivision are inaccurate, but since they fluctuate around the correct values, they are useful for the fitting. Several fitting functions are tested, and exponential function in the form of $a \exp(b\omega) + c \exp(d\omega)$ is found to yield the best fit. Fig. 5 shows the added mass and radiation damping of the oscillating body. Fig. 6 shows the added mass, radiation damping, and the parameters $G(\omega) = \Re\{Y(\omega)\}$, $B(\omega) = \Im\{Y(\omega)\}$, $C(\omega) = \Re\{H(\omega)\}$, and $J(\omega) = \Im\{H(\omega)\}$ of the OWC. Definitions of the parameters $G$, $B$, $C$, $J$ and the procedures to derive them are given in [14]. However, instead of using relation (11) in [14], here we use the following relation to derive $B(\omega)$:

$$B(\omega) = -\int_{0}^{\infty} y(t) \sin(\omega t) dt,$$

where

$$y(t) = \frac{2}{\pi} \int_{0}^{\infty} G(\omega) \cos(\omega t) d\omega. \quad (10)$$

Also, $C(\omega)$ is obtained as follows:

$$C(\omega) = C(\infty) + \int_{0}^{\infty} h(t) \cos(\omega t) dt,$$  \quad (11)

where

$$h(t) = -\frac{2}{\pi} \int_{0}^{\infty} J(\omega) \sin(\omega t) d\omega.$$  \quad (12)

Here any integration over the interior free surface is approximated by a sum over 100 field points with uniform spacings on the interior free surface. Any integration from $\beta = 0$ to 2$\pi$, where $\beta$ is the incident wave propagation angle, is approximated by a sum over a discrete set of incident wave propagation angles with uniform interval of 5 degrees.

C. Impulse response functions

For direct convolution integration, which is described in Section IV-A, it is essential to have an accurate IRF for each mode of motion. The IRFs can be evaluated directly by solving the time-domain boundary value problem or indirectly from the frequency-domain data. Here we will discuss the latter option since softwares which solve the boundary value problem in time domain are less widely known than those which solve the problem in frequency domain.

For an oscillating body, the radiation IRF for a given degree of freedom can be obtained from the corresponding frequency-domain data as follows (see, e.g. [15]):

$$k(t) = \frac{2}{\pi} \int_{0}^{\infty} R(\omega) \cos(\omega t) d\omega,$$  \quad (13)

where $R(\omega)$ is the radiation damping, or, alternatively,

$$k(t) = -\frac{2}{\pi} \int_{0}^{\infty} \omega [m(\omega) - m(\infty)] \sin(\omega t) d\omega,$$  \quad (14)

where $m(\omega)$ is the added mass. For an OWC, the radiation admittance IRF $y(t)$ and the radiation coupling IRF $h(t)$ can be obtained similarly (see (10) and (12)).

For most body geometries, the hydrodynamic data are not available in analytical forms, and are normally evaluated by a numerical panel method. The integration in (13) or (14) then has to be evaluated numerically over a finite frequency range. This presents a challenge for accurate evaluation of IRF.
from frequency-domain data. The necessity of a high enough upper truncation limit of integration has been highlighted by several authors [5]–[7], [16], [17]. Low truncation frequency is known to result in inaccuracies in the evaluated IRF. To obtain hydrodynamic data at high frequencies by panel methods, however, would require very small panels, which is not practical. High-frequency values should be obtained by other means. One way to do this is by extrapolation using a fitting function to fit the tail of the data. We use this approach in our study, where an exponential function is used for the fitting. Trapezoidal integration method is used to evaluate the integral, where finer frequency resolution is obtained by interpolation.

The radiation IRF of the oscillating body evaluated using (13), the radiation admittance IRF \( y(t) \) of the OWC, and the radiation coupling IRF \( h(t) \) of the floating OWC are shown in Fig. 7. In contrast to the radiation IRF of the oscillating body, the radiation IRFs of the OWC decay very slowly due to the narrow bandwidth of the corresponding frequency-domain parameters. This behaviour has been reported, e.g. in [18]. Fig. 8 shows comparisons of the convolution terms \( k(t) \ast u(t) \) of the oscillating body, \( k(t) \ast u(t), y(t) \ast p(t) \), and \( h(t) \ast u(t) \) of the OWC with their respective frequency-domain equivalents for given \( u(t) \) and \( p(t) \). The very good agreement verifies the accuracy of our method. Improved agreement may be obtained by using finer frequency resolution for the computation of the hydrodynamic parameters.

IV. TIME-DOMAIN MODELS

We compare three categories of time-domain models, namely the direct convolution integration model, the constant hydrodynamic parameter model, and the state-space model.

A. Direct convolution integration

In the direct convolution integration model the convolution terms in the equations of motion are not replaced by any approximations and are integrated directly at each time step. This requires, firstly, precomputation of the IRF values at specified time intervals, where linear interpolation is used if the simulation time step is shorter, and, secondly, storage of past response (velocity and/or pressure). With accurate IRFs and sufficiently small simulation time step, a direct convolution integration model should give accurate simulation results despite the considerable computational burden involved.

B. Constant hydrodynamic coefficients

In the constant hydrodynamic coefficient model the frequency-dependent coefficients are replaced by constant coefficients, whose values are taken to be those at the wave spectral peak frequency \( \omega_p \).

The equation of motion for the oscillating single body then becomes

\[
F_{c}(t) = \left[ m_\infty + m(\omega_p) \right] \ddot{u}(t) + R(\omega_p) u(t) + (S_b + S) s(t) + R_C \ \text{sgn} \ u(t) + R_u u(t) |u(t)|
\]

\[
+ r R_u u(t), \quad (15)
\]
the equation of motion for the fixed OWC becomes
\[ Q_e(t) = G(\omega_p)p(t) + B'(\omega_p) \int_0^t p(t)\,dt + Q_e(t) + Q_e(t) + (R_c + 1/R_a) u(t), \]
and the equations of motion for the floating OWC become
\[ F_c(t) = [m_m + m(\omega_p)] \ddot{u}(t) + R(\omega_p) u(t) - C(\omega_p)p(t) - J'(\omega_p)\dot{p}(t) + (S_b + S) s(t) + R_C \text{sgn} u(t) + R_{\mu} u(t)|u(t)| - \tau p(t), \]
\[ Q_e(t) = G(\omega_p)p(t) + B'(\omega_p) \int_0^t p(t)\,dt + C(\omega_p)u(t) + J'(\omega_p)\dot{u}(t) + Q_e(t) + Q_e(t) + R u(t) + (R_c + 1/R_a) p(t). \]
In the above, \( J' = J/\omega \) and \( B' = -\omega B \).

C. State-space representation

In the state-space model the convolution term
\[ \mu(t) = k(t) * u(t) = \int_0^t k(t - \tau)u(\tau)\,d\tau \]
is replaced by a set of coupled linear ordinary differential equations, which may be expressed in matrix form (see, e.g. [19]):
\[ \dot{x}(t) = \hat{A}x(t) + \hat{B}u(t) \]
\[ \dot{\mu}(t) = \hat{C}x(t) \]
where \( x(t) \) is the state vector, the number of components of which corresponds to the order of the state-space model,
and $\hat{A}$, $\hat{B}$, $\hat{C}$ are constant matrices. We use the frequency-domain identification approach following the algorithm detailed in [19], [20]. As mentioned earlier, the method uses frequency-domain hydrodynamic data for identification. The approach is to fit a rational transfer function

$$K(s) = \frac{P(s)}{Q(s)} = \frac{p_n s^n + p_{n-1} s^{n-1} + \ldots + p_0}{s^n + q_{n-1} s^{n-1} + \ldots + q_0},$$

(22)

where $s = i\omega$, to the FRF $K(\omega)$, $Y(\omega)$, or $H(\omega)$ depending on the problem considered. Further constraints on the model have been derived in [6] based on the properties of the FRF and its corresponding IFR. A least-squares fitting method is applied to find the coefficients $p_i$ and $q_i$, and once the coefficients $p_i$ and $q_i$ are obtained, the matrices $\hat{A}$, $\hat{B}$, and $\hat{C}$ can be constructed using any of the standard canonical forms. We will examine the use of different model orders, starting from a minimum order of 2.

V. SIMULATION

A. Excitation force and excitation volume flow

Comparisons will be made of the different models under both monochromatic wave and polychromatic wave excitations. The excitation force and/or excitation volume flow time series are generated before the simulation and stored as data files to be read during the simulation. The generation of excitation force and/or excitation volume flow time series in monochromatic waves is straightforward. For polychromatic waves, the method is described as follows.

First, we obtain the spectral density of the excitation force (likewise for the excitation volume flow):

$$S_{F_i}(\omega) = |f_{e_i}(\omega)|^2 S(\omega),$$

(23)

where $f_{e_i}(\omega)$ is the computed complex excitation force for a unit incident wave and $S(\omega)$ is the given wave spectrum. We use a JONSWAP spectrum with peakedness parameter of 3 in our simulations. The excitation force is then given as

$$F_e(t) = \sum_{n=0}^{N/2} \left( a_n \cos \phi_n + b_n \sin \phi_n \right) \cos \omega_n t + \sum_{n=0}^{N/2} \left( -a_n \sin \phi_n + b_n \cos \phi_n \right) \sin \omega_n t,$$

(24)

where $a_n$ and $b_n$ are generated from a Gaussian distribution with variance $S_{F_i}(\omega_n) \Delta \omega$ [21]. Here, $N$ is the number of values in the time series, determined by the required length of the series $T$ and the time interval between values $\Delta t$. Also, $\omega_n = n \Delta \omega$, where $\Delta \omega = 2\pi / T$. In addition, $\phi_n$ is the phase (in radians) of $f_{e_i}(\omega_n)$. Values of $\omega_n$ may be larger than the largest frequency for which $f_{e_i}$ is computed. Modulus and phase of $f_{e_i}$ for these frequencies may therefore be extrapolated separately using fitting functions as in the extrapolation of added mass and radiation damping data for the modulus, an exponential function in the form of $a \exp(b \omega) + c \exp(d \omega)$ is used, while for the phase, a power function in the form of $aw^b$ is used. The necessity of having accurate $f_{e_i}$ values beyond 6 rad/s, however, is of lower importance in this case because typical wave spectra have negligible values beyond 6 rad/s. Thus, it may be practical to even assume zero values for $f_{e_i}$ beyond 6 rad/s.

The sum in Eqn. (24) may be identically evaluated by an inverse Fast Fourier Transform at a fraction of computer time. This is implemented in our simulation. The initial part of the resulting time series is filtered by a cosine taper window so as to avoid exciting any lightly damped modes in the system.

B. Integration methods

For the direct convolution integration model, simulations are carried out with fixed time step. We compare different time integration methods and examine the effect of different time steps on accuracy and efficiency. The methods compared are the Euler’s method (ode1), the improved Euler’s (Heun’s) method (ode2), Runge-Kutta 3 (ode3), and Runge-Kutta 4 (ode4) methods. Simulations for the direct convolution integration model are carried out using a computing package [22].

For the constant coefficient and the state-space models, an adaptive Runge-Kutta-Fehlberg solver is used for the oscillating body device, while an adaptive Vode Adams solver is used for the OWC device. Whenever the adaptive methods are not successful, the fixed step solvers are used. Both the absolute and relative integration error tolerances are set to $1 \times 10^{-7}$. Simulations for the constant coefficient and state-space models are carried out using a modelling and simulation package [23]. All simulations are run in a 2.53 GHz, 2.96 GB RAM CPU.

C. Treatment of Coulomb damping

In the simulations, Coulomb damping force is modelled as follows:

$$F_C(t_i) = \begin{cases} R_C \frac{u(t_i)}{u_C} & \text{if } u_C < u(t_i) < u_C \\ R_C \frac{\text{sgn } u(t_i)}{u_C} & \text{otherwise} \end{cases},$$

(25)

where $u_C$ is a small velocity threshold. We choose $u_C = 4 \times 10^{-4}$ rad/s for the oscillating body and $u_C = 4 \times 10^{-4}$ m/s for the floating OWC.

VI. RESULTS AND DISCUSSIONS

For each device, we compare responses obtained from the different time-domain models under monochromatic and polychromatic wave excitations. We compare both the case where all nonlinear terms are set to zero and the case where the nonlinear terms are varied. For the former, responses are compared to those from the frequency-domain model, and we measure the error (in %) which is defined as

$$e_{FD} = 100 \frac{|\hat{u} - u_{FD}|}{|u_{FD}|},$$

(26)

where $\hat{u}$ is the complex body velocity obtained from state-space or direct convolution model and $u_{FD}$ is the complex body velocity obtained from frequency-domain model.
For the latter, the assessed response is compared to the ‘true’ response, and we assess the error (in %) which is defined as

$$e = \frac{100}{N} \sum_{i=1}^{N} \frac{|\hat{q}(t_i) - q(t_i)|}{\max q(t_i)},$$

where $\hat{q}(t_i)$ and $q(t_i)$ are the instantaneous assessed response and the ‘true’ response, respectively, at time $i$. The response $q(t)$ can be the converted power, the body velocity for the oscillating body device, or the chamber pressure for OWC devices. The ‘true’ response is defined as the converged response obtained from direct convolution integration using sufficiently small simulation time step. When nonlinear terms are zero, expression (27) is approximately equivalent to (26).

For comparisons in polychromatic waves, in addition to comparing instantaneous values of power, velocity, and/or pressure, we also compare the maximum and mean values of converted power, as well as the root mean square velocity and/or pressure.

A. Oscillating single body

Fig. 9 compares the body velocity obtained from state-space and frequency-domain models of the oscillating single-body device when the external stiffness $S$ is set to 100 Nm, the transformation factor $r$ is set to 1, the load resistance $R_L$ is set to $6 \times 10^5$ kg m$^2$ s$^{-1}$, and the nonlinear terms are set to zero. The wave amplitude is 1 m. Four different state-space model orders are compared: 2, 3, 4, and 7. From the figures we see that increasing model order improves the model accuracy for the range of wave frequencies considered. For this particular case, however, a model order even as low as 3 keeps the error within 2% and increasing the model order up to 7 does not improve the accuracy significantly. It should be noted that the error for a given model order is dependent on the specified tolerance in the adaptive solver. The purpose of the comparison is to give an indication of how the different model orders perform relative to one another.

Fig. 10 compares the performance of different time integration methods in the direct convolution integration model of the oscillating single-body device for the same set of parameters. The top figure compares the time step required for each time-integration method to keep the error defined in (26) within 2%. Our assumption is that the error increases with time step length. The bottom figure compares the computation time (averaged from three runs) required for 200-second simulation using the time steps of the top figure. In general, shorter time steps are required for smaller wave periods to keep the same degree of accuracy. The Euler’s (ode1) method requires much longer computation time than that of the other methods for smaller wave periods. The performance of the improved Euler’s (Heun’s) method (ode2), Runge-Kutta 3 (ode3), and Runge-Kutta 4 (ode4) methods are comparable, with ode4 being the best performing method for smaller wave periods and ode3 for larger wave periods.

To give an idea of how a state-space model compares with the direct convolution integration model in terms of efficiency,
we perform simulations of both models—still with nonlinear terms set to zero—using the same fixed step solver (Runge-Kutta 4) with the same time step and implemented in the same computing package [22]. An order of 3 is used for the state-space model. The result is shown in Fig. 11. We see that the computation time for the state-space model varies linearly with the simulation length, while for the direct convolution integration model it varies as a quadratic function of the simulation length. A simple explanation to this is the fact that for a direct convolution integration model the number of computations to be performed at every time step corresponds to the simulation length or the number of time steps up to that time. This means that the number of computations to be performed at every time step grows as an arithmetic progression. The total computation time then is proportional to the sum of this series. On the other hand, for a state-space model the number of computations to be performed at every time step is constant so that the total computation time is just proportional to the number of time steps or the simulation length.

Now we consider cases where nonlinearities are introduced. We first present results from monochromatic wave excitations, and then from polychromatic wave excitations. For the monochromatic case, we consider a wave period $T$ of 8 s. The load resistance $R_u$ is set to $5 \times 10^4$ kg m$^2$ s$^{-1}$. The simulation length is 100 s. For the oscillating body, the nonlinear terms we have included in the model are the Coulomb and quadratic damping forces. Table I compares the error in velocity and power (according to (27)) from the direct convolution integration model with different time integration methods (ode2, ode3, and ode4), for different values of Coulomb damping, while the quadratic damping $R_q$ is set to zero. We see that the larger the Coulomb damping, the larger is the error for a given integration method and time step. This implies that the introduction of Coulomb damping carries with it a requirement for shorter time step to achieve the same degree of accuracy. Also, all the three integration methods exhibit similar order of accuracy, with ode4 generally being the most accurate, followed by ode3 and ode2. It should be noted however that for the same time step, ode4 requires a longer computation time than the other two methods. For instance, for the same step time of 0.01 s, we note that ode4 requires approximately 35 s to complete a 100-second simulation length, while ode3 requires approximately 25 s and ode2 requires only 17 s.

With state-space model, simulation using adaptive time step solvers can be done with ease. The performance of state-space and constant-coefficient models simulated with adaptive time step solver (see Section V-B above) for the same set of parameters is summarized in Table II. For the state-space models, increasing the model order improves the accuracy of the solutions although the lowest model order of 2 already gives very good accuracy. The low percentage of errors also confirms the excellent agreement between state-space and direct convolution integration models when Coulomb damping is introduced. Moreover, we note the following: (1) longer computation time is required for larger Coulomb damping to meet the same error tolerance, (2) increasing the model order improves the accuracy and has little effect on the computation time, and (3) overall, the required computation time is much less than that required by the direct convolution models to achieve similar degree of accuracy.

The constant-coefficient model, on the other hand, requires the least computation time—although still comparable to that of the state-space model—but has larger errors. And this is so even for monochromatic wave excitation considered here, as already pointed out in [4]. With larger Coulomb damping, however, the error reduces. This may be surprising at first, but it can be explained by the fact that as other forces begin to dominate, the radiation force becomes relatively less important so that a constant-coefficient model of the radiation force provides an acceptable approximation [24].

The effect of quadratic damping, as it turns out, is not as significant as that of Coulomb damping, as shown in Table III, where now $R_q = 0$. For the direct convolution integration models we use a time step of 0.1 s as this is comparable with the state-space and constant coefficient models in terms of computation time. Compared to the previous case of increasing Coulomb damping, increasing quadratic damping does not introduce significantly larger errors. Also, the errors for the
constant-coefficient model are shown to be small.

We shall now move on to polychromatic wave excitations. We consider two spectral peak periods \(T_p = 5\) and \(8\) s, and significant wave height \(H_s = 2\) m. The simulation length is \(100\) s. The load resistance \(R_u\) is set to \(5 \times 10^4\) kg m\(^2\) s\(^{-1}\). The result is shown in Table IV. Two combinations of \(R_C\) and \(R_q\) are considered. We compare the velocity and power errors calculated according to (27), as well as the errors in mean power, maximum power, and root mean square velocity. These are listed in sequence in the table. For the direct convolution integration model, a time step of 0.1 s is used for all the fixed step solvers (ode 2 to 4). From the table we see that among the fixed step solvers, ode4 is generally the most accurate for the same time step, followed by ode2 and ode3. It is worth noting that this is somewhat different from the monochromatic case, with ode2 generally being the least accurate. As in the monochromatic case, the state-space model proves to be superior both in terms of accuracy and efficiency. The performance of constant-coefficient model is significantly poorer than the other models. For cases with higher load resistance (not shown here), however, the constant-coefficient model has smaller errors for the reason given previously.

### B. Fixed oscillating water column

For the fixed OWC, the nonlinear terms we have included are the terms associated with air compressibility and relief valve characteristics. For brevity, we only present comparisons for polychromatic case, with \(T_p = 5\) and \(8\) s, and significant wave height \(H_s = 2\) m. We set \(R_u = 0.9\) kg m\(^{-4}\) s\(^{-1}\) and \(R_e = 1\) m\(^3\) kg\(^{-1}\). The result is shown in Table V. Two combinations of parameters \(p_{cl}^0\), \(p_{op}^0\), and \(V_0\) are considered, as shown in the table. We compare the pressure and power errors calculated from (27), and also the errors in mean power, maximum power, and root mean square pressure.

The fixed OWC model exhibits stiff dynamics arising mainly from the air compressibility relationship (6). Thus, for the direct convolution integration model, very small time step is necessary to achieve stability of the numerical solution. Even smaller time step is required for the second combination of \(p_{cl}^0\), \(p_{op}^0\), and \(V_0\) as the response exhibits Coulomb-type nonlinearity due to the operation of the relief valve. For this reason we do not investigate the effects of step size for the direct convolution integration model. It may be possible to use more sophisticated implicit fixed step solvers more suited for stiff problems, but this is beyond the scope of this study.

From the table we see that the state-space models give very good results. Nearly perfect accuracy is obtained by a model order of 4, whereas a model order of 2 already gives excellent accuracy. The constant-coefficient model is shown to have relatively good performance, probably because the radiation admittance has small values relative to \(1/R_u\) and \(R_e\). We may expect poorer accuracy for larger \(R_u\) and smaller \(R_e\).

### C. Floating oscillating water column

When the floating OWC is considered, four convolution terms are present (see (1) and (2)). Thus we expect the computational burden to be multiplied. For the state-space model, different orders are used for the various convolution terms. We consider two set of model orders, one with model orders of 4, 2, and 3 for the radiation, radiation admittance, and radiation coupling terms, respectively, and the other with model orders of 6, 4, and 5. We call the former SS\text{low} and the latter SS\text{high}. In addition, all four nonlinear terms are present, namely the Coulomb and quadratic damping terms, as well as the terms related to air compressibility and relief valve operation. As in the fixed OWC, we only consider the polychromatic case, with \(T_p = 5\) and \(8\) s, and significant wave height \(H_s = 2\) m. We set \(R_u = 0.9\) kg m\(^{-4}\) s\(^{-1}\), \(R_e = 1\) m\(^3\) kg\(^{-1}\), \(p_{cl}^0 = 40\) Pa, \(p_{op}^0 = 41\) Pa, \(V_0 = 500\) m\(^3\), \(R_C = 3 \times 10^5\) kg m\(^2\) s\(^{-1}\), and \(R_q = 8 \times 10^2\) kg m\(^2\). Thus, we only vary the spectral peak period \(T_p\). We compare the velocity, pressure, and power errors calculated from (27), and also the errors in mean power, maximum power, root mean square velocity, and root mean square pressure. Table VI summarizes the result.

As in the fixed OWC, the floating OWC model also exhibits stiff dynamics. Therefore we do not investigate the effects of step size for the direct convolution integration model. Similar to the fixed OWC case, the state-space models give very good results, with the higher-order model giving better accuracy than the lower-order one. The constant-coefficient model performs poorer than the state-space models, but the errors are seen to be acceptable. This is again dependent on the relative magnitude of the radiation terms compared to the other terms.

<table>
<thead>
<tr>
<th>Device</th>
<th>Velocity (power) error in % for the oscillating single-body device, (T = 8) s, (R_u = 5 \times 10^4) kg m(^2) s(^{-1})</th>
<th>(R_C) [kg m(^2) s(^{-1})]</th>
</tr>
</thead>
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<tr>
<td></td>
<td>(2 \times 10^4)</td>
<td>(8 \times 10^4)</td>
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</tbody>
</table>

| SS2    | 0.48 (0.47) | 0.34 (0.26) | 0.22 (0.18) |
| SS3    | 0.10 (0.12) | 0.20 (0.19) | 0.14 (0.13) |
| SS4    | 0.08 (0.11) | 0.19 (0.19) | 0.14 (0.13) |
| SS7    | 0.03 (0.04) | 0.03 (0.04) | 0.03 (0.04) |
| constant coeffs. | 19.04 (22.68) | 10.72 (11.80) | 6.52 (7.88) |

<table>
<thead>
<tr>
<th>Device</th>
<th>Velocity (power) error in % for the oscillating single-body device, (T = 8) s, (R_u = 5 \times 10^4) kg m(^2) s(^{-1})</th>
<th>(R_q) [kg m(^2)]</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>(2 \times 10^4)</td>
<td>(15 \times 10^4)</td>
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</table>

| ode2 \(\Delta t = 0.1\) s | 0.58 (0.57) | 0.58 (0.57) | 0.59 (0.58) |
| ode3 \(\Delta t = 0.1\) s | 0.11 (0.11) | 0.11 (0.11) | 0.11 (0.12) |
| ode4 \(\Delta t = 0.1\) s | 0.14 (0.17) | 0.15 (0.18) | 0.16 (0.20) |
| SS3    | 0.03 (0.03) | 0.03 (0.03) | 0.03 (0.03) |
| constant coeffs. | 0.27 (0.29) | 1.07 (1.34) | 2.31 (3.04) |
Different time-domain models according to their convolution approximations have been compared. The direct convolution integration model numerically integrates the convolution without any approximations. The simulation error is largely controlled by the simulation time step used. The state-space model approximates the convolution term by a set of coupled linear ordinary differential equations. The state-space model is shown to be highly accurate and efficient. A fixed time step of 0.1 s for the improved Euler’s (Heun’s) method is acceptable for most cases. Smaller time step is necessary when Coulomb-type nonlinearities are present or when the simulation error is large. The improved Euler’s (Heun’s) method offers significant saving in computation time. Even a model approximation of the desired outputs for cases when the number of extra state variables introduced. The constant-coefficient model replaces the frequency-dependent hydrodynamic parameters by constants assuming the values of the parameters at the incident wave spectral peak frequency. Three generic models of wave energy devices have been used to represent major features of WECSs. The fixed OWC and the oscillating single body have distinct features both in terms of hydrodynamics and the nonlinearities involved. The floating OWC can be seen as a combination of a fixed OWC and an oscillating body, coupled through additional hydrodynamic coupling parameters.

A direct convolution integration model is slow, but its accuracy is guaranteed for sufficiently small integration time step provided that the IRFs are accurate. When evaluating the IRFs from frequency-domain data, extrapolation to high frequencies and interpolation for finer frequency resolution are practical ways to ensure accuracy of the IRFs. Some fixed step solvers have been compared. The improved Euler’s (Heun’s) method can be a good choice for a balance between accuracy and efficiency. A fixed time step of 0.1 s for the improved Euler’s and Runge-Kutta 3 and 4 methods is acceptable for most cases with oscillating body devices. Smaller time step is necessary when Coulomb-type nonlinearities are present or when the model exhibits stiff dynamics due to fluid compressibility.

A state-space model is shown to be highly accurate and offer significant saving in computation time. Even a model order as low as 2 has good performance for most cases.

A constant-coefficient model is useful to give a quick approximation of the desired outputs for cases when the simulation error is large. The improved Euler’s (Heun’s) method offers significant saving in computation time. Even a model approximation of the desired outputs for cases when the number of extra state variables introduced. The constant-coefficient model replaces the frequency-dependent hydrodynamic parameters by constants assuming the values of the parameters at the incident wave spectral peak frequency. Three generic models of wave energy devices have been used to represent major features of WECSs. The fixed OWC and the oscillating single body have distinct features both in terms of hydrodynamics and the nonlinearities involved. The floating OWC can be seen as a combination of a fixed OWC and an oscillating body, coupled through additional hydrodynamic coupling parameters.

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radiation force is relatively smaller than the other forces in the system.

ACKNOWLEDGEMENTS

This study was carried out as part of the Statkraft Ocean Energy Research Program, sponsored by Statkraft (www.statkraft.no). This support is gratefully acknowledged.

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