Integration-based approach to evaluate the sub-threshold slope of MOSFETs

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1. Introduction

The study of the sub-threshold behavior of MOSFET transfer characteristics has been a topic of continuous interest for a long time [1–3]. This attention is presently heightened because of the growing number of applications that depend on the device's operation in the sub-threshold region. Under sub-threshold conditions, current conduction flowing in most MOSFETs is dominated by diffusive transport and controlled by the maximum barrier (minimum potential) along the channel. Such behavior of the drain current can be modeled by an exponential function of the gate voltage of the form [4]:

\[ I_D = I_{th} \exp \left( \frac{V_{GS}}{nV_{th}} \right), \]

where

\[ I_{th} = \frac{W}{L} I_a \exp \left( \frac{-V_M}{nV_{th}} \right) \left( 1 - \exp \left( \frac{-V_{DS}}{V_{th}} \right) \right), \]

where \( W \) is the channel width, \( L \) is the channel length, \( V_{th} = kT/q \) is the thermal voltage, \( I_a \) and \( V_M \) are constant parameters for current and voltage, respectively, and \( n \) is the sub-threshold ideality factor.

It is common to characterize sub-threshold conduction by means of the derivative of the logarithm of the drain current with respect to gate voltage, ordinarily referred to as the Transconductance-to-Current-Ratio (TCR) [5]:

\[ \frac{d \ln(I_D)}{dV_{GS}} = \frac{1}{I_D} \frac{dI_D}{dV_{GS}} = \frac{g_m}{I_D} \equiv \text{TCR} = \frac{1}{nV_{th}}, \]

where the value of this ideality factor is normally larger than unity [4].

The last equality in (3) refers to the sub-threshold region, where (1) holds true. It is also customary in the literature to speak of a sub-threshold slope factor, \( S \), sometimes known as Sub-threshold Swing (SS), expressed in units of millivolts of gate voltage change per decade of drain current change (mV/dec), and defined as:

\[ S = \ln(10) \left[ \frac{\ln(10)}{g_m} \right] = \frac{I_D \ln(10)}{B_n} = \frac{\ln(10)}{TCR}. \]

It is obvious from (3) and (4) that this sub-threshold slope factor, or swing, may be also understood as the inverse of TRC in the sub-threshold region. An alternative way to describe the transfer characteristics sub-threshold behavior is through the sub-threshold ideality factor, \( n \), using (3):

\[ n = \frac{1}{V_{th}} \frac{1}{\ln(10)} \frac{B_n}{I_D} = \frac{1}{V_{th}TCR}, \]

where the value of this ideality factor is normally larger than unity [4].

Being the sub-threshold slope factor, \( S \), an essential analysis tool to appraise MOSFETs’ sub-threshold performance and reliability [6–8], it is crucial to be able to extract its value from measured data in an as accurate and dependable way as possible. However, the traditional and obvious method of numerically calculating the derivative of the drain current with respect to gate voltage frequently falls short of these requirements. The measurement of the very small current values typically encountered in the sub-threshold region, especially in more modern devices, is often hampered by measurement error and noise, which is further amplified by the differentiation process inherent to the traditional method.

At first glance the TCR method seems to be computationally simple as it involves the numerical differentiation. However, the TCR method has a fundamental shortcoming. It frequently needs,
as any differentiation-based method, some kind of noise reduction pre-processing to lessen the effect of data noise. Fourier transform filtering, and multiple measurement averaging, are just two of the different noise reduction pre-processing steps commonly used.

To avoid such pre-processing steps and expedite the procedure we propose in this note the alternative of using simple integration-based methods to extract parameter $S$. The intention is to do two things at once: “smoothing” the data and extracting the parameter. The very mathematical nature of these methods directly contributes in diminishing the effect of measurement error and noise. The effectiveness of integration-based methods, such as those presented here, has been proven already in the extraction of other electron devices’ model parameters [9–11]. In what follows we will show how such simple methods may be advantageously applied for calculating the sub-threshold slope factor of MOSFETs.

2. Methods

The following auxiliary function can be readily obtained by numerical integration of the sub-threshold transfer data measured at a constant drain-to-source voltage:

$$H_1(V_{GS}, I_D) = \frac{V_{CS} I_D(V_{GS})dV_{CS}}{I_D(V_{GS})-I_D(V_{GS} = 0)},$$

(6)

where $V_{CS}$ is the running independent variable, and we have allowed, for simplicity’s sake, the mathematical license of not utilizing a dummy integration variable.

The type of auxiliary function represented by (6) was originally proposed in 1999 to extract the model parameters of PN junctions as an alternative to (6): assuming that we are applying (6) to sub-threshold current through transfer data, we substitute (1) into (6) and perform the integration to get:

$$H_1(V_{GS}, I_D) = \frac{n v_{th} I_D \left[ \exp \left( \frac{V_{CS}}{n v_{th}} \right) - 1 \right]}{I_D \left[ \exp \left( \frac{V_{GS}}{n v_{th}} \right) - 1 \right]}.$$  

(7)

Thus,

$$H_1(V_{GS}, I_D) = n v_{th} = \frac{1}{TCR} = \frac{S}{\ln(10)}.$$  

(8)

Therefore, $n$, $TCR$ or $S$ may be quickly calculated, without resorting to the noise-exacerbating differentiation, by the use of auxiliary function $H_1$.

Although Eq. (6) is based on numerical integration of the sub-threshold transfer data, which acts as a low-pass filter, it still contains the possibly noisy raw current data in the denominator. If additional reduction of the effect of data noise were needed, the idea suggested by (6) may be taken one step further. Another function, $H_2$, based on successive integration, can be defined to be used as an alternative to (6):

$$H_2(V_{CS}, I_D) = \frac{n v_{th} I_D \left[n v_{th} \exp \left( \frac{V_{CS}}{n v_{th}} \right) - 1 \right]}{n v_{th} I_D \left[ \exp \left( \frac{V_{CS}}{n v_{th}} \right) - 1 \right] - I_D V_{CS}}.$$  

(10)

Thus,

$$H_2(V_{CS}, I_D) = n v_{th} = \frac{1}{TCR} = \frac{S}{\ln(10)}.$$  

(11)

Therefore, either function $H_1$ or $H_2$ may be used to characterize conduction in the sub-threshold region as alternatives to traditional sub-threshold slope factor extraction processes based on differentiation.

3. Application examples

To illustrate the applicability of functions $H_1$ and $H_2$ they are used to characterize the sub-threshold slope factor of two experimental planar MOSFETs fabricated by two different technologies, with 2 and 4 nm thick gate oxides and 90 and 185 nm mask channel lengths, respectively. The traditional derivative method is also applied for comparison purposes. Fig. 1 shows the transfer characteristics of the shorter channel length device, measured with a gate voltage step of 10 mV at a fixed $V_{DS} = 10$ mV. These data are numerically differentiated to calculate $TCR$, and also numerically integrated, once to calculate function $H_1$, and twice to calculate function $H_2$. The results of performing the operations indicated in (3), (6), and (9) are presented in Fig. 2. Fig. 3 shows the corresponding sub-threshold slope factor, $S$, calculated using the traditional Transconductance-to-Current Ratio ($TCR$) method and the proposed functions $H_1$ and $H_2$.

Fig. 4 shows the transfer characteristics of the longer channel length device, measured with a gate voltage step of 10 mV at a high fixed drain bias of $V_{DS} = 2$ V, appropriate for this technology. Fig. 5 presents the results of performing the operations indicated in (3), (6), and (9). The corresponding sub-threshold slope factor, $S$, calculated using the $TCR$ method and the proposed $H_1$ and $H_2$ methods is presented in Fig. 6.

Figs. 2 and 5 illustrate the considerable noise reduction ability of the proposed functions in contrast to the traditional $TCR$ method based on the derivative of the current. The sub-threshold slope factors calculated by the three methods are equivalent in the sub-threshold region where the dependence of the drain current on gate voltage is supposed to be exponential, as Figs. 3 and 6 indicate. However, it is obvious the advantage of using $H_1$ or $H_2$ compared to $TCR$ regarding noise reduction. Of course, the equivalence holds true only in the sub-threshold region, because as gate voltage in-
creases approaching and surpassing the threshold voltage, the drain current deviates from being exponentially dependent on gate bias, and functions $H_1$ and $H_2$ are no longer equivalent to each other or to the TCR.

4. Conclusions

We have proposed the use of simple integration-based tools to examine the sub-threshold slope factor behavior of MOSFETs, as an alternative means to the traditional differentiation-based TCR method. Although numerical differentiation and integration operations are basically comparable regarding computational intensity, the proposed integration-based methods have the advantage over the TCR method that they concurrently lessen the effect of data noise, whereas the TCR method worsens the effect of data noise, and therefore requires a noise reduction pre-processing step. Two auxiliary functions have been presented for this purpose. The effectiveness of these simple integration-based functions was illustrated and compared to the traditional Transconductance-to-Current Ratio method. Both functions were applied to the measured transfer characteristics of two experimental MOSFETs of different planar technology, gate oxide thickness, and mask channel length. The extraction methods were applied to the transfer characteristics measured at both low and high drain biases. The results clearly demonstrate the advantage of the proposed methods over the traditional TCR method extraction procedure regarding inherent measurement noise reduction.
References