

# ON THE ATMOSPHERIC DISPERSION AND GAUSSIAN PLUME MODEL

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*Abstract:* - As an air pollutant is transported from a source to a potential receptor the pollutant disperses into the surrounding air so that it arrives at a much lower concentration than it was on leaving the source. Strict environmental regulations worldwide resulted in an ever growing concern about the validity and reliability of air quality dispersion models. The present work is a try to evaluate the applicability of dispersion models from an industrial source. Two examples of the air quality dispersion models are considered here; the classical Gaussian plume model by Sutton and PRISE (Plume Rise) model by Henderson-Sellers and Allen.

*Key Words:* - Dispersion – Gaussian model – Air pollution – Atmospheric pollution

## 1 Introduction

The concentration of an air pollutant at a given place is a function of a number of variables, including the emission rate, the distance of the receptor from the source, and the atmospheric conditions. The most important atmospheric conditions are wind speed, wind direction, and the vertical temperature structure of the local atmosphere. If the temperature decreases with height at a rate higher than the adiabatic lapse rate, the atmosphere is in unstable equilibrium and vertical motions are enhanced. This is to keep pollution concentrations moderate or weak at ground level. But, if the temperature decreases with height at a rate lower than the adiabatic lapse rate (stable atmosphere) or increases with height (inversion), vertical motions are reduced or damped. This will lead to potentially high pollution concentrations.

Atmospheric air quality dispersion models are usually used to estimate just how much reduction has occurred during the transport of pollutant from an industrial source, and consequently to project the pollution concentration at ground level. Dispersion models usually incorporate meteorological, terrain, physical and chemical characteristics of the effluent and source design to simulate the formation and transport of pollutant plumes. Strict environmental regulations worldwide is behind the growing concern about the validity and reliability of air quality dispersion models (e.g; [1] – [10]).

The objective of the present study was to evaluate the applicability of dispersion models from an industrial source. Two examples of the air quality dispersion models are considered here; the classical

Gaussian plume model by Sutton [11] and PRISE (Plume Rise) model by Henderson-Sellers and Allen [12].

## 2 Dispersion Models

### 2.1 Gaussian Plume Model

Gaussian plume model uses a realistic description of dispersion, where it represents an analytical solution to the diffusion equation for idealized circumstances. The model assumes that the atmospheric turbulence is both stationary and homogeneous. In reality, none of these conditions is fully satisfied. However, Gaussian plume model has been successfully used for rural configurations. Extensive validation has been done on tracer experiments conducted in Kincaid and Prairie grass ([13] and [14]). Gaussian model has also been tested against tracer experiments in urban surroundings (e.g Indianapolis experiment, in [15]). The model is still the method of choice for many (e.g. [16] and [17]), especially for the prediction of yearly averaged concentration. It is the most widely used plume model and is the basis for most of the computer models distributed by the EPA.

In the Gaussian plume dispersion model the concentration of pollution downwind from a source is treated as spreading outward from the centerline of the plume following a normal statistical distribution. The plume spreads in both the horizontal and vertical directions.

In the model, determining the pollutant concentrations at ground-level beneath an elevated plume involves two main steps; first, the height to which the plume rises at a given downwind distance

from the plume source is calculated. The calculated plume rise is added to the height of the plume's source point to obtain the so-called "effective stack height". Second, the ground-level pollutant concentration beneath the plume at the given downwind distance is predicted using the Gaussian dispersion equation.

The Gaussian dispersion equation can be written as:

$$C(x, y, z) = \frac{Q}{2\pi\sigma_y\sigma_z u} \exp\left(-\frac{y^2}{2\sigma_y^2}\right) \times \left\{ \exp\left(-\frac{(z-H)^2}{2\sigma_z^2}\right) + \exp\left(-\frac{(z+H)^2}{2\sigma_z^2}\right) \right\} \quad (1)$$

which was developed by [11], where C is the concentration, Q is the emission rate of the pollutant from the source, u is the wind speed which defines the direction x. y is the horizontal distance perpendicular to the wind direction, z is the vertical direction, H is the effective height of the plume (considering the additional height Δh to which the hot gases rise above the physical height of the source h); i.e., H = h + Δh, and σ<sub>y</sub> & σ<sub>z</sub> are the parameters of the normal distributions in y and z directions, usually called the dispersion coefficients in y and z directions respectively. A definition sketch of the plume dispersion is shown in figure (1).

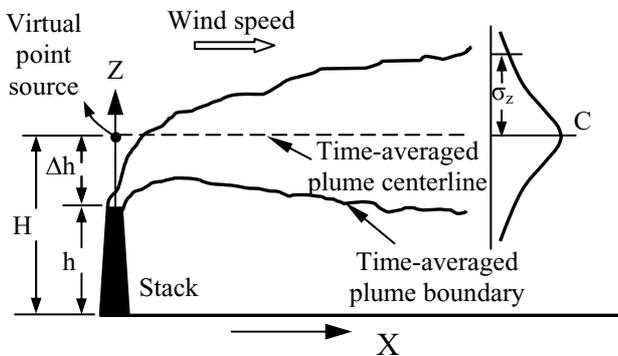


Fig. 1 Plume dispersion: definition sketch

In this equation, the ground is usually assumed to be a perfect reflector and its presence is represented by a mirror image source placed below ground. For a receptor at the ground surface, or a source located at the ground (z=0), the previous equation reduces to:

$$C(x, y, 0) = \frac{Q}{\pi\sigma_y\sigma_z u} \exp\left(-\frac{y^2}{2\sigma_y^2} - \frac{H^2}{2\sigma_z^2}\right) \quad (2)$$

In analyzing the Gaussian plume model, the following assumptions are usually made:

1) Continuous emission and negligible diffusion in the direction of travel.

- 2) The material diffused is a stable gas or aerosol, with a negligible deposition rate.
- 3) Mass is conserved through reflection at surfaces.
- 4) Background pollution is negligible.
- 5) Steady-state conditions.
- 6) Constant wind speed and direction with time and elevation.
- 7) Negligible wind shear effect on horizontal diffusion.
- 8) The dispersion parameters are assumed to be functions of x (and hence u alone).
- 9) The terrain is relatively flat, open country.

Plume rise Δh plays an important role in determining ground-level concentrations for real sources. The plume rise schemes of Briggs [18] are recommended by EPA, and they are the commonly used schemes. These schemes express the final rise height of the buoyant release as a function of, among other parameters, the buoyancy flux, the mean wind speed at the stack top, and the friction velocity.

Gaussian plume models are applicable for downwind distance, x > 100 m, because near the source concentration approaches infinity [19]. Accordingly, many researchers imposed a lower limit on σ<sub>y</sub>(x) and σ<sub>z</sub>(x), or an upper limit on the near-source concentration.

The dispersion coefficients, σ, define the spread of the plume. As with the normal distribution, 67% of the pollutant is assumed to be within ±σ of the centerline of the plume. Thus a plume may be described as being approximately four to six σ wide. The value of σ is determined by the magnitude of the turbulence in the atmosphere. The larger eddies, and larger values of σ, will be observed during periods when the atmosphere is unstable. The smaller eddies, and smaller values of σ, will be observed when the atmosphere is stable.

Measurements of σ have been made under a variety of atmospheric conditions. The measurements of σ used in virtually all the models are those published by Turner [20] (called the "Pasquill-Gifford coefficients") from data taken in open, rural surroundings. Because of their origin they are appropriate for dispersion estimates in rural settings but less so for urban areas. The greater surface roughness and greater release of heat at the surface means that atmospheric conditions in urban areas are seldom as stable as in rural areas.

The measurements of the Pasquill-Gifford coefficients were made over periods of 10 to 20 minutes and are strictly applicable only to such short time periods. In order to calculate long-term (e.g., annual) average concentrations, it is necessary to take into account the wind speed, direction, and atmospheric stability over the entire period.

The physical description of the Gaussian plume model is based on the traditional discrete stability categories (Pasquill-Turner stability classes). The atmosphere is generally described as being in six stability classes, labeled A through F. Classes A through C are unstable conditions, class D is neutral, and classes E and F are stable. The most frequently observed classes are C, D, and E. Application and analysis of the Gaussian plume model are given in the following sections.

### 2.1.1 Downwind Ground-level Concentration

The ground-level concentrations directly downwind are of interest, since pollution concentration will be highest along that axis. With  $y = 0$ , equation (2) will be simplified to the following downwind ground-level form:

$$C(x,0,0) = \frac{Q}{\pi\sigma_y\sigma_z u} \exp\left(-\frac{H^2}{2\sigma_z^2}\right) \quad (3)$$

Using equation (3), the effect of variations in the key parameters (atmospheric stability, wind speed, ambient temperature, stack height, gas exit velocity, and gas exit temperature) on the ground concentrations are calculated and shown as a composite plot in figure (2). The base case used in the calculations is given in the figure. Figure (2a) shows that the unstable atmosphere produces the highest peak downwind concentration. The turbulence in the unstable atmosphere brings the plume to the ground very quickly, resulting in high peak values near the stack. Farther downwind, however, concentrations drop off very quickly. The stable atmosphere, on the other hand, has a much lower peak. However, beyond a considerable distance downwind, the concentration is higher than that for the unstable atmosphere and continues to be appreciable in the downwind direction. The plume is also quite sensitive to changes in stack height, as can be seen from figure (2d). Lowering the stack from 30 to 10 m causes the peak to be more than double in concentration. Figures (2b, 2c, 2e, and 2f) demonstrate the impact of varying the wind speed, ambient temperature, gas exit velocity, and gas exit temperature on downwind ground level concentrations. The four parameters affect the plume rise, and the effective stack height  $H$ .

A 3D surface plot for figure (2d) along with a contour map for the ground-level concentration are constructed using equation (2) and given in figure (3) to better visualize the distribution of the ground-level concentration in the  $x$ - $y$  plane, and the effect of the stack height on that distribution.

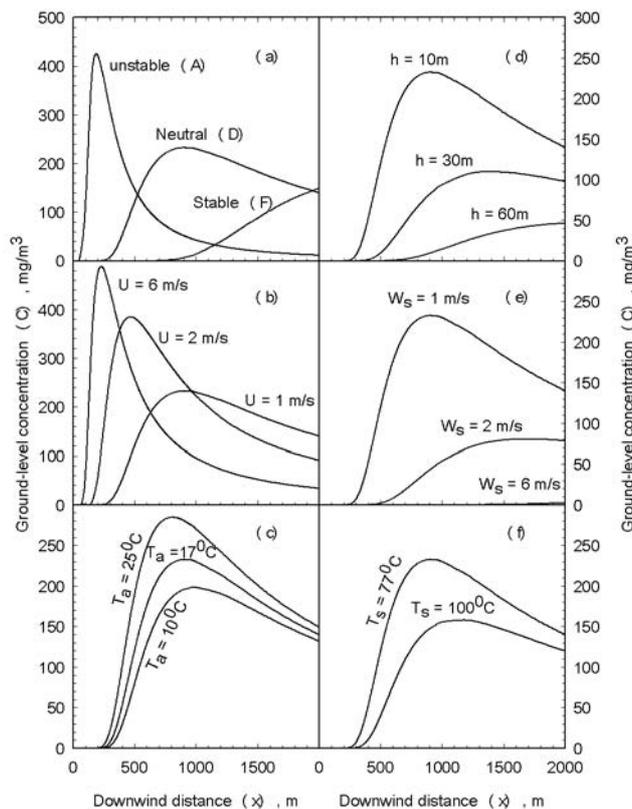
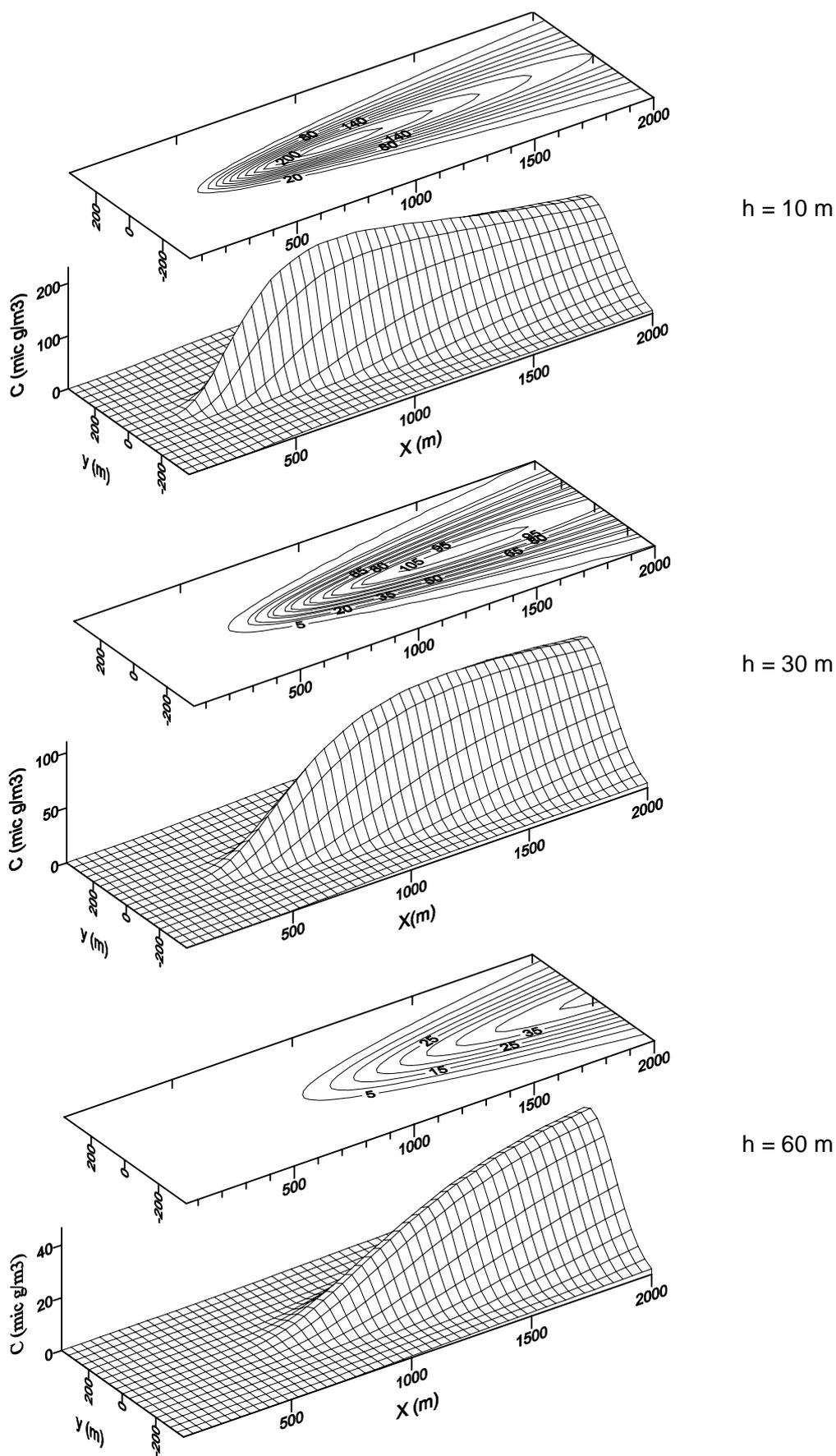


Fig. 2 Effect of key parameters on the downwind ground-level concentrations  
 $h=10$  m,  $W_s = 1$  m/s,  $U = 1$  m/s,  $Q = 5.787$  g/s,  $T_s = 77^\circ\text{C}$ ,  $T_a = 17^\circ\text{C}$ , Stability class: neutral

It is to be seen from the figures that the plume center-line concentration drops off while the ground-level concentration goes higher. This continues until both concentrations asymptote to the same value; an indication to the uniform distribution of the pollutant concentration with height ( $z$ ). It is seen that the concentration distribution of the unstable atmosphere becomes uniform much closer to the source than that of the stable atmosphere. This is due to the much more atmospheric turbulence in the unstable atmosphere.

### 2.1.2 Sensitivity Study

Some idea of the sensitivity of the Gaussian plume model to the model parameters can be gained from figure (6). Based on a control run (gas exit velocity = 1 m/s, gas exit temperature = 300 K, stack exit diameter = 2.5 m, stack height = 50 m, wind speed = 1 m/s, and ambient temperature = 281 K) each parameter was changed over a reasonable range. The resulting graphs of the plume rise ( $\Delta h$ ) against each parameter in turn gives an indication of the sensitivity of the model to one particular parameter.



**Fig. 3** 3D-surface plot and contour map of the downwind Ground-level concentrations

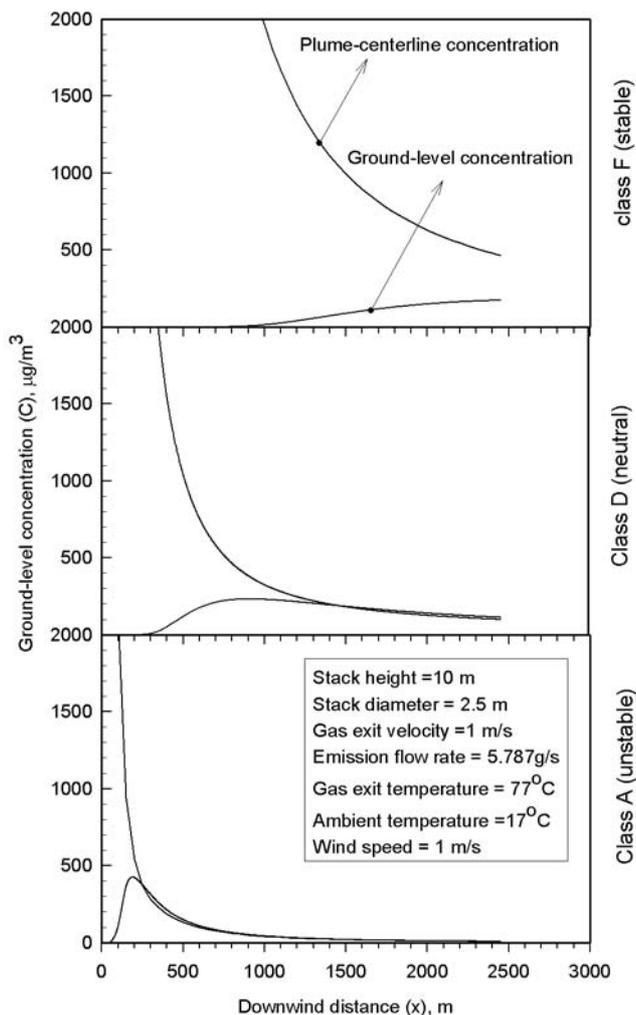


Fig. 4 Effect of atmospheric stability on plume-centerline and ground-level concentrations

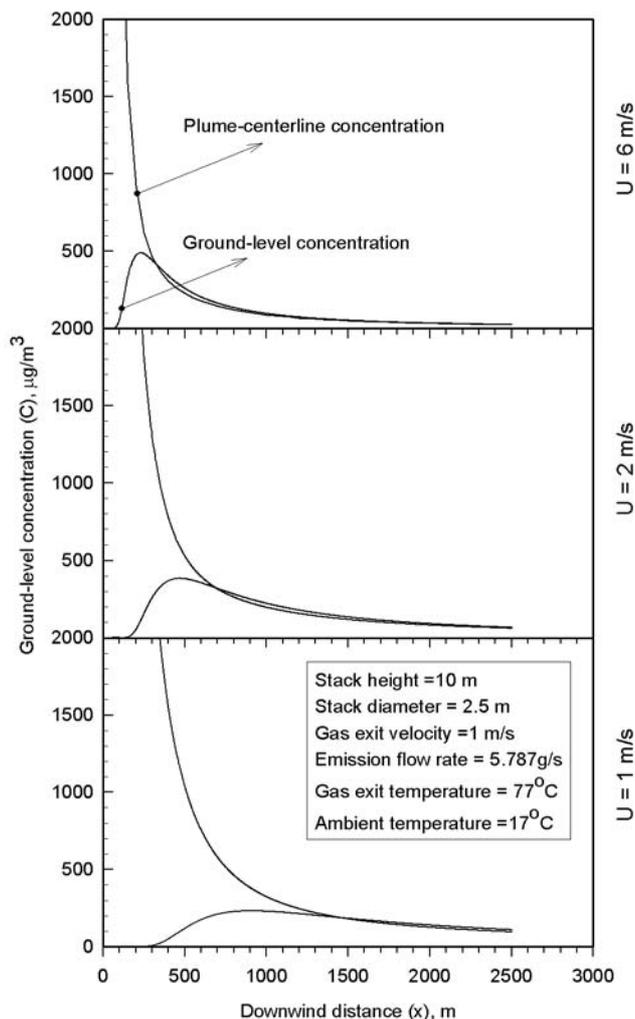


Fig. 5 Effect of wind speed on plume-centerline and ground-level concentrations

The effects of meteorological factors may be seen in figures (6a, 6b, and 6c). Figure (6a) shows that the atmospheric stability affects the plume rise ( $\Delta h$ ); the stable atmosphere is seen to produce the biggest plume rise. Figure (6b) demonstrates that the substantial plume rise for a wind speed of 1 m/s ( $\Delta h = 15$  m/s) is virtually eliminated for wind speeds higher than 10 m/s., while figure (6c) reflects the importance of ambient temperature on the plume rise. The ambient temperature level is less important in some regions compared to others. In Canada, for example, the ambient temperature may be lower than  $-20$  °C. Figures (6d and 6e) show the effect of changes in the two effluent characteristics: velocity ( $W_s$ ) and temperature ( $T_s$ ). Changes in  $W_s$  from 1 to 20 m/s show changes of over 120 m in plume rise as can be seen from figure (6d), while figure (6e) shows that a range of 180 K in effluent temperature gives a plume rise change of over 55 m.

Another sensitivity study was performed by assuming reasonable degrees of error (10%) in some of the key parameters used in the Gaussian model, and determining the propagated end-result effects of those errors on the calculated ground-level pollutant concentrations. The key parameters considered are atmospheric stability,  $u$ ,  $T_a$ ,  $H$ ,  $W_s$ ,  $T_s$ , and emission rate. The results of the sensitivity study (as a percentage variation in ground-level concentrations versus downwind distance) show that over or under predictions in the ground-level concentration occurred due to a change of 10% of the key parameter; with a maximum of 100% propagated under prediction. Thus, propagation of seemingly small errors in the Gaussian model parameters can cause very large variations in the model's predictions. These results were used to construct figure (7), which shows the percentage of variations in the predicted ground-level concentrations at a point close to the source (400 m) and at another point, but, farther downwind (5000 m). Close to the source, it is seen that stability class and stack height have the highest impact on concentration predictions. At further

distances downwind, concentrations are mainly influenced by stability class, wind speed, and source emission rate. These results confirm that the model is sensitive to the atmospheric parameters; mainly atmospheric stability and wind speed, as well as effective stack height and emission rate.

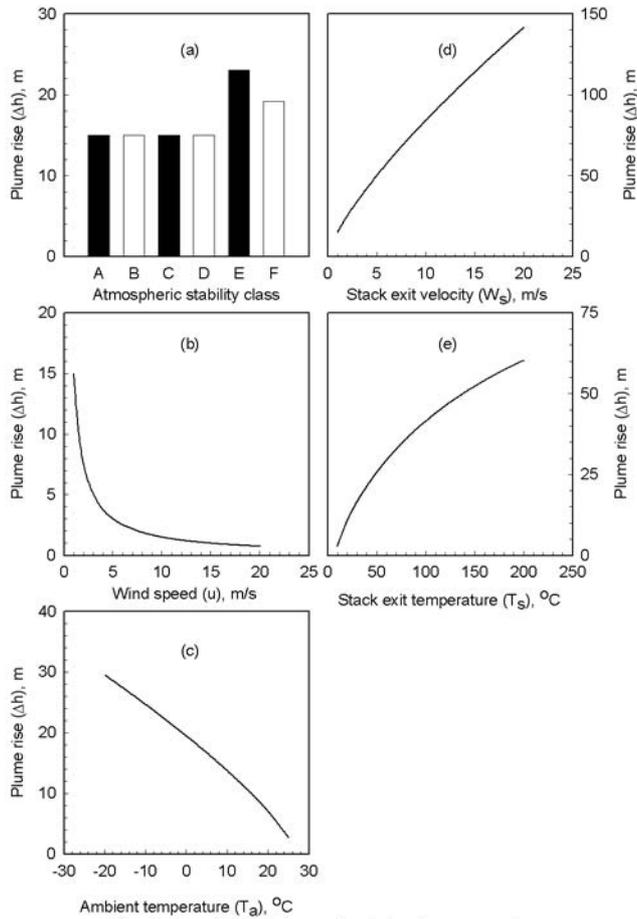


Fig. 6 Sensitivity of plume rise to the key parameters

(h=50 m, W<sub>s</sub> = 1m/s, U = 1 m/s, Q = 5.787g/s, T<sub>s</sub> = 27°C, T<sub>a</sub> = 8°C, Stability class: neutral)

## 2.2 Plume Rise and Dispersion Model

The plume rise and dispersion model (PRISE), see Henderson-Sellers and Allen [12], calculates all of the phases (rising, bending over the (quasi-) equilibrium dispersion) of the behavior of the plume emitted by a stack in atmosphere in one continuous formulation, taking fully into account the ambient meteorological (hydraulic) conditions. The atmosphere is parameterized as consisting of two layers: a neutral layer with a lapse rate equal to the dry adiabatic and an overlying stable layer (extending to infinity). Three wind speed profiles are available: constant with height, logarithmic and logarithmic based on a user-input roughness length. The model is of the integral type, employing a curvilinear coordinate system as shown in figure (8). The model is derived by integrating the conservation equations of mass, momentum, density deficiency, and energy over the cross-

sectional profile of the plume. For a round (point source) plume, the model integral equations are given as:

$$\frac{d}{ds} \int_0^\infty u_s \, ndn = bv_e \quad (4)$$

$$\frac{d}{ds} \int_0^\infty u_s^2 \, ndn = g \int_0^\infty \frac{(\rho_a - \rho)}{\rho_o} \sin \phi \, ndn \quad (5)$$

$$g \int_0^\infty \frac{(\rho_a - \rho)}{\rho_o} \cos \phi \, ndn = \text{constant} \quad (6)$$

$$\frac{d}{ds} \int_0^\infty u_s (\rho_a - \rho) \, ndn = -\frac{d\rho_a}{dz} \int_0^\infty u_s \sin \phi \, ndn \quad (7)$$

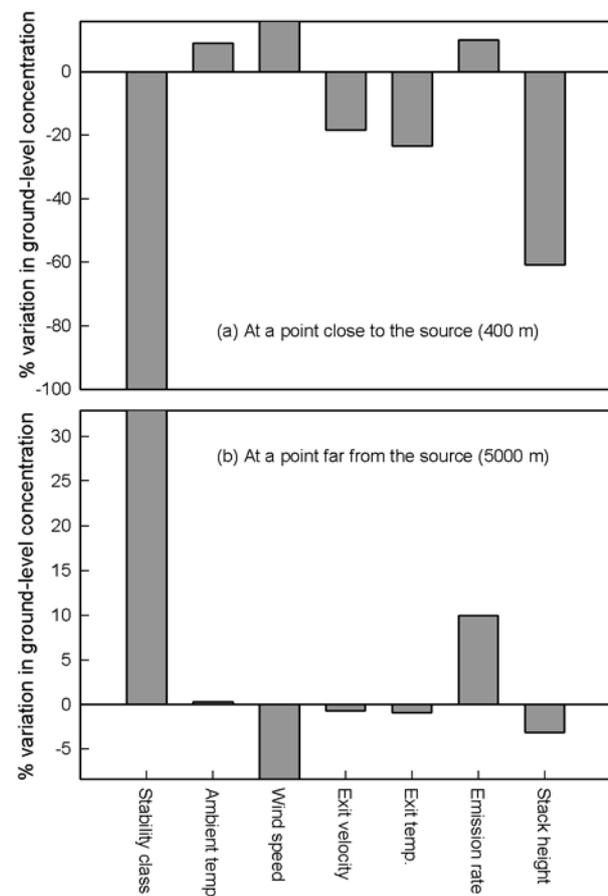


Fig. 7 Expected variations in ground-level concentrations due to a change of 10% in the model parameters

h=50 m, W<sub>s</sub> = 1m/s, U = 1 m/s, Q = 5.787g/s, T<sub>s</sub> = 27 °C, T<sub>a</sub> = 8°C, Stability: neutral

In these equations, the coordinates are (s, n), u<sub>s</sub> is the axial velocity, v<sub>e</sub> is the entrainment velocity, φ is the angle to the horizontal, b is the radial length scale, and ρ<sub>o</sub> is the density at s = 0.0. Using ideal gas law and Boussinesq approximation, (ρ<sub>a</sub> - ρ)/ρ<sub>o</sub> can be converted in terms of temperature as (T - T<sub>a</sub>)/T<sub>a</sub>. The assumptions employed in the PRISE model are:

- 1) All quantities are time-averaged over a period of several minutes.

- 2) The plume is assumed in a steady state.
- 3) Aerodynamic effects (downwash, zone of flow establishment, and pressure gradient terms) are neglected.
- 4) Temperature gradient exists in the vertical directional only.

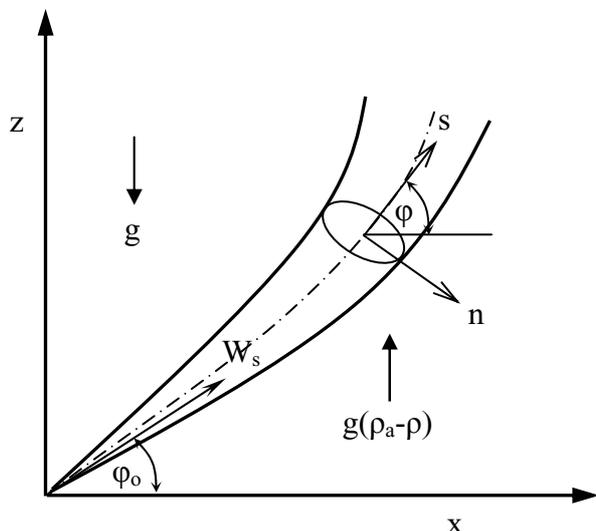


Fig. 8 Forced (buoyant) plume: definition sketch and curvilinear coordinate system

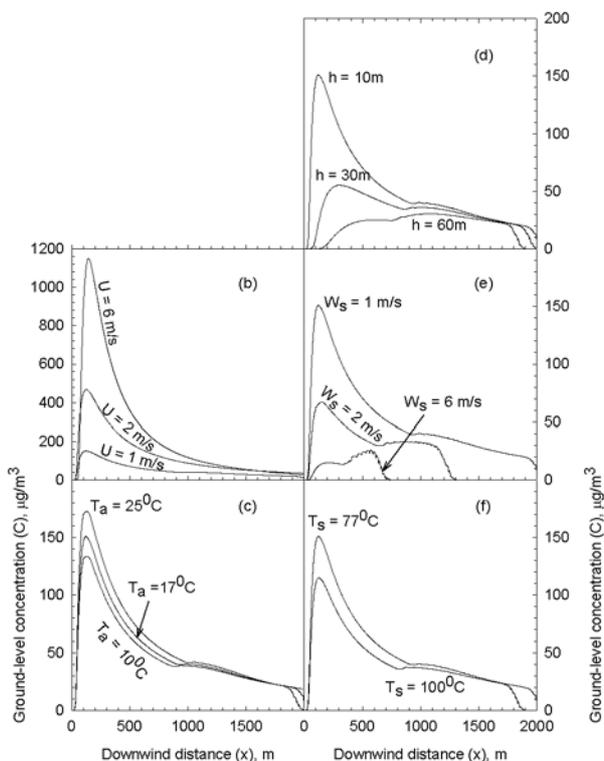


Fig. 9 Effect of key parameters on the ground-level concentrations (PRISE)

$h=10\text{ m}, W_s = 1\text{ m/s}, U = 1\text{ m/s}, Q = 5.787\text{ g/s}, T_s = 77^\circ\text{C}, T_a = 17^\circ\text{C}$ , Stability class: neutral

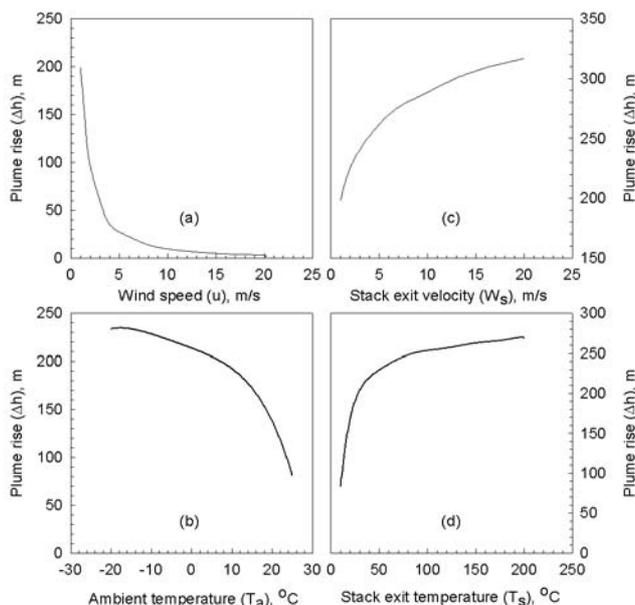


Fig. 10 Sensitivity of plume rise to the key parameters (PRISE)

$h=50\text{ m}, W_s = 1\text{ m/s}, U = 1\text{ m/s}, Q = 5.787\text{ g/s}, T_s = 27^\circ\text{C}, T_a = 8^\circ\text{C}$ , Stability class: neutral

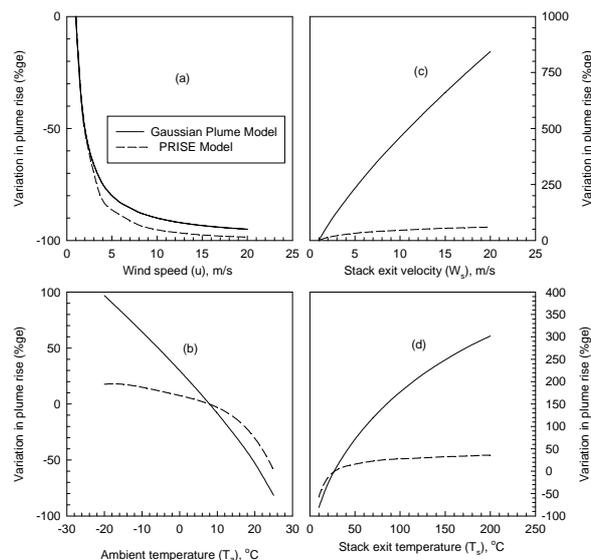


Fig. 11 Sensitivity of plume rise to the key parameters

$h=50\text{ m}, W_s = 1\text{ m/s}, U = 1\text{ m/s}, Q = 5.787\text{ g/s}, T_s = 27^\circ\text{C}, T_a = 8^\circ\text{C}$ , Stability class: neutral

The trajectory equations are found from:

$$\frac{dx}{ds} = \cos\phi$$

$$\frac{dz}{ds} = \sin\phi \tag{8}$$

The model equations are usually solved using a finite difference technique with a grid length defined by an incremental distance  $\Delta s$ . In the present work, the algorithm of Henderson-Sellers and Allen was adapted and implemented in a computer code. More

details about the model may be found in Henderson-Sellers and Allen.

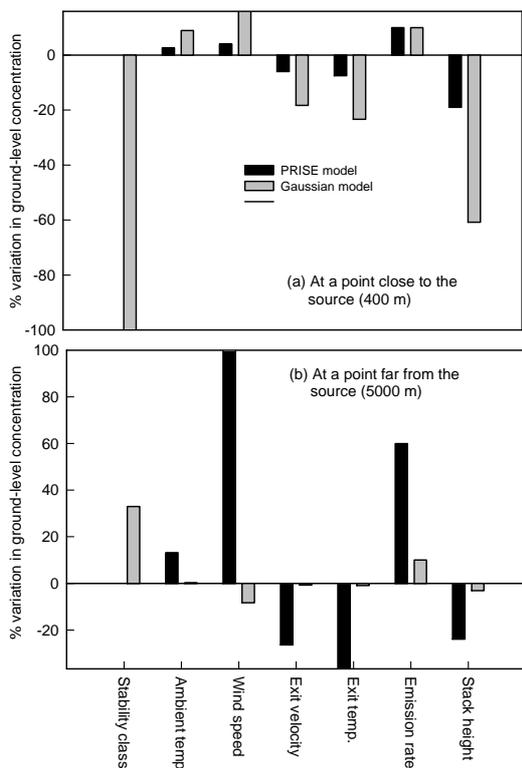


Fig. 12 Expected variations in ground-level concentrations due to a change of 10% in the model parameters

$h=50$  m,  $W_s=1$  m/s,  $U=1$  m/s,  $Q=5.787$  g/s,  $T_s=27$  °C,  $T_a=8$  °C, Stability class: ne

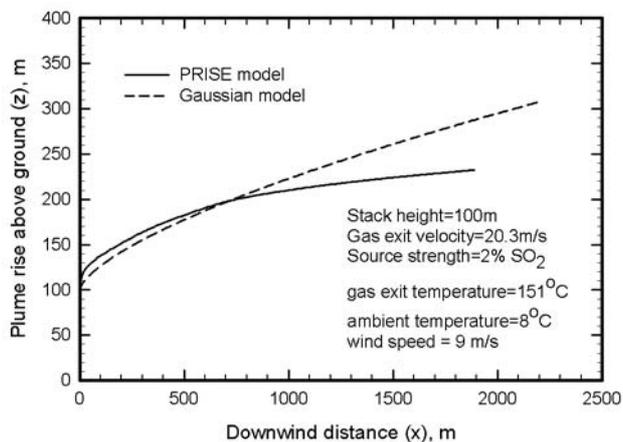


Fig. 13 Comparison of plume trajectory for Gaussian and PRISE models

### 3. Conclusion

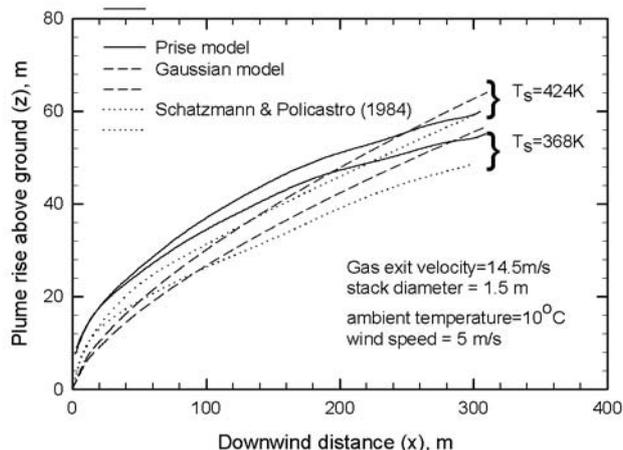


Fig. 14 Plume trajectories for Gaussian and PRISE models, compared with data of Schatzmann & Policastro (1984)

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