A Decision Directed Square-Root Free Inverse QR-Decomposition based Groupwise Recursive Channel Estimator for SFBC-OFDM Systems

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Abstract — In this paper, a decision directed square-root free inverse QR-decomposition based groupwise recursive (SIQR-GR) channel estimation scheme is proposed for space-frequency block coded orthogonal frequency division multiplexing (SFBC-OFDM) systems. The proposed SIQR-GR channel estimation algorithm employs sequences of scaled Givens rotations to recursively processes data decisions corresponding to the SFBC groups within each OFDM block. As a result of the groupwise recursive processing, the proposed SIQR-GR scheme yields improved system performance under doubly-selective (i.e., frequency-selective and fast fading) channels. To evaluate the performance advantages of SIQR-GR, we compare the normalized mean square error (NMSE) and symbol error rate (SER) performances of SIQR-GR to those corresponding to previously proposed channel estimation schemes. Furthermore, we also present computational complexity comparisons between SIQR-GR and the previously proposed schemes. These comparisons show an excellent performance-complexity tradeoff achieved by SIQR-GR over the previously proposed solutions.

I. INTRODUCTION

Transmit diversity is one of the key technologies that is expected to play a major role in improving transmission reliability with high spectral efficiency in future generation broadband wireless networks. Two transmit diversity techniques proposed for combating the detrimental effects of multipath fading are space-frequency block coded orthogonal frequency division multiplexing (SFBC-OFDM) [1] and space-time block coded orthogonal frequency division multiplexing (STBC-OFDM) [2]. In STBC-OFDM systems, space-time block coded data symbols are transmitted over multiple adjacent OFDM blocks [2]. Alternatively, in SFBC-OFDM, space-frequency block coded (SFBC) data symbols are applied over groups of adjacent sub-carriers (referred to as SFBC groups, henceforth) within a single OFDM block [1]. Since space-frequency encoded data symbols are transmitted across a single OFDM block in SFBC-OFDM systems, these systems are shown in [1] to outperform STBC-OFDM under fast fading channels.

In practice, to perform coherent detection and to achieve enhanced transmission reliability, SFBC-OFDM systems require accurate channel state information (CSI) at the receiver. Channel estimation methods based on the least squares (LS) [3] and the minimum mean square error (MMSE) [4] criteria are popular means of acquiring CSI at the receiver. When applied to SFBC-OFDM systems in the decision directed mode, the LS and MMSE channel estimators first use outdated CSI estimates to decode the data symbols transmitted through a given OFDM block $n$. The decoded data symbols are then used to attain the CSI estimate corresponding to the $n^{th}$ OFDM block. Since the CSI estimates are updated after decoding the data symbols from each OFDM block, we will henceforth refer to the LS and MMSE channel estimators as block-based LS (BLS) and block-based MMSE (BMMSE) schemes, respectively. It should be noted that when BLS and BMMSE are employed in fast fading environments, the mismatch between the outdated CSI estimate (i.e., the one that is used to decode the data symbols from OFDM block $n$) and the true CSI corresponding to the $n^{th}$ OFDM block can be significant [3]. As a result, the application of the BLS and BMMSE schemes under fast fading channels would translate into poor system performance and reduced transmission reliability. Additionally, the computational complexities associated with BLS and BMMSE are also high since both BLS and BMMSE require the implementation of matrix inversions.

Another class of channel estimators that has received recent attention is based on the expectation-maximization (EM) algorithm. In [5], Wautelet et al. propose various EM based channel estimation algorithms for multiple-input multiple-output wireless channels. Performance comparisons of their proposed channel estimation algorithms are also provided in [5] for both frequency-flat and frequency-selective channels. However, the algorithms and the comparisons presented in [5] assume the channel to be slow-fading. In [6], an iterative EM-based maximum a posteriori (EM-MAP) channel estimation scheme employing the truncated Karhunen-Loeve expansion is proposed for SFBC-OFDM systems in doubly-selective (i.e., frequency-selective and fast fading) channels. In each iteration of the EM-MAP scheme, the a posteriori probabilities of all the data symbols transmitted through a given OFDM block are first computed using the CSI estimate from the previous iteration. Next, the computed a posteriori probabilities are used to refine the CSI estimate. The EM-MAP scheme iterates between these two steps until convergence is achieved. Following convergence, the CSI estimate from the last iteration is used to perform coherent detection. To reduce channel estimation complexity, the EM-MAP scheme employs a quasi-
static assumption in the frequency domain. This quasi-static assumption requires the channel frequency response to be static over the number of adjacent sub-carriers spanning one SFBC group. However, the above quasi-static assumption is generally not accurate for SFBC-OFDM systems employing SFBCs with large block lengths and/or operating under highly frequency-selective fading channels. As a result, the EM-MAP scheme suffers notable performance losses due to the inaccuracies associated with the quasi-static assumption.

In this paper, we propose a low complexity decision directed channel estimation scheme for SFBC-OFDM systems with improved performance under doubly-selective fading channels. Firstly, we propose a square-root free inverse QR-decomposition based groupwise recursive (SIQR-GR) channel estimation algorithm employing sequences of scaled Givens rotations. The proposed SIQR-GR scheme recursively processes data decisions corresponding to the SFBC groups within each OFDM block to improve system performance under doubly-selective fading channels. Furthermore, the avoidance of square-roots and the utilization of scaled Givens rotations ensure that the proposed SIQR-GR scheme is computationally efficient. We then compute the normalized mean square error (NMSE) and symbol error rate (SER) performances of SIQR-GR to those corresponding to previously proposed channel estimation schemes. Additionally, complexity comparisons are also drawn between SIQR-GR and the previously proposed schemes. These comparisons show an excellent performance-complexity tradeoff achieved by SIQR-GR over the previous solutions.

The rest of the paper is organized as follows. Section II presents the SFBC-OFDM system model. Next, in Section III, we propose the decision directed SIQR-GR channel estimator. This is followed in Section IV by performance and computational complexity comparisons between SIQR-GR and previously proposed schemes. Finally, Section V concludes the paper.

**Notations:** The transpose and Hermitian transpose are represented by \( (\cdot)^T \) and \( (\cdot)^H \), respectively. A diagonal matrix with entries \( \{y_1, y_2, \ldots, y_N\} \) is given by \( \text{diag}\{y_1, y_2, \ldots, y_N\} \), and the \( i \)th column of any matrix \( A \) is denoted as \( [A]_i \). All-zero and identity matrices of arbitrary dimensions are represented by \( 0 \) and \( I \), respectively. Lastly, \( E[\bullet] \) denotes statistical expectation.

**II. SYSTEM MODEL**

As shown in Fig. 1, we consider a SFBC-OFDM system that employs \( N_{TX} \) transmit (Tx) antennas, one receive (Rx) antenna, and \( N_{SC} \) sub-carriers per OFDM block. Firstly, let \( x_n \triangleq [x_{n,1}, x_{n,2}, \ldots, x_{n,M}]^T \) denote the vector containing the \( M \) QPSK symbols to be transmitted through the \( n \)th OFDM block. In the space-frequency encoder, the elements of \( x_n \) are segmented into \( K \) symbol groups with the \( k \)th group defined as \( x_{n,k} \triangleq [[x_{n,(k-1)M/k+1}, x_{n,(k-1)M/k+2}, \ldots, x_{n,M/k}]^T]. \) The symbol group \( x_{n,k} \) is then mapped onto a \( (N_{SC}/K)\times N_{TX} \) SFBC matrix \( S_{n,k} \). Following space-frequency encoding, the SFBC matrices \( S_{n,k} \) are used to form the concatenated matrix \( S_n = [S_{n,1}^T, S_{n,2}^T, \ldots, S_{n,K}^T]^T \). The \( i \)th column \( [S_n]_i \) of \( S_n \) then undergoes OFDM modulation and Tx pulse shape filtering before being transmitted through Tx antenna \( i \) (\( i = 1, 2, \ldots, N_{TX} \)).

Throughout the paper, we assume that the channel responses between different Tx-Rx antenna pairs are spatially uncorrelated and have identical power delay profiles. The \( P \)-path channel response corresponding to the link between Tx antenna \( i \) and the Rx antenna is modeled as

\[
c_i(t, \tau) = \sum_{p=1}^{P} c_{i,p}(n) \delta(\tau - \tau_p),
\]

where \( \tau_p \) and \( \delta(\tau) \) represent the delay of the \( p \)th path and the Dirac delta function, respectively. The path gains \( c_{i,p}(\tau) \) (\( p = 1, 2, \ldots, P \)) are modeled as independent circularly complex Gaussian random variables having zero-mean and variances \( \sigma_p^2 \) (\( p = 1, 2, \ldots, P \)). Assuming the channel fluctuations over the transmit duration of one OFDM block to be negligible, the \( i \)th \( (l = 0, \ldots, L-1) \) tap of the discrete-time channel impulse response (CIR) between Tx antenna \( i \) and the Rx antenna is given by [4]

\[
h_{i,l} = \sum_{p=1}^{P} c_{i,p}(n) g(T_5 - \tau_p),
\]

where \( T_5 \) and \( T_6 \) respectively represent the symbol sampling interval and the transmit duration of one OFDM block, and \( g(\tau) \) denotes the combined impulse response of the Tx and Rx filters. Next, let us define the CIR vector \( h_i^{(l)} = [h_{i,0}^{(l)}, h_{i,1}^{(l)}, \ldots, h_{i,L-1}^{(l)}]^T \). Then, the \( (l, \ell') \)th element of covariance matrix \( C \triangleq E[h_i^{(l)}(h_i^{(\ell')}^H)] \) can be expressed as [4]

\[
[C]_{ll'} = \sum_{p=1}^{P} \sigma_p^2 g(T_5 - \tau_p)g(T_5 - \tau_{l'}). \tag{3}
\]

Using the first-order Markov model, the time-variation of \( h_i^{(l)} \) over adjacent OFDM blocks can be modeled as [7]

\[
h_{i} = \alpha h_{i,-1} + \sqrt{1-\alpha^2} C^{1/2}\omega_t, \tag{4}
\]

where \( C^{1/2} \) denotes the Cholesky factor of \( C \), and \( \omega_t (\forall t) \) represent independent vectors consisting of independent and identically distributed (IID) Gaussian random variables with mean zero and variance unity. Furthermore, the time-correlation parameter \( \alpha = J_0(2\pi f_d T_6) \), wherein \( f_d \) and \( J_0(\bullet) \) denote the maximum Doppler frequency and the zero-order Bessel function of the first kind, respectively.

After OFDM demodulation at the receiver, the \( (N_{SC}/K)\times 1 \) received signal vector corresponding to the \( k \)th SFBC group can be expressed as

\[
r_{n,k} = A_{n,k} h_n + z_{n,k}, \tag{5}
\]

where the noise vector \( z_{n,k} \) constitutes of zero-mean IID Gaussian samples with a variance of \( \sigma^2_t \), and
\( \mathbf{h}_k \triangleq (h_{k1}^T \ h_{k2}^T \ \cdots \ h_{kn}^T)^T \). Furthermore, matrix \( \mathbf{A}_{x,k} \) in (5) is defined as

\[
\mathbf{A}_{x,k} \triangleq \begin{bmatrix} \mathbf{W}_1 & \mathbf{W}_2 & \cdots & \mathbf{W}_e \end{bmatrix},
\]

(6)

where \( \mathbf{W}_i \triangleq [\mathbf{w}_{i-1}N_c/K \mathbf{w}_{i-1}(1-N_c/K) \mathbf{w}_{i-2}N_c/K \cdots \mathbf{w}_{i-k}N_c/K]^H \) with \( \mathbf{w}_i = (N_c)^{-\frac{1}{2}} [1, e^{i2\pi m_i/K}, \ldots, e^{i2\pi (m_{i-k-1})/K}]^T \).

Throughout the paper, we assume a communication scenario where data transmissions occur in bursts of \( N_D \) OFDM blocks. To initialize the channel estimation algorithms, a preamble group (i.e., \( \mathbf{x}_{n,k} \)) is a forgetting factor, and \( \hat{\mathbf{A}}_{x,k} \) is obtained by replacing \( \hat{\mathbf{S}}_{x,k} \) in (6) with \( \hat{\mathbf{S}}_{x,k} \) (note that \( \mathbf{S}_{x,k} \) here represents the SFBC matrix corresponding to \( \mathbf{x}_{n,k} \)). Furthermore, \( \mathbf{Q}_{x,k} \) in (8) represents a sequence of standard Givens rotations [8] that annihilate the elements of \( \hat{\mathbf{A}}_{x,k} \) by forcing the lower \((N_{sc}/K) \times (N_T L)\) elements of the post-rotation matrix to zero.

Now, using the unitary structure of \( \mathbf{Q}_{x,k} \) (i.e., \( \mathbf{Q}_{x,k}^H \mathbf{Q}_{x,k} = \mathbf{I} \)) and the matrix inversion lemma in (8), it can be shown that

\[
\mathbf{R}_{x,k}^{-1} \mathbf{T}_{x,k}^H + \mathbf{T}_{x,k} \mathbf{R}_{x,k}^{-1} = \beta^2 \mathbf{R}_{x,k+1}^{-1},
\]

(9)

where

\[
\mathbf{T}_{x,k} \triangleq \beta^{-1} \mathbf{R}_{x,k+1}^{-1} \mathbf{R}_{x,k}^{-1} \hat{\mathbf{A}}_{x,k} \mathbf{U}_{x,k}.
\]

Note that in (10), \( \mathbf{U}_{x,k} \) is defined such that

\[
\mathbf{U}_{x,k} \mathbf{U}_{x,k}^H = [I + \beta^2 \hat{\mathbf{A}}_{x,k} \mathbf{R}_{x,k+1}^{-1} \mathbf{R}_{x,k}^{-1} \hat{\mathbf{A}}_{x,k}^H].
\]

We can now rewrite (9) in factored form as

\[
[\mathbf{R}_{x,k}^{-1} \mathbf{T}_{x,k}] [\mathbf{R}_{x,k}^{-1} \mathbf{T}_{x,k}^H] = \beta^{-1} \beta^{-1} \mathbf{R}_{x,k+1}^{-1} \mathbf{R}_{x,k+1}^{-1} [\mathbf{U}_{x,k} \mathbf{U}_{x,k}^H],
\]

(12)

where \( \mathbf{U}_{x,k} \mathbf{U}_{x,k}^H \) is a sequence of standard Givens rotations satisfying the condition \( \mathbf{U}_{x,k}^T \mathbf{U}_{x,k} = \mathbf{U}_{x,k} \mathbf{U}_{x,k}^H = \mathbf{I} \).

One approach for recursively updating \( \mathbf{h}_{n,k} \) is through a groupwise recursive update formula for \( \mathbf{R}_{x,k}^{-1} \) derived from (12). In this approach, \( \mathbf{h}_{n,k} = \mathbf{R}_{x,k} \mathbf{p}_{n,k} \) where \( \mathbf{p}_{n,k} \) satisfies

\[
\mathbf{Q}_{x,k} \begin{bmatrix} \beta \mathbf{p}_{n,k} \\ \mathbf{r}_{n,k} \end{bmatrix} = \begin{bmatrix} \mathbf{p}_{n,k} \\ \mathbf{d}_{n,k} \end{bmatrix},
\]

(13)

where \( \mathbf{d}_{n,k} \) is an \((N_{sc}/K) \times 1\) vector. However, the abovementioned approach is computationally expensive since the standard Givens rotation matrices \( \Theta_{x,k} \) and \( \mathbf{Q}_{x,k} \) require the implementation of several square-root operations. This is because square-root operations are computationally demanding in terms of power consumption and chip area requirements [9]. Hence, to attain low complexity channel estimation, we strive to avoid square-root operations by introducing the following square-root factorizations:

\[
\mathbf{R}_{x,k}^{-1} \mathbf{T}_{x,k}^H = \mathbf{G}_{x,k}^{-1/2} \mathbf{F}_{x,k}^{-1/2},
\]

(14)

\[
\mathbf{T}_{x,k} \mathbf{R}_{x,k}^{-1} = \mathbf{G}_{x,k}^{-1/2} \mathbf{F}_{x,k}^{-1/2},
\]

(15)

where \( \mathbf{F}_{x,k} \) is as defined in (7), and \( \mathbf{G}_{x,k} \) is a real diagonal matrix with dimension \((N_{sc}/K) \times (N_{sc}/K)\). Now, by using (14)-(15) in (12), \( \mathbf{R}_{x,k} \) and \( \mathbf{F}_{x,k} \) can be recursively updated as

\[
\begin{bmatrix} \mathbf{F}_{x,k}^{-1/2} & 0 \\ 0 & \mathbf{G}_{x,k}^{-1/2} \end{bmatrix} = \begin{bmatrix} \Theta_{x,k} \hat{\mathbf{A}}_{x,k} \mathbf{U}_{x,k} \end{bmatrix} \begin{bmatrix} \mathbf{F}_{x,k+1}^{-1/2} & 0 \\ 0 & \mathbf{G}_{x,k+1}^{-1/2} \end{bmatrix} \begin{bmatrix} \beta^{-1} \mathbf{R}_{x,k+1}^{-1} \\ 0 \end{bmatrix}.
\]

(16)

Note that the updates in (16) are performed square-root free via a sequence of scaled Givens rotations (see [10] and references therein). For compactness, we next rewrite (16) as

\[
\begin{bmatrix} \mathbf{R}_{x,k}^{-1} \mathbf{T}_{x,k}^H \\ \mathbf{T}_{x,k} \mathbf{R}_{x,k}^{-1} \end{bmatrix} = \Theta_{x,k} \begin{bmatrix} \beta^{-1} \mathbf{R}_{x,k+1}^{-1} \\ 0 \end{bmatrix},
\]

(17)

where \( \Theta_{x,k} \) is a sequence of scaled Givens rotations which is related to the standard Givens rotations matrix \( \Theta_{x,k} \) by

\[
\Theta_{x,k} = \begin{bmatrix} \mathbf{F}_{x,k}^{-1/2} & 0 \\ 0 & \mathbf{G}_{x,k}^{-1/2} \end{bmatrix} \Theta_{x,k} \begin{bmatrix} \mathbf{F}_{x,k+1}^{-1/2} & 0 \\ 0 & \mathbf{G}_{x,k+1}^{-1/2} \end{bmatrix} \begin{bmatrix} \beta^{-1} \mathbf{R}_{x,k+1}^{-1} \\ 0 \end{bmatrix}.
\]

(18)

Hence, using (18) and noting the fact that \( \Theta_{x,k} \) is unitary (i.e., \( \Theta_{x,k}^T \Theta_{x,k} = \mathbf{I} \)), the scaled Givens rotations matrix \( \Theta_{x,k} \) is shown to satisfy the condition
Then, substituting (20) into (17) and (19), we have
\[
G_{\text{v},k} \left[ F_{k-1} \right] \Theta_{n,k} = \left[ 0 \right] \Theta_{n,k} = \left[ 0 \right] G_{n,n},
\]
(19)

Then, substituting (20) into (17) and (19), we have
\[
\Theta_{n,k} = \left[ \Theta_{n,k}^{11} \Theta_{n,k}^{12} \right],
\]
(20)

However, from (14)-(15) and (10), \(T_{n,k}^{*} \) can be alternatively expressed as
\[
T_{n,k}^{*} = \beta^{-1} U_{n,k}^{H} \hat{A}_{n,k} R_{n,k-1}^{H} F_{n,k-1}^{H},
\]
(24)

where
\[
U_{n,k}^{H} \hat{U}_{n,k}^{H} = G_{n,k}^{-1}.
\]
(25)

Having derived \( \Theta_{n,k}^{11} \), we now proceed towards solving for \( \Theta_{n,k}^{12} \). First, by substituting (14) and (25) in (11), it can be shown that
\[
U_{n,k}^{H} G_{n,k} U_{n,k}^{H} - I = \beta^{-2} \hat{A}_{n,k} R_{n,k-1}^{H} F_{n,k-1}^{H},
\]
(26)

Next, substituting (26) into (22), we have
\[
\beta^{-2} U_{n,k}^{H} \hat{A}_{n,k} R_{n,k-1}^{H} F_{n,k-1}^{H} + \Theta_{n,k}^{12} \Theta_{n,k}^{H} = G_{n,k}.
\]
(28)

Further, substituting (27) into (28), it can be shown that
\[
G_{n,k} = G_{n,k} - U_{n,k}^{H} U_{n,k}^{H} + \Theta_{n,k}^{12} \Theta_{n,k}^{H}.
\]
(29)

Thus, in order to satisfy (29), we must have
\[
\Theta_{n,k}^{12} = U_{n,k}^{H}.
\]
(30)

Now, using the solutions of \( \Theta_{n,k}^{11} \) and \( \Theta_{n,k}^{12} \) along with (21) and (27), it can be shown that
\[
\left[ \Theta_{n,k}^{11} \Theta_{n,k}^{12} \right] \beta^{2} R_{n,k-1}^{H} A_{n,k}^{H} = \Theta_{n,k}^{11} \beta^{2} R_{n,k-1}^{H} A_{n,k}^{H} = G_{n,k} U_{n,k}^{H}.
\]
(31)

Hence, from (31), \( \Theta_{n,k}^{11} \) can be chosen as a sequence of scaled Givens rotations that annihilates the elements of \( \beta^{-2} R_{n,k-1}^{H} A_{n,k}^{H} \) by forcing the upper \((N_{L}L) \times (N_{L}/K)\) elements of the post-rotation matrix to zero.

C. Recursive Update Procedure for \( p_{n,k} \)

Next, we derive the recursive update procedure for the \((N_{L}L) \times 1\) vector \( p_{n,k} \) satisfying
\[
\hat{p}_{n,k} = F_{n,k}^{H} p_{n,k},
\]
(32)

where \( F_{n,k} \) and \( \hat{p}_{n,k} \) are defined in (7) and (13), respectively. Let us now partition the unitary matrix \( \hat{Q}_{n,k} \) defined in (8) as
\[
\hat{Q}_{n,k} = \begin{bmatrix} \hat{Q}_{n,k}^{11} & \hat{Q}_{n,k}^{12} \\ \hat{Q}_{n,k}^{21} & \hat{Q}_{n,k}^{22} \end{bmatrix}
\]

Then, using (32)-(33) in (13), we have
\[
\begin{bmatrix} F_{n,k}^{H} \hat{Q}_{n,k}^{11} F_{n,k}^{H} & F_{n,k}^{H} \hat{Q}_{n,k}^{12} \\ G_{n,k}^{H} \hat{Q}_{n,k}^{21} F_{n,k}^{H} & G_{n,k}^{H} \hat{Q}_{n,k}^{22} \\ \beta^{-1} p_{n,k-1} \end{bmatrix} = \begin{bmatrix} p_{n,k} \\ d_{n,k} \end{bmatrix},
\]
(34)

where \( d_{n,k} \) indicates the unitary structure of \( \hat{d}_{n,k} \) and \( G_{n,k} \) are defined in (13) and (15), respectively. Furthermore, using (8), (33), and (11), it can be shown that
\[
\hat{Q}_{n,k}^{11} = -\beta^{-1} \hat{Q}_{n,k}^{22} \hat{A}_{n,k} R_{n,k-1}^{H},
\]
(35)

\[
\hat{Q}_{n,k}^{12} = \hat{Q}_{n,k}^{21} = \hat{Q}_{n,k}^{22} = 0.
\]
(36)

By substituting (35) into (37), we then have
\[
\hat{Q}_{n,k}^{22} = -\beta^{-1} \hat{Q}_{n,k}^{22} \hat{A}_{n,k} R_{n,k-1}^{H}.
\]
(37)

Thereupon, simplifying (36) using (35) and (11) yields
\[
\hat{Q}_{n,k}^{22} = \hat{U}_{n,k}^{H} \hat{U}_{n,k}^{H} \hat{Q}_{n,k}^{22} = 0.
\]
(39)

Hence, from (39), it is easily seen that
\[
\hat{Q}_{n,k}^{22} = \hat{U}_{n,k}^{H}.
\]
(40)

We next post-multiply both sides of (35) by \( \beta^{-1} R_{n,k-1}^{H} \hat{A}_{n,k}^{H} \) and simplify the resulting expression using (11) to attain
\[
\hat{Q}_{n,k}^{11} \left[ \beta^{-1} R_{n,k-1}^{H} \hat{A}_{n,k}^{H} \right] = -\hat{Q}_{n,k}^{12} R_{n,k-1}^{H} \hat{A}_{n,k}^{H} + \hat{Q}_{n,k}^{22} \hat{U}_{n,k}^{H}.
\]
(41)

Since \( \hat{Q}_{n,k}^{11} = \hat{U}_{n,k}^{H} \) from (40), we can rewrite (41) as
\[
-\beta^{-1} \hat{Q}_{n,k}^{11} R_{n,k-1}^{H} \hat{A}_{n,k}^{H} + \hat{Q}_{n,k}^{22} = \hat{U}_{n,k}^{H}.
\]
(42)

Now, combining (38) and (42), we have
\[
\begin{bmatrix} \hat{Q}_{n,k}^{11} & \hat{Q}_{n,k}^{12} \\ \hat{G}_{n,k}^{H} \hat{Q}_{n,k}^{21} & \hat{G}_{n,k}^{H} \hat{Q}_{n,k}^{22} \end{bmatrix} \left[ \beta^{-1} R_{n,k-1}^{H} \hat{A}_{n,k}^{H} \right] = \begin{bmatrix} 0 \\ \hat{G}_{n,k}^{H} U_{n,k}^{H} \end{bmatrix}.
\]
(43)

where we have used the substitutions \( \hat{R}_{n,k}^{H} = F_{n,k-1}^{H} R_{n,k-1}^{H} \) and \( \hat{U}_{n,k}^{H} = G_{n,k}^{H} U_{n,k}^{H} \) from (14) and (25), respectively. Additionally, pre-multiplying both sides of (43) by the real diagonal matrix
\[
\begin{bmatrix} F_{n,k}^{H} & 0 \\ 0 & G_{n,k}^{H} \end{bmatrix}
\]

yields
\[
\begin{bmatrix} F_{n,k}^{H} \hat{Q}_{n,k}^{11} F_{n,k}^{H} & F_{n,k}^{H} \hat{Q}_{n,k}^{12} \\ -F_{n,k}^{H} \hat{G}_{n,k}^{H} \hat{Q}_{n,k}^{21} F_{n,k}^{H} & G_{n,k}^{H} \hat{Q}_{n,k}^{22} \end{bmatrix} \left[ \beta^{-1} R_{n,k-1}^{H} \hat{A}_{n,k}^{H} \right] = \begin{bmatrix} 0 \\ \hat{G}_{n,k}^{H} U_{n,k}^{H} \end{bmatrix}.
\]
(44)

Now, comparing (44) to (31), we deduce the following equalities:
\[
\Theta_{n,k}^{11} = F_{n,k}^{H} \hat{Q}_{n,k}^{11} F_{n,k}^{H},
\]
(45)

\[
\Theta_{n,k}^{12} = -F_{n,k}^{H} \hat{Q}_{n,k}^{12},
\]
(46)

\[
\Theta_{n,k}^{21} = -G_{n,k}^{H} \hat{Q}_{n,k}^{21} F_{n,k}^{H},
\]
(47)

\[
\Theta_{n,k}^{22} = G_{n,k}^{H} \hat{Q}_{n,k}^{22}.
\]
(48)

Hence, substituting (45)-(48) in (34), \( p_{n,k} \) can be recursively updated via the scaled Givens rotations matrix \( \Theta_{n,k} \) as
In (54), the matrices , we can rewrite (7) as

and (19), respectively; and the vector ,

simulations, a SFBC-OFDM system with 256

the required channel statistics are assumed to be known

channel estimators. In the cases of BMMSE and EM-MAP,

computational complexity of SIQR-GR to those associated

This is because the EM-MAP scheme is based on the quasi-

static assumption that the channel frequency responses

Further, it should be noted that since , is an

real diagonal matrix, the groupwise recursive update of in the proposed SIQR-GR scheme only requires

real scalar divisions and avoids matrix inversions.

IV. NUMERICAL RESULTS

In this section, we compare the performance and computational complexity of SIQR-GR to those associated with three existing schemes. The existing schemes included in the comparisons are the BLS, the BMMSE, and the EM-MAP channel estimators. In the cases of BMMSE and EM-MAP, the required channel statistics are assumed to be known perfectly at the receiver. Furthermore, we assume 3 EM iterations for simulations involving EM-MAP. Throughout the simulations, a SFBC-OFDM system with , is utilized for space-frequency encoding. Given the CIR vector estimate, the hard decision vectors ( ) are computed by the low complexity successive interference cancellation receiver proposed in [10].

Fig. 2 shows the NMSE performance comparison between SIQR-GR and the existing schemes for a six-ray (i.e., ) typically-urban channel (TUC) [12]. Compared to SIQR-GR, we note that both the BLS and BMMSE channel estimators exhibit much higher NMSE values. For instance, at high SNRs, SIQR-GR achieves NMSE reductions in excess of 3.5 dB over BLS and 3.0 dB over BMMSE. Since BLS and BMMSE employ block based processing, they compute the hard decision vectors , , , , , , using the outdated CIR vector estimate . Consequently, both BLS and BMMSE suffer from performance loss due to the modeling mismatch between the outdated CIR vector estimate and the true CIR vector . By utilizing a groupwise recursive processing approach, the proposed SIQR-GR channel estimator progressively reduces the modeling mismatch with increasing SFB group index . As a result, SIQR-GR achieves superior NMSE reductions over BLS and BMMSE.

Additionally, from Fig. 2, we see that SIQR-GR yields an NMSE reduction of 5.6 dB over EM-MAP when . This is because the EM-MAP scheme is based on the quasi-static assumption that the channel frequency responses corresponding to all Tx-Rx antenna pairs are constant over the ( ) adjacent sub-carriers within each SFBC group. Since the quasi-static assumption is inaccurate for frequency-selective channels with high delay spreads, the EM-MAP scheme performs poorly when compared to SIQR-GR. It should be noted, however, that the EM-MAP scheme is more robust against varying maximum Doppler frequency values. This is due to the fact that EM-MAP first computes the a posteriori probabilities of all the data symbols transmitted through each OFDM block and then makes use of these probabilities to update the channel estimate (also note that in the case of EM-MAP, the hard decision vectors , , , , , , are computed using the updated channel estimate). As a result, EM-MAP marginally outperforms SIQR-GR by approximately 0.2 dB when .

Next, we compare the SER performances of SIQR-GR and the three existing schemes in Fig. 3. Also shown in Figs. 3 is the SER performance achieved when the CSI is perfectly known at the receiver. First, we note that when , SIQR-GR performs within 1.6 dB of the perfect CSI case. Furthermore, when , SIQR-GR achieves SNR gains of 3.5 dB over BLS, 2.8 dB over BMMSE, and 6.0 dB over EM-MAP. It is further noted from Fig. 3 that when , all channel estimation schemes form error floors in the high SNR region. Most notably, SIQR-GR reduces the error floors of BLS and BMMSE by factors of and , respectively.

Finally, we present a comparison between the approximate computational complexity requirements of the proposed SIQR-GR channel estimator and those of the three existing schemes under consideration. We quantify the approximate complexity of each scheme by counting the number of additions, multiplications, divisions, and square-roots required for channel estimation within each OFDM block. Throughout this section, the floating point operation (flop) counts for complex addition, complex multiplication, and complex division are respectively chosen as 2, 6, and 6. Additionally, each square-
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In this paper, we propose the decision directed SIQR-GR channel estimator based on groupwise recursive processing for SFBC-OFDM systems. The proposed SIQR-GR scheme employs sequences of scaled Givens rotations to achieve low complexity decision directed CIR vector estimation. NMSE performance comparisons reveal superior NMSE performances achieved by SIQR-GR over existing channel estimation methods. Via SER comparisons, we demonstrate that SIQR-GR attains significant SNR gains and error floor reductions over the existing schemes under different channel conditions. In addition, the proposed SIQR-GR channel estimator is shown to be computationally much simpler than the existing schemes.

V. CONCLUSIONS

In this paper, we propose the decision directed SIQR-GR

![Graph](image1)

Fig. 2. NMSE performance comparisons with existing schemes over a six-ray TUC with $L=8$. (Note that the EM-MAP performance difference between the cases of $f_d=100$ Hz and $f_d=200$ Hz is unnoticeable).

![Graph](image2)

Fig. 3. SER performance comparisons with existing schemes over a six-ray TUC with $L=8$. (Note that the EM-MAP performance difference between the cases of $f_d=100$ Hz and $f_d=200$ Hz is unnoticeable).

root operation is counted as 10 flops, and all other real operations considered are counted as 1 flop. The approximate complexity requirements for the case of the TUC are given in Table I. Also shown in the table are the flop count reductions achieved by the proposed SIQR-GR channel estimator over the three existing schemes. From Table I, we see that SIQR-GR achieves flop count reductions of 4.65 % and 10.28 % over BLS and BMMSE, respectively. The corresponding flop count reduction achieved over EM-MAP is 38.39 %. Finally, it should be noted that the flop counts presented for BMMSE and EM-MAP in Table I do not take into account the complexities associated with the estimation of the required channel statistics. Therefore, if these complexities are taken into account, the actual flop counts for BMMSE and EM-MAP will be even higher. This will further enhance the percentile flop count reduction achieved by SIQR-GR over BMMSE and EM-MAP.

### TABLE I

<table>
<thead>
<tr>
<th>Channel Estimator</th>
<th>Complexity (in flops)</th>
<th>Flop Count Reduction Achieved by SIQR-GR</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIQR-GR</td>
<td>3,600,128</td>
<td>--</td>
</tr>
<tr>
<td>BLS</td>
<td>3,775,616</td>
<td>4.65 %</td>
</tr>
<tr>
<td>BMMSE</td>
<td>4,012,416</td>
<td>10.28 %</td>
</tr>
<tr>
<td>EM-MAP</td>
<td>5,843,376</td>
<td>38.39 %</td>
</tr>
</tbody>
</table>

![Graph](image3)

![Graph](image4)

APPENDIX

```
\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Channel Estimator & Complexity (in flops) & Flop Count Reduction Achieved by SIQR-GR \\
\hline
SIQR-GR           & 3,600,128             & --                                       \\
BLS               & 3,775,616             & 4.65 %                                   \\
BMMSE             & 4,012,416             & 10.28 %                                  \\
EM-MAP            & 5,843,376             & 38.39 %                                  \\
\hline
\end{tabular}
\caption{Approximate Channel Estimation Complexity Requirements for a Single OFDM Block in the Typically-Urban Channel}
\end{table}
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REFERENCES


