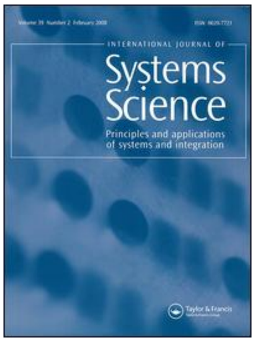

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New H_∞ dynamic observer design for time-delay systems subject to disturbances

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ABSTRACT

This paper presents a new approach to design an H_∞ dynamic observer (DO) for multi-delayed linear systems subject to $\mathcal{L}_2$-norm disturbances. This observer generalises the existing results on the proportional observer (PO), the proportional integral observer (PIO), and the DO. The proposed design approach is derived from the solution of LMIs based on the parametrization results of algebraic constraints. These algebraic constraints can be easily obtained from the unbiasedness conditions of the estimation error. The obtained results are illustrated by a numerical example to show the performances of the proposed observer.

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H_∞ dynamic observer; multi-delayed systems; disturbances; unbiasedness conditions; estimation error

1. Introduction

The observer design problem has gained high interest in the literature, due to the fact that the state cannot be measured frequently (Sun et al., 2018). Then, it is necessary to use an observer to estimate the system state when the state cannot be determined by direct measurement. For this reason, the fuzzy observer-based controller for a class of nonlinear systems is considered in Qiu et al. (2019) and Li et al. (2016). In the daily life, the disturbance is very common. In control process, not only the reference input can affect the output, but also the disturbance can result in negative influences on the output. Therefore, in the observer design, the disturbance is taken into account by many authors (Chang et al., 2011; Chen et al., 2016; El Haiek et al., 2017). All the observers introduced before are the kind of POs, which are not capable to handle the static error. Consequently, the PIO is introduced by duality to the PI controller to achieve the desired performances (Kühne et al., 2017; Xu et al., 2014; Youssef et al., 2017, 2014). We still also need an observer that offers more extra degrees of freedom over the PIO, which can be shown to be useful in many cases.

To present an alternative state estimation structure, a new form of the observer, called DO, is developed (Li & Yang, 2012; Park et al., 2002). The proposed observer can be considered as more general than PO and PIO, and these latter can be considered as particular cases of this form. Different from the PO and the PIO, the DO contains a dynamic gain in the observer design. Then, the present paper concerns a new form of an H_∞ DO which has a more general form than those presented in the literature. The proposed observer is formed by an H_∞ theory, a dynamical part, and a static part to estimate the state of multi-delayed linear systems in the presence of disturbances. Without estimating the disturbances, the H_∞ DO offers a direct method to reject the negative effect of disturbances.

On the other hand, time-delays appear in many kinds of control systems and their presence can be source of performances degradation and instability (Ech-charqy et al., 2018; El Aiss et al., 2017; El Fezazi et al., 2019, 2020, 2017; El Haoussi et al., 2011; Lamrabet, El Fezazi et al., 2020; Lamrabet, Tissir et al., 2020; Lamrabet et al., 2019). Due to the fact that many practical systems present time-delays, it becomes important to study the stability issues regarding this kind of systems. For this reason, Lyapunov–Krasovskii (L–K) stability theory is used in many works to design the state observers for delayed systems and sufficient conditions are provided (El Aiss

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et al., 2019; El Fezazi, 2019; El Fezazi et al., 2019; Kao et al., 2016; Ma et al., 2017; Thuan et al., 2012). Then, the design problem of an H_∞ DO for multi-delayed systems has not been fully investigated so far, and still remains an open and unsolved problem. This topic will be investigated further in this paper. These results can be also extended for the systems subject to finite frequency disturbances following some ideas (El-Amrani, Boukili, El Hajjaji et al., 2018; El-Amrani, Boukili, Hmamed et al., 2018; Wang et al., 2018a, 2018b) and using specific lemma (Kalman–Yakubovich–Popov Lemma).

The unbiasedness conditions of the estimation error, the H_∞ theory, and the DO are explicitly taken into account in this paper using a new approach to perfect the observer characteristics in order to develop a practical observer where no assumption of online measurement of the delay is required. An effective H_∞ DO is then designed for multi-delayed systems subject to $\mathcal{L}_2-$norm disturbances. Then, less conservative results are obtained using free matrices, with only one of them restricted to be positive definite, in order to achieve the desired H_∞ performance. Based on the parametrization of algebraic constraints, the proposed observer design is derived from the solution of LMI. These constraints are easily obtained from the analysis of the estimation error. The performance and effectiveness of the proposed method are then validated through detailed simulations.

The paper is organised as follows. The H_∞ DO design problem is formulated in Section 2. In Section 3, The algebraic constraints are derived through the analysis of estimation error. Then, a theorem is presented to solve the design problem. The design procedure of the proposed observer is presented in Section 4. In Section 5, numerical examples are provided to illustrate the efficiency of the proposed approach. Finally, Some conclusions are drawn in Section 6.

The following notations are used throughout the paper: $\Re^n$ is the n dimensional Euclidean space, $\mathcal{L}_2[0,\infty)$ is the space of square integrable vectors on $[0,\infty)$, $\|\varpi\|_2$ is the $\mathcal{L}_2-$norm of ϖ where $\|\varpi(t)\|_2^2 = \int_0^\infty \varpi^T(t).\varpi(t)\,\mathrm{d}t$ and $w(t) \in \mathcal{L}_2[0,\infty)$, $rank(\nu)$ is the rank of matrix ν, I and 0 are respectively, the identity and zero matrices of appropriate dimensions, $*$ is the symmetric block in a matrix, and the superscript ^{T} is the matrix transposition.

2. Problem formulation and preliminaries

Consider the following system:

$$\begin{aligned} \dot{x}(t) &= A_0x(t) + \sum_{i=1}^{m} A_ix(t-\tau_i(t)) \\ &\quad + Bu(t) + B_ww(t) \\ y(t) &= C_yx(t) \end{aligned} \tag{1}$$

where $x(t) \in \Re^{n_x}$ is the state vector, $u(t) \in \Re^{n_u}$ is the input vector, $w(t) \in \Re^{n_w}$ is the disturbance vector of finite energy, and $y(t) \in \Re^{n_y}$ is the output vector. Matrices $A_0,\ldots,A_m$, B, B_w, and C_y are known and of appropriate dimensions. On the other hand, the delays $\tau_1(t),\ldots,\tau_m(t)$ are assumed to be time dependent and satisfy $0 \le \tau_1(t) \le h_{\tau_1}, 0 \le \dot{\tau}_1(t) \le d_1 < 1, \ldots, 0 \le \tau_m(t) \le h_{\tau_m}, 0 \le \dot{\tau}_m(t) \le d_m < 1$.

Then, let us consider the following DO:

$$\begin{aligned} \dot{z}(t) &= N_0z(t) + \sum_{i=1}^{m} N_iz(t-\tau_i(t)) + J_0y(t) \\ &\quad + \sum_{i=1}^{m} J_iy(t-\tau_i(t)) + Hu(t) + M_0v(t) \\ &\quad + \sum_{i=1}^{m} M_iv(t-\tau_i(t)) \\ \dot{v}(t) &= P_0z(t) + \sum_{i=1}^{m} P_iz(t-\tau_i(t)) + Q_0y(t) \\ &\quad + \sum_{i=1}^{m} Q_iy(t-\tau_i(t)) + G_0v(t) \\ &\quad + \sum_{i=1}^{m} G_iv(t-\tau_i(t)) \\ \hat{x}(t) &= Rz(t) + Sy(t) \end{aligned} \tag{2}$$

where $z(t) \in \Re^{n_z}$, $v(t) \in \Re^{n_z}$, and $\hat{x}(t) \in \Re^{n_x}$ are the state vector of the observer, the auxiliary state vector, and the estimation of the state vector, respectively. Matrices $N_0,\ldots,N_m$, $J_0,\ldots,J_m$, H, $M_0,\ldots,M_m$, $P_0,\ldots,P_m$, $Q_0,\ldots,Q_m$, $G_0,\ldots,G_m$, R, and S are unknown to be determined and of appropriate dimensions. The auxiliary vector v is similar to the additional term $w(t)$ in PIO, which is used to realise the steady state performance.

Remark 2.1:

- The observer (2) is in a generalised form and generalises the existing ones. In fact:
  - If $M_0 = \cdots = M_m = 0,\ P_0 = \cdots = P_m = 0,\ Q_0 = \cdots = Q_m = 0,\ G_0 = \cdots = G_m = 0,\ S = 0,$ and $R = I$, the observer (2) reduces to the following full order PO:

$$\dot{\hat{x}}(t) = N_0\hat{x}(t) + \sum_{i=1}^{m} N_i\hat{x}(t-\tau_i(t)) + J_0 y(t) + \sum_{i=1}^{m} J_i y(t-\tau_i(t)) + Hu(t)$$

  - If $Q_0 = \cdots = Q_m = I,\ G_0 = \cdots = G_m = 0,$ $R = I$, and $S = 0$, we obtain the following PIO:

$$\dot{\hat{x}}(t) = N_0\hat{x}(t) + \sum_{i=1}^{m} N_i\hat{x}(t-\tau_i(t)) + J_0 y(t) + \sum_{i=1}^{m} J_i y(t-\tau_i(t)) + Hu(t) + M_0 v(t) + \sum_{i=1}^{m} M_i v(t-\tau_i(t))$$

$$\dot{v}(t) = P_0\hat{x}(t) + \sum_{i=1}^{m} P_i\hat{x}(t-\tau_i(t)) + y(t) + \sum_{i=1}^{m} y(t-\tau_i(t))$$

  - If $R = I$ and $S = 0$, the following standard DO is obtained:

$$\dot{\hat{x}}(t) = N_0\hat{x}(t) + \sum_{i=1}^{m} N_i\hat{x}(t-\tau_i(t)) + J_0 y(t) + \sum_{i=1}^{m} J_i y(t-\tau_i(t)) + Hu(t) + M_0 v(t) + \sum_{i=1}^{m} M_i v(t-\tau_i(t))$$

$$\dot{v}(t) = P_0\hat{x}(t) + \sum_{i=1}^{m} P_i\hat{x}(t-\tau_i(t)) + Q_0 y(t) + \sum_{i=1}^{m} Q_i y(t-\tau_i(t)) + G_0 v(t) + \sum_{i=1}^{m} G_i v(t-\tau_i(t))$$

- Assuming $rank(C_y) = p$, if $q = n - p$, we obtain the reduced order version of the observer (2), and if $q = n$, we obtain the full order one.

Remark 2.2: The maximisation problem of the disturbance rejection is addressed in this work in order to minimise the $\mathcal{L}_2$−gain (measured with respect to some norm) between $w(t)$ and $e(t)$ (the error estimate) considering the $\mathcal{L}_2$−norm disturbance. This norm is given by $\|w(t)\|_2^2 \le \delta^{-1} \le \infty$ where δ is a scalar (Wang et al., 2018c).

Definition 2.1: $\mathbb{A}$ is called a Hurwitz matrix if every eigenvalue of $\mathbb{A}$ has strictly negative real part, that is, $\mathfrak{Re}[\lambda_i] < 0$ for each eigenvalue λ_i.

The following useful lemma will be used in this paper:

Lemma 2.1 (Gu et al., 2003): *Jensen Inequality: For any scalar $b > a$, the following inequality holds:*

$$(b-a)\int_a^b \xi^T(s)R_1\xi(s)\,ds \ge \left(\int_a^b \xi(s)\,ds\right)^T R_1 \left(\int_a^b \xi(s)\,ds\right)$$

The problem of the H_∞ DO design is focused to find the matrices $N_0,\ldots,N_m$, $J_0,\ldots,J_m$, H, $M_0,\ldots,M_m$, $P_0,\ldots,P_m$, $Q_0,\ldots,Q_m$, $G_0,\ldots,G_m$, R, and S such that the ratio between the norm of the estimation error and that of the disturbance is given by

$$(\|e(t)\|_2^2/\|w(t)\|_2^2) < \gamma \tag{3}$$

where γ is a specified value.

3. Main results

In this section, we will present a method to design an H_∞ DO for the multi-delayed system (1). Then, this observer is derived from the solution of LMI formulations based on some algebraic constraints and parametrization works.

3.1. Unbiasedness conditions

To present the parametrization of the observer, we define firstly a new error variable $\varepsilon(t) = z(t) - Tx(t)$ where the matrix T is an arbitrary matrix. Then, the following lemma is given:

Lemma 3.1: *The system* (2) *is a DO for the system* (1) *if there exists T such that the following constraints are satisfied:*

$$\begin{aligned} N_0T + J_0C_y - TA_0 &= 0 \\ &\vdots \\ N_mT + J_mC_y - TA_m &= 0 \\ H &= TB \\ P_0T + Q_0C_y &= 0 \\ &\vdots \\ P_mT + Q_mC_y &= 0 \\ RT + SC_y &= I \end{aligned} \tag{4}$$

and the matrices (5) *are Hurwitz.*

$$\mathbb{A}_0 = \begin{bmatrix} N_0 & M_0 \\ P_0 & G_0 \end{bmatrix}, \ldots, \mathbb{A}_m = \begin{bmatrix} N_m & M_m \\ P_m & G_m \end{bmatrix} \tag{5}$$

Proof: The dynamic of the error $\varepsilon(t)$ is given by

$$\begin{aligned} \dot{\varepsilon}(t) &= \dot{z}(t) - T\dot{x}(t) \\ &= N_0\varepsilon(t) + \sum_{i=1}^{m} N_i\varepsilon(t - \tau_i(t)) \\ &\quad + (N_0T + J_0C_y - TA_0)x(t) \\ &\quad + \sum_{i=1}^{m}(N_iT + J_iC_y - TA_i)x(t - \tau_i(t)) \\ &\quad + (H - TB)u(t) + M_0v(t) \\ &\quad + \sum_{i=1}^{m} M_iv(t - \tau_i(t)) - TB_ww(t) \end{aligned}$$

Then, we obtain:

$$\begin{aligned} \dot{v}(t) &= P_0\varepsilon(t) + \sum_{i=1}^{m} P_i\varepsilon(t - \tau_i(t)) \\ &\quad + (P_0T + Q_0C_y)x(t) + \sum_{i=1}^{m}(P_iT + Q_iC_y) \\ &\quad \times x(t - \tau_i(t)) + G_0v(t) + \sum_{i=1}^{m} G_iv(t - \tau_i(t)) \\ \hat{x}(t) &= R\varepsilon(t) + (RT + SC_y)x(t) \end{aligned}$$

One can clearly see that the dynamics of the error $\varepsilon(t)$ and the auxiliary state $v(t)$ are independent of $x(t)$ and $u(t)$ if the following constraints are satisfied:

$$\begin{aligned} N_0T + J_0C_y - TA_0 &= 0 \\ &\vdots \\ N_mT + J_mC_y - TA_m &= 0 \\ H - TB &= 0 \\ P_0T + Q_0C_y &= 0 \\ &\vdots \\ P_mT + Q_mC_y &= 0 \end{aligned}$$

The expression of the estimation error is $e(t) = \hat{x}(t) - x(t) = R\varepsilon(t)$ if $RT + SC_y = I$. Then, we have:

$$\begin{aligned} \dot{\varepsilon}(t) &= N_0\varepsilon(t) + \sum_{i=1}^{m} N_i\varepsilon(t - \tau_i(t)) + M_0v(t) \\ &\quad + \sum_{i=1}^{m} M_iv(t - \tau_i(t)) - TB_ww(t) \\ \dot{v}(t) &= P_0\varepsilon(t) + \sum_{i=1}^{m} P_i\varepsilon(t - \tau_i(t)) + G_0v(t) \\ &\quad + \sum_{i=1}^{m} G_iv(t - \tau_i(t)) \\ e(t) &= R\varepsilon(t) \end{aligned}$$

In this case, the augmented system can be represented as follows:

$$\begin{aligned} \dot{\xi}(t) &= \mathbb{A}_0\xi(t) + \sum_{i=1}^{m} \mathbb{A}_i\xi(t - \tau_i(t)) + \mathbb{B}_ww(t) \\ e(t) &= \mathbb{C}_e\xi(t) \end{aligned} \tag{6}$$

where

$$\xi(t) = \begin{bmatrix} \varepsilon(t) \\ v(t) \end{bmatrix}, \quad \mathbb{A}_0 = \begin{bmatrix} N_0 & M_0 \\ P_0 & G_0 \end{bmatrix}, \ldots,$$
$$\mathbb{A}_m = \begin{bmatrix} N_m & M_m \\ P_m & G_m \end{bmatrix},$$
$$\mathbb{B}_w = \begin{bmatrix} -TB_w \\ 0 \end{bmatrix}, \quad \mathbb{C}_e = \begin{bmatrix} R & 0 \end{bmatrix}$$

Clearly, one can say that $e(t) \to 0$ when both $\varepsilon(t) \to 0$ and $v(t) \to 0$ if and only if $\mathbb{A}_0, \ldots, \mathbb{A}_m$ are Hurwitz. Then, the trajectories of the augmented system (6) converge asymptotically to the origin. Finally, the proof of Lemma 3.1 is completed. ■

3.2. Synthesis of the DO

Our goal now is to determine the parameters of the functional observer $N_0, \ldots, N_m$, $J_0, \ldots, J_m$, H, $M_0, \ldots, M_m$, $P_0, \ldots, P_m$, $Q_0, \ldots, Q_m$, $G_0, \ldots, G_m$, R, and S.

From Equation (4), it is easy to have:

$$\begin{bmatrix} P_0 & Q_0 \\ \vdots & \\ P_m & Q_m \\ R & S \end{bmatrix} \begin{bmatrix} T \\ C_y \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ I \end{bmatrix} \tag{7}$$

Equation (7) has a solution if and only if Boutat-Baddas et al. (2019) and Gao et al. (2016)

$$rank \begin{bmatrix} T \\ C_y \\ 0 \\ \vdots \\ 0 \\ I \end{bmatrix} = rank \begin{bmatrix} T \\ C_y \end{bmatrix} = n$$

Let E be an arbitrary matrix of full row rank such that

$$rank \begin{bmatrix} E \\ C_y \end{bmatrix} = rank \begin{bmatrix} T \\ C_y \end{bmatrix} = n \tag{8}$$

Then, there exists matrices T and K such that the following condition is verified (Boutat-Baddas et al., 2019; Gao et al., 2016):

$$\begin{bmatrix} T \\ C_y \end{bmatrix} = \begin{bmatrix} I & -K \\ 0 & I \end{bmatrix} \begin{bmatrix} E \\ C_y \end{bmatrix}$$

Notice that $T = E - KC_y$ where

$$\begin{bmatrix} T & K \end{bmatrix} \begin{bmatrix} I \\ C_y \end{bmatrix} = E \tag{9}$$

Then, Equation (9) has a solution if

$$rank \begin{bmatrix} I \\ C_y \\ E \end{bmatrix} = rank \begin{bmatrix} I \\ C_y \end{bmatrix}$$

From Equation (9), we obtain (Boutat-Baddas et al., 2019):

$$T = E \begin{bmatrix} I \\ C_y \end{bmatrix}^+ \begin{bmatrix} I \\ 0 \end{bmatrix}, \quad K = E \begin{bmatrix} I \\ C_y \end{bmatrix}^+ \begin{bmatrix} 0 \\ I \end{bmatrix} \tag{10}$$

On the other hand, Equation (7) becomes

$$\begin{bmatrix} P_0 & Q_0 \\ \vdots & \\ P_m & Q_m \\ R & S \end{bmatrix} \begin{bmatrix} I & -K \\ 0 & I \end{bmatrix} \begin{bmatrix} E \\ C_y \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ I \end{bmatrix} \tag{11}$$

Considering the arbitrary matrix Z, a general solution to (11) is given by Gao et al. (2016)

$$\begin{bmatrix} P_0 & Q_0 \\ \vdots & \\ P_m & Q_m \\ R & S \end{bmatrix} = \left\{ \begin{bmatrix} 0 \\ \vdots \\ 0 \\ I \end{bmatrix} \begin{bmatrix} E \\ C_y \end{bmatrix}^+ - Z\left(I - \begin{bmatrix} E \\ C_y \end{bmatrix} \begin{bmatrix} E \\ C_y \end{bmatrix}^+ \right) \right\} \begin{bmatrix} I & K \\ 0 & I \end{bmatrix} \tag{12}$$

Then, the matrices $P_0, \ldots, P_m$, $Q_0, \ldots, Q_m$, R, and S are defined as follows:

$$\begin{aligned} P_0 &= -Z_0\beta_1, \ldots, P_m = -Z_m\beta_1, \\ Q_0 &= -Z_0\beta_2, \ldots, Q_m = -Z_m\beta_2, \\ R &= \alpha_1 - Z_{m+1}\beta_1, \quad S = \alpha_2 - Z_{m+1}\beta_2 \end{aligned} \tag{13}$$

where

$$\begin{aligned} Z_0 &= \begin{bmatrix} I & 0 & \ldots & 0 \end{bmatrix} Z, \ldots, Z_{m+1} \\ &= \begin{bmatrix} 0 & \ldots & 0 & I \end{bmatrix} Z, \\ \Sigma &= \begin{bmatrix} E \\ C_y \end{bmatrix}, \quad \alpha_1 = \Sigma^+ \begin{bmatrix} I \\ 0 \end{bmatrix}, \quad \alpha_2 = \Sigma^+ \begin{bmatrix} K \\ I \end{bmatrix}, \\ \beta_1 &= (I - \Sigma\Sigma^+) \begin{bmatrix} I \\ 0 \end{bmatrix}, \quad \beta_2 = (I - \Sigma\Sigma^+) \begin{bmatrix} K \\ I \end{bmatrix} \end{aligned} \tag{14}$$

Using $T = E - KC_y$ and taking into account Equation (4), it is easy to obtain:

$$\begin{bmatrix} N_0 & K_0 \end{bmatrix} \Sigma = \theta_0, \ldots, \begin{bmatrix} N_m & K_m \end{bmatrix} \Sigma = \theta_m \tag{15}$$

where

$$J_0 = N_0K + K_0, \dots, J_m = N_mK + K_m,$$
$$\theta_0 = TA_0, \dots, \theta_m = TA_m \tag{16}$$

As given in (12), a general solution to (15) is given by the Equation (17) considering the arbitrary matrices $Z_{A_0}, \dots, Z_{A_m}$.

$$\begin{bmatrix} N_0 & K_0 \end{bmatrix} = \theta_0\Sigma^+ - Z_{A_0}(I - \Sigma\Sigma^+), \dots, \begin{bmatrix} N_m & K_m \end{bmatrix} = \theta_m\Sigma^+ - Z_{A_m}(I - \Sigma\Sigma^+) \tag{17}$$

Then, the matrices $N_0, \dots, N_m$ and $K_0, \dots, K_m$ are defined as follows:

$$N_0 = \theta_0\alpha_1 - Z_{A_0}\beta_1, \dots, N_m = \theta_m\alpha_1 - Z_{A_m}\beta_1,$$
$$K_0 = \theta_0\alpha_3 - Z_{A_0}\beta_3, \dots, K_m = \theta_m\alpha_3 - Z_{A_m}\beta_3 \tag{18}$$

where

$$\alpha_3 = \Sigma^+\begin{bmatrix} 0 \\ I \end{bmatrix}, \quad \beta_3 = (I - \Sigma\Sigma^+)\begin{bmatrix} 0 \\ I \end{bmatrix} \tag{19}$$

From the results above, the matrices $\mathbb{A}_0, \dots, \mathbb{A}_m$ become

$$\mathbb{A}_0 = \overline{\mathbb{A}}_0 - \mathbb{Z}_0\mathbb{I}, \dots, \mathbb{A}_m = \overline{\mathbb{A}}_m - \mathbb{Z}_m\mathbb{I}$$

where

$$\overline{\mathbb{A}}_0 = \begin{bmatrix} \theta_0\alpha_1 & 0 \\ 0 & 0 \end{bmatrix}, \dots, \overline{\mathbb{A}}_m = \begin{bmatrix} \theta_m\alpha_1 & 0 \\ 0 & 0 \end{bmatrix},$$
$$\mathbb{I} = \begin{bmatrix} \beta_1 & 0 \\ 0 & -I \end{bmatrix},$$
$$\mathbb{Z}_0 = \begin{bmatrix} Z_{A_0} & M_0 \\ Z_0 & G_0 \end{bmatrix}, \dots, \mathbb{Z}_m = \begin{bmatrix} Z_{A_m} & M_m \\ Z_m & G_m \end{bmatrix}$$

In this case, the augmented system (6) becomes as follows:

$$\dot{\xi}(t) = (\overline{\mathbb{A}}_0 - \mathbb{Z}_0\mathbb{I})\xi(t) + \sum_{i=1}^{m}(\overline{\mathbb{A}}_i - \mathbb{Z}_i\mathbb{I})\xi(t - \tau_i(t)) + \mathbb{B}_w w(t) \tag{20}$$
$$e(t) = \mathbb{C}_e\xi(t)$$

3.3. An LMI synthesis condition

At this stage, we can obtain the more general solution and provide an LMI condition. Then, the design problem is reduced to study the system (20), i.e. to determine the parameter matrices $\mathbb{Z}_0, \dots, \mathbb{Z}_m$ which can be obtained from the following theorem:

Theorem 3.1: *The system* (20) *is asymptotically stable and verify the H_∞ performance if there exist symmetric positive definite matrices P_1, $Q_1, \dots, Q_m$, $R_1, \dots, R_m$, appropriately sized matrices P_2, $Y_0, \dots, Y_m$, $P_{11}, \dots, P_{1m}, P_{21}, \dots, P_{2m}, P_{31}, \dots, P_{3m}, \dots, P_{km}, P_{31}, \dots, P_{k1}, \dots, P_{km}$ where $k = m + 2$, and a scalar α satisfying the following condition:*

$$\begin{bmatrix}
\Xi_{11} & \Xi_{12} & \Pi_{11} & \dots & \Pi_{1m} & -P_{11}^T & \dots & -P_{1m}^T & P_2^T\mathbb{B}_w & \mathbb{C}_e^T \\
* & \Xi_{22} & \Pi_{21} & \dots & \Pi_{2m} & -P_{21}^T & \dots & -P_{2m}^T & \alpha P_2^T\mathbb{B}_w & 0 \\
* & * & \Pi_{31} & \dots & \Pi_{3m} & -P_{31}^T & \dots & -P_{3m}^T & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
* & * & * & \dots & \Pi_{km} & -P_{k1}^T & \dots & -P_{km}^T & 0 & 0 \\
* & * & * & \dots & * & \dfrac{-R_1}{h_{\tau_1}} & \dots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
* & * & * & \dots & * & * & \dots & \dfrac{-R_m}{h_{\tau_m}} & 0 & 0 \\
* & * & * & \dots & * & * & \dots & * & -I & 0 \\
* & * & * & \dots & * & * & \dots & * & * & -\gamma I
\end{bmatrix} < 0 \tag{21}$$

where

$$\Xi_{11} = \overline{\mathbb{A}}_0^T P_2 - \mathbb{I}^T Y_0^T + P_2^T\overline{\mathbb{A}}_0 - Y_0\mathbb{I} + P_{11} + \cdots + P_{1m} + P_{11}^T + \dots + P_{1m}^T + \sum_{i=1}^{m} Q_i,$$
$$\Xi_{12} = \alpha\overline{\mathbb{A}}_0^T P_2 - \alpha\mathbb{I}^T Y_0^T + P_1^T - P_2^T + P_{21} + \cdots + P_{2m},$$
$$\Xi_{22} = -\alpha P_2 - \alpha P_2^T + \sum_{i=1}^{m} h_{\tau_i}R_i,$$
$$\Pi_{11} = P_2^T\overline{\mathbb{A}}_1 - Y_1\mathbb{I} - P_{11}^T + P_{31} + \cdots + P_{3m},$$
$$\Pi_{21} = \alpha P_2^T\overline{\mathbb{A}}_1 - \alpha Y_1\mathbb{I} - P_{21}^T,$$

$$\Pi_{31} = -P_{31} - P_{31}^T - (1-d_1)Q_1,$$
$$\Pi_{1m} = P_2^T\overline{\mathbb{A}}_m - Y_m\mathbb{I} - P_{1m}^T + P_{k1} + \cdots + P_{km},$$
$$\Pi_{2m} = \alpha P_2^T\overline{\mathbb{A}}_m - \alpha Y_m\mathbb{I} - P_{2m}^T,$$
$$\Pi_{3m} = -P_{k1} - P_{3m}^T,$$
$$\Pi_{km} = -P_{km} - P_{km}^T - (1-d_m)Q_m,$$

and $\mathbb{Z}_0 = P_2^{-T}Y_0, \dots, \mathbb{Z}_m = P_2^{-T}Y_m$.

Proof: To prove this theorem, let us consider the following L–K functional:

$$V(t) = \xi^T(t)P_1\xi(t) + \sum_{i=1}^{m}\left(\int_{t-\tau_i(t)}^{t}\xi^T(s)Q_i\xi(s)\,\mathrm{d}s + \int_{-\tau_i(t)}^{0}\int_{t+\theta}^{t}\dot{\xi}^T(s)R_i\dot{\xi}(s)\,\mathrm{d}s\,\mathrm{d}\theta\right) \tag{22}$$

Taking into account the derivative of the proposed L–K functional (22), we have:

$$\dot{V}(t) = 2\xi^T(t)P_1\dot{\xi}(t) + \sum_{i=1}^{m}\Big(\xi^T(t)Q_i\xi(t) - \xi^T(t-\tau_i(t))Q_i\xi(t-\tau_i(t)) + \tau_i(t)\dot{\xi}^T(t)R_i\dot{\xi}(t) - \int_{t-\tau_i(t)}^{t}\dot{\xi}^T(s)R_i\dot{\xi}(s)\,\mathrm{d}s\Big)$$

From Equation (20) and the Newton–Leibniz formula, we can write:

$$2\xi^T(t)P_1\dot{\xi}(t) = 2\begin{bmatrix}\xi(t)\\ \dot{\xi}(t)\\ \xi(t-\tau_1(t))\\ \vdots\\ \xi(t-\tau_m(t))\end{bmatrix}^T \times \begin{bmatrix}P_1^T & P_2^T & P_{11}^T & \dots & P_{1m}^T\\ 0 & P_3^T & P_{21}^T & \dots & P_{2m}^T\\ 0 & 0 & P_{31}^T & \dots & P_{3m}^T\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & P_{k1}^T & \dots & P_{km}^T\end{bmatrix}\begin{bmatrix}\dot{\xi}(t)\\ 0\\ 0\\ \vdots\\ 0\end{bmatrix}$$

$$= 2\begin{bmatrix}\xi(t)\\ \dot{\xi}(t)\\ \xi(t-\tau_1(t))\\ \vdots\\ \xi(t-\tau_m(t))\end{bmatrix}^T\begin{bmatrix}P_1^T & P_2^T & P_{11}^T & \dots & P_{1m}^T\\ 0 & P_3^T & P_{21}^T & \dots & P_{2m}^T\\ 0 & 0 & P_{31}^T & \dots & P_{3m}^T\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & P_{k1}^T & \dots & P_{km}^T\end{bmatrix} \times \begin{bmatrix}\dot{\xi}(t)\\ \Upsilon\\ \xi(t)-\xi(t-\tau_1(t))-\int_{t-\tau_1(t)}^{t}\dot{\xi}(s)\,\mathrm{d}s\\ \vdots\\ \xi(t)-\xi(t-\tau_m(t))-\int_{t-\tau_m(t)}^{t}\dot{\xi}(s)\,\mathrm{d}s\end{bmatrix}$$

where $\Upsilon = -\dot{\xi}(t) + (\overline{\mathbb{A}}_0 - \mathbb{Z}_0\mathbb{I})\xi(t) + \sum_{i=1}^{m}(\overline{\mathbb{A}}_i - \mathbb{Z}_i\mathbb{I})\xi(t-\tau_i(t)) + \mathbb{B}_w w(t)$.

Then, we have:

$$2\xi^T(t)P_1\dot{\xi}(t) = 2\begin{bmatrix}\xi(t)\\ \dot{\xi}(t)\\ \xi(t-\tau_1(t))\\ \vdots\\ \xi(t-\tau_m(t))\end{bmatrix}^T\begin{bmatrix}P_1^T & P_2^T & P_{11}^T & \dots & P_{1m}^T\\ 0 & P_3^T & P_{21}^T & \dots & P_{2m}^T\\ 0 & 0 & P_{31}^T & \dots & P_{3m}^T\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & P_{k1}^T & \dots & P_{km}^T\end{bmatrix} \times \left(\begin{bmatrix}0 & I & 0 & \dots & 0\\ \overline{\mathbb{A}}_0 - \mathbb{Z}_0\mathbb{I} & -I & \overline{\mathbb{A}}_1 - \mathbb{Z}_1\mathbb{I} & \dots & \overline{\mathbb{A}}_m - \mathbb{Z}_m\mathbb{I}\\ I & 0 & -I & \dots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ I & 0 & 0 & \dots & -I\end{bmatrix} \times \begin{bmatrix}\xi(t)\\ \dot{\xi}(t)\\ \xi(t-\tau_1(t))\\ \vdots\\ \xi(t-\tau_m(t))\end{bmatrix} + \begin{bmatrix}0\\ 0\\ -I\\ \vdots\\ 0\end{bmatrix}\int_{t-\tau_1(t)}^{t}\dot{\xi}(s)\,\mathrm{d}s + \dots\right.$$

$$+\begin{bmatrix}0\\0\\0\\\vdots\\-I\end{bmatrix}\int_{t-\tau_m(t)}^{t}\dot{\xi}(s)\,\mathrm{d}s+\begin{bmatrix}0\\\mathbb{B}_w\\0\\\vdots\\0\end{bmatrix}w(t)\Bigg)$$

$$=\eta^T(t)\left(\Psi\eta(t)+2\Psi_1\int_{t-\tau_1(t)}^{t}\dot{\xi}(s)\,\mathrm{d}s+\cdots+2\Psi_2\int_{t-\tau_m(t)}^{t}\dot{\xi}(s)\,\mathrm{d}s+2\Psi_3 w(t)\right) \tag{23}$$

where

$$\Psi=\begin{bmatrix}\Omega_{11}&\Omega_{12}&\Psi_{11}&\dots&\Psi_{1m}\\ *&\Omega_{22}&\Psi_{21}&\dots&\Psi_{2m}\\ *&*&\Psi_{31}&\dots&\Psi_{3m}\\ \vdots&\vdots&\vdots&\ddots&\vdots\\ *&*&*&\dots&\Psi_{km}\end{bmatrix},$$

$$\Psi_1=\begin{bmatrix}-P_{11}^T\\-P_{21}^T\\-P_{31}^T\\\vdots\\-P_{k1}^T\end{bmatrix},$$

$$\Psi_2=\begin{bmatrix}-P_{1m}^T\\-P_{2m}^T\\-P_{3m}^T\\\vdots\\-P_{km}^T\end{bmatrix},\quad \Psi_3=\begin{bmatrix}P_2^T\mathbb{B}_w\\P_3^T\mathbb{B}_w\\0\\\vdots\\0\end{bmatrix},$$

and

$$\Omega_{11}=(\overline{\mathbb{A}}_0-\mathbb{Z}_0\mathbb{I})^TP_2+P_2^T(\overline{\mathbb{A}}_0-\mathbb{Z}_0\mathbb{I})+P_{11}+\cdots+P_{1m}+P_{11}^T+\cdots+P_{1m}^T,$$
$$\Omega_{12}=(\overline{\mathbb{A}}_0-\mathbb{Z}_0\mathbb{I})^TP_3+P_1^T-P_2^T+P_{21}+\cdots+P_{2m},\quad \Omega_{22}=-P_3-P_3^T,$$
$$\Psi_{11}=P_2^T(\overline{\mathbb{A}}_1-\mathbb{Z}_1\mathbb{I})-P_{11}^T+P_{31}+\cdots+P_{3m},\quad \Psi_{21}=P_3^T(\overline{\mathbb{A}}_1-\mathbb{Z}_1\mathbb{I})-P_{21}^T,$$
$$\Psi_{31}=-P_{31}-P_{31}^T,\quad \Psi_{1m}=P_2^T(\overline{\mathbb{A}}_m-\mathbb{Z}_m\mathbb{I})-P_{1m}^T+P_{k1}+\cdots+P_{km},$$
$$\Psi_{2m}=P_3^T(\overline{\mathbb{A}}_m-\mathbb{Z}_m\mathbb{I})-P_{2m}^T,$$
$$\Psi_{3m}=-P_{k1}-P_{3m}^T,\quad \Psi_{km}=-P_{km}-P_{km}^T.$$

Using Equation (23), the Schur complement, and Lemma 2.1, we obtain:

$$\dot{V}(t)+\frac{1}{\gamma}e^T(t)e(t)-w^T(t)w(t)$$
$$\le\begin{bmatrix}\eta(t)\\\int_{t-\tau_1(t)}^t\dot{\xi}(s)\,\mathrm{d}s\\\vdots\\\int_{t-\tau_m(t)}^t\dot{\xi}(s)\,\mathrm{d}s\\w(t)\end{bmatrix}^T\begin{bmatrix}\Pi&\Psi_1&\dots&\Psi_2&\Psi_3\\ *&\frac{-R_1}{h_{\tau_1}}&\dots&0&0\\ \vdots&\vdots&\ddots&\vdots&\vdots\\ *&*&\dots&\frac{-R_m}{h_{\tau_m}}&0\\ *&*&\dots&*&-I\end{bmatrix}\times\begin{bmatrix}\eta(t)\\\int_{t-\tau_1(t)}^t\dot{\xi}(s)\,\mathrm{d}s\\\vdots\\\int_{t-\tau_m(t)}^t\dot{\xi}(s)\,\mathrm{d}s\\w(t)\end{bmatrix} \tag{24}$$

where

$$\Pi=\begin{bmatrix}\Omega_{11}+\sum_{i=1}^m Q_i+\frac{1}{\gamma}\mathbb{C}_e^T\mathbb{C}_e&\Omega_{12}&\Psi_{11}&\dots&\Psi_{1m}\\ *&\Omega_{22}+\sum_{i=1}^m h_{\tau_i}R_i&\Psi_{21}&\dots&\Psi_{2m}\\ *&*&\Pi_{31}&\dots&\Pi_{3m}\\ \vdots&\vdots&\vdots&\ddots&\vdots\\ *&*&*&\dots&\Pi_{km}\end{bmatrix}$$

It is clear that if $\Pi<0$, then

$$\dot{V}(t)+\frac{1}{\gamma}e^T(t)e(t)-w^T(t)w(t)<0 \tag{25}$$

Introducing some changes of variables: $P_3=\alpha P_2$, $P_2^T\mathbb{Z}_0=Y_0,\dots,P_2^T\mathbb{Z}_m=Y_m$, and using the Schur complement to Equation (24), we obtain the condition (21).

Since the matrix (21) holds (using $w(t)=0$), Equation (25) implies that $\dot{V}(t)<0$ and the system (20) is asymptotically stable. Now, integrating both sides of Equation (25) from 0 to ∞ where $w(t)\neq 0$, we have:

$$V(\infty)-V(0)+\int_0^\infty\left(\frac{1}{\gamma}e^T(t)e(t)-w^T(t)w(t)\right)\mathrm{d}t<0$$

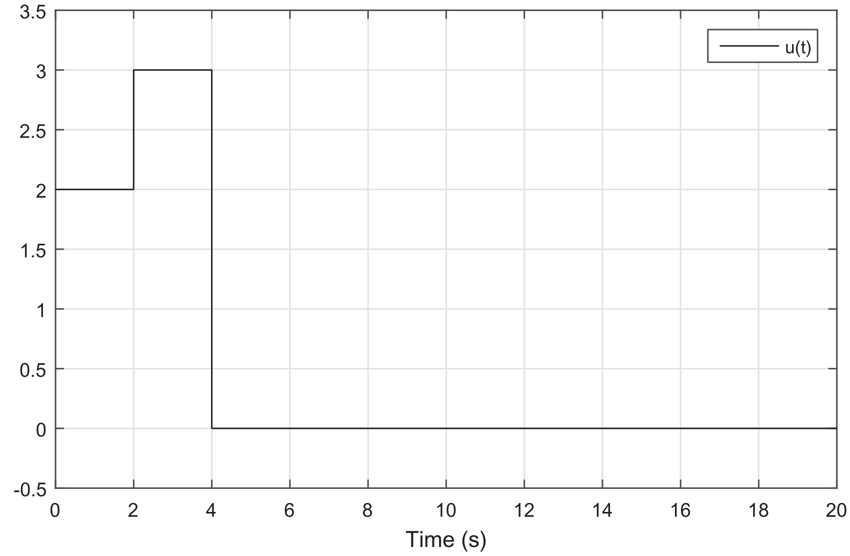

Figure 1. Control input used in the simulations.

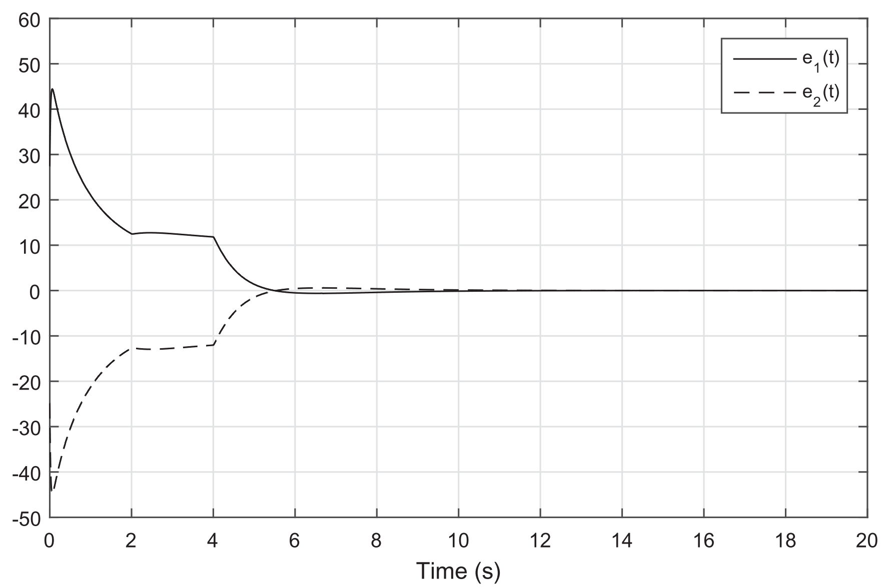

Figure 2. Evolution of the error variables.

Then, we can conclude that Equation (3) is verified thanks to the asymptotic stability ($V(\infty) = 0$) and the zero initial condition ($V(0) = 0$). The proof is finally completed. ■

Remark 3.1: It must be pointed out that when we were deriving Theorem 3.1 we took into account that P_1 contains the free matrices P_1, P_2, P_3, $P_{11}, \ldots, P_{1m}$, $P_{21}, \ldots, P_{2m}$, $P_{31}, \ldots, P_{3m}, \ldots, P_{km}$, $P_{31}, \ldots, P_{k1}, \ldots$, P_{km}. For this reason, our results is more general than the ones of the literature (Da Silva et al., 2011; El Fezazi et al., 2017; Zabari & Tissir, 2014) since they provide more degree of freedom. On the other hand, we can use a numerical optimisation algorithm to find the optimal value of α.

4. Design algorithm

With all these results, the design procedure of the proposed observer can be obtained as follows:

(1) Choose the matrix E according to the condition (8);
(2) Compute the matrices T and K from Equation (10);

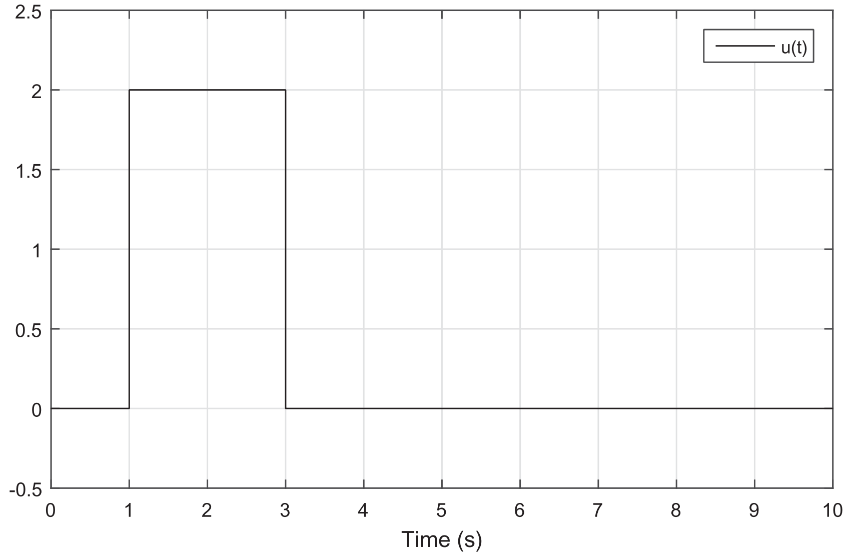

Figure 3. Control input used in the simulations.

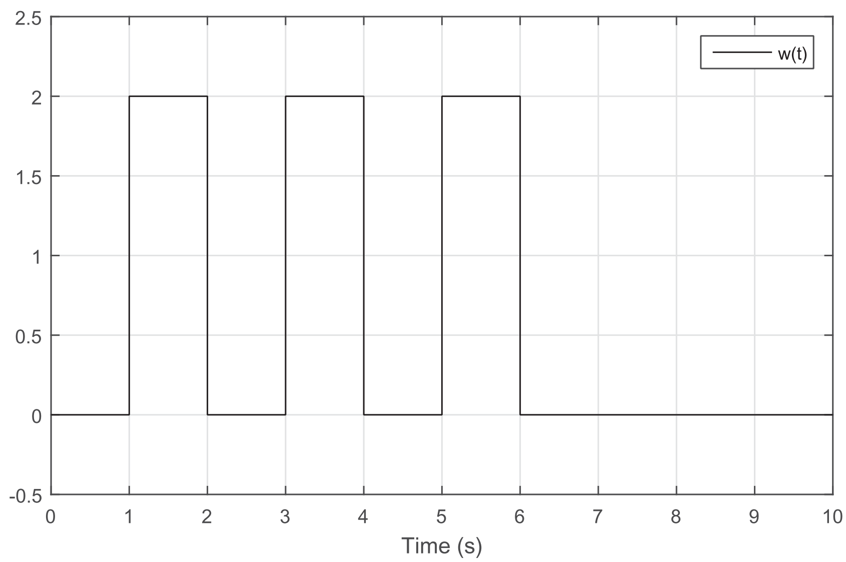

Figure 4. Disturbance used in the simulations.

(3) Compute α_1, α_2, α_3, β_1, β_2, and β_3 from the conditions (14) and (19);

(4) Compute the matrices $\mathbb{Z}_0, \ldots, \mathbb{Z}_m$ by solving LMI (21);

(5) Deduce all the matrices $N_0, \ldots, N_m$, $J_0, \ldots, J_m$, H, $M_0, \ldots, M_m$, $P_0, \ldots, P_m$, $Q_0, \ldots, Q_m$, $G_0, \ldots,$ G_m, R, and S from the conditions (4), (13), (16), and (18).

5. Illustrative examples

The feasibility and performance of the observer design approach proposed in this paper are illustrated in this section by two numerical examples.

Example 5.1: Consider the delayed system (1) where $m = 1$ and:

$$A_0 = \begin{bmatrix} -0.2 & 0 \\ 0 & -0.4 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -0.5 & 0.2 \\ 0 & -0.3 \end{bmatrix},$$

$$B_0 = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix},$$

$$C_y = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad E = \begin{bmatrix} 0.3 & 0.25 \\ 0.15 & 0.4 \end{bmatrix},$$

$$\alpha = 0.2, \quad d_1 = 0.1.$$

Then, Theorem 3.1 is solvable for $h_{\tau_1} = 10$ using the MATLAB LMI toolbox.

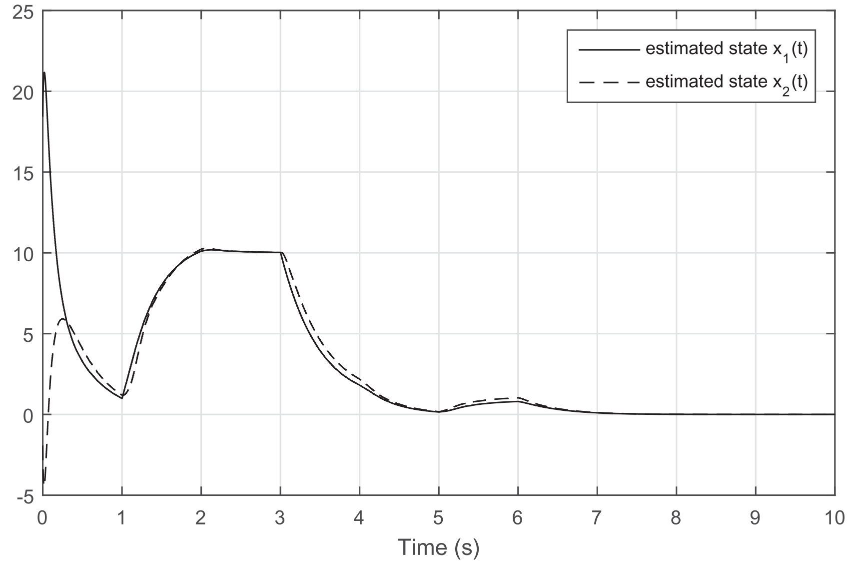

Figure 5. Evolution of the state variables (H_∞ DO).

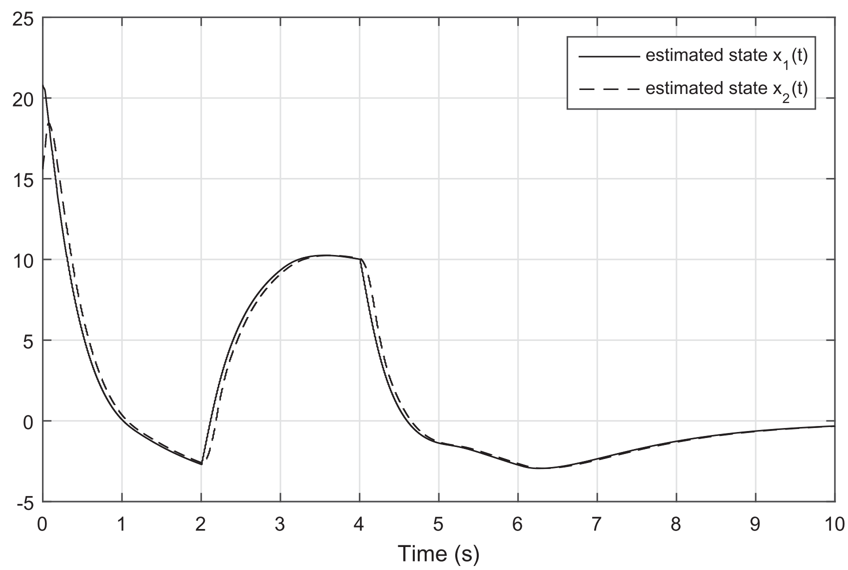

Figure 6. Evolution of the state variables (PO).

The simulation results are shown in Figures 1 and 2. Figure 1 represents the control input and Figure 2 represents the error variables evolution where the initial states used in this example are $x_0 = [-15 \quad 15]^T$.

From these figures we can see that the trajectories tend to zero asymptotically. Then, the simulation results show the ability of our generalised observer to have good performances.

Example 5.2: The system (1) is reconsidered in this example where its matrices are given by ($m = 3$):

$$A_0 = \begin{bmatrix} -0.1 & -0.4 \\ 9 & -9 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -0.2 & -0.3 \\ 2 & -2 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -0.4 & -0.1 \\ 4 & -4 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -0.3 & -0.2 \\ 3 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 10 \\ 0 \end{bmatrix},$$

$$B_w = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C_y = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

The use of LMI tools of MATLAB, the application of the conditions obtained in this article, and the choice of the matrix $E = \left[\begin{smallmatrix} 0.5 & 0.2 \\ 0.4 & 0.7 \end{smallmatrix}\right]$, allow us to obtain $\gamma = 0.0193$ where $h_{\tau_1} = h_{\tau_2} = h_{\tau_3} = 0.1$, $\alpha = 0.9$, and $d_1 = d_2 = d_3 = 0.1$.

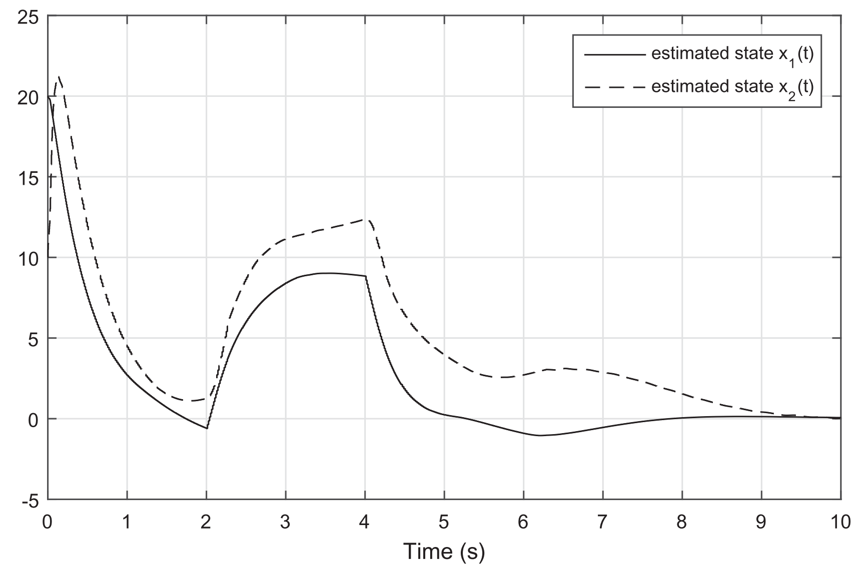

Figure 7. Evolution of the state variables (PIO).

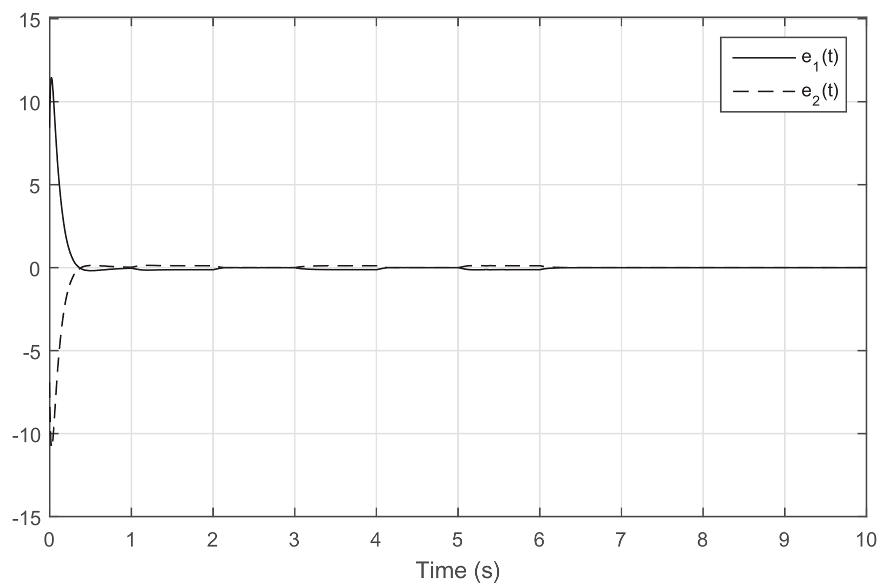

Figure 8. Evolution of the error (H_∞ DO).

In order to study the convergence performances of our observer, we make some simulations with the H_∞ DO, the PO, and the PIO. Then, we compare the proposed observer with the designed PO and PIO, in the presence of disturbance. Thus, the simulation results are given in Figures 5–10 to represent the evolution of the state and error variables where the control input and the disturbance are given in Figures 3 and 4, respectively. The initial states used in these simulations are $x_0 = [10 \quad 5]^T$.

From the figures, we can clearly see that our methodology affords better performances with a shorter convergence time and rejects the disturbance at a faster rate. In general, the H_∞ DO adopted in this paper has better behaviour than the existing observers PO and PIO and finally the validity of our approach is confirmed.

6. Conclusion

The design problem of an H_∞ DO that is a generalisation of the PO, the PIO, and the DO for multi-delayed systems affected by disturbances is studied in this paper. This problem is solved from the solution of an LMI, based on the parametrization results of

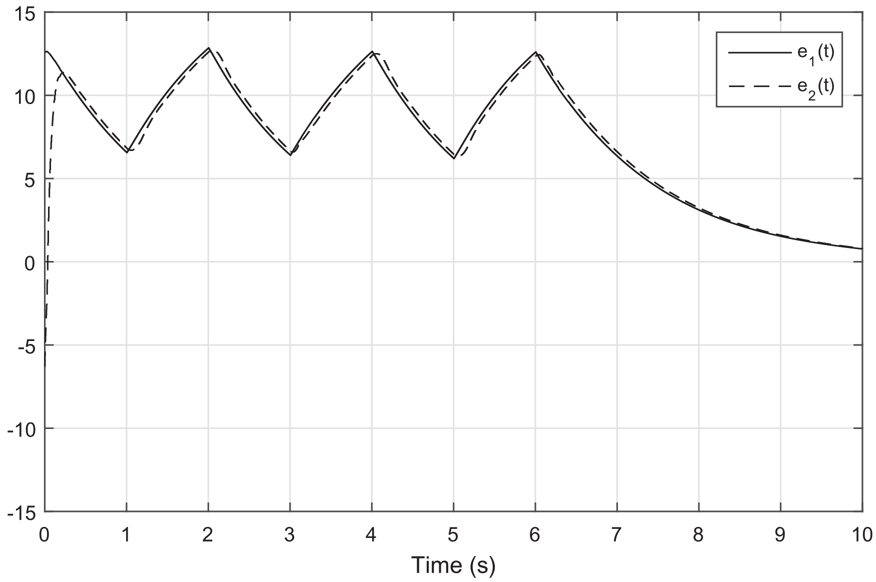

Figure 9. Evolution of the error (PO).

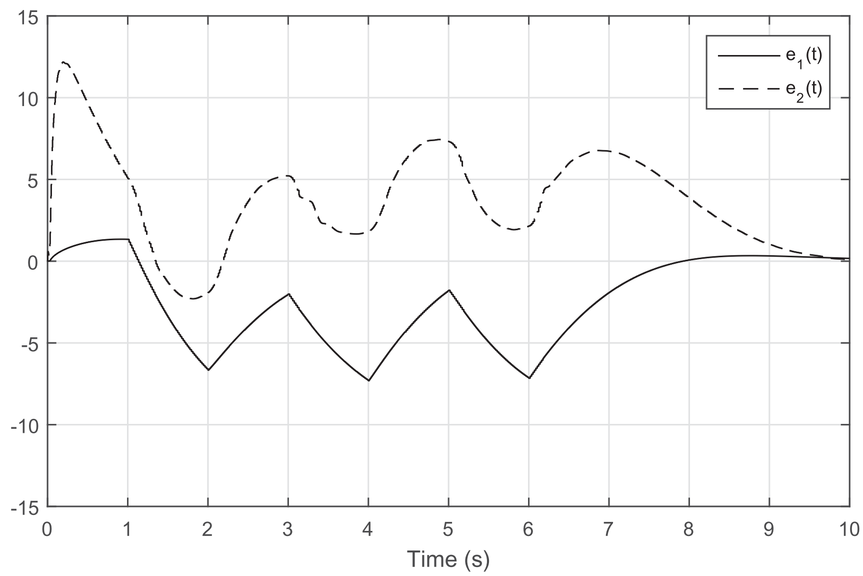

Figure 10. Evolution of the error (PIO).

algebraic constraints obtained from the unbiased estimation error. The observer design approach presented in this paper has a wide range of applicability, as the class of systems investigated appears in many process control applications.

The proposed approach opens new lines of research: In the near future, we aim to extend the proposed methodology to related systems such as singular, T-S fuzzy, etc.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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