Revisiting the vegetation hot spot modeling: Case of Poisson/Binomial leaf distributions

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A B S T R A C T

The accuracy of spaceborne/airborne sensor measurements in the solar domain keeps increasing over time. High resolution, multi-directional and hyperspectral image acquisitions start to be abundant. With regard to the multi-angular remote sensing data, the hot spot, i.e. the exact backscattering direction of direct sunlight together with its neighboring directions, is of special interest. Accurate hot spot models have to be used to adequately simulate the hot spot signature and to allow reliable inversion of multi-angular data. In this paper, we propose a physical hot spot model (Leaf Spatial Distribution based Model, LSDM) assuming that for a given point inside the vegetation to be sunlit (respectively, observed) it should be located within a cylinder free from leaf centers. The cylinder is oriented to the sun (respectively, sensor) direction. Assuming a leaf random, regular or clumped spatial distributions, the gap probabilities in the sun and sensor directions are expressed as a function of these cylinder volumes. Based on the same hypothesis, the bidirectional gap probability is estimated as a function of the total volume of the two cylinders. The evaluation of the needed common volume of two cylinders having different radii is reduced to calculation of some elliptic integrals. Finally, the hot spot signature is estimated based on the bidirectional gap probability distribution. Different model versions with different leaf spatial distribution functions are compared. Particularly, it is shown that compared to the random distribution, the regular (the clumped, respectively) distribution increases (decreases, respectively) the reflectance factor of the canopy.

1. Introduction

The results of spaceborne multi-angular remote sensing measurements have been available at least during the last 15 years. New sensors such as CHRIS, which is an imaging spectrometer carried on board the space platform PROBA, allow high resolution multi-angular and hyperspectral acquisitions. In terms of multi-angular observations, this sensor can be pointed off-nadir in both along-track and across-track directions. The sophistication of such instruments keeps increasing over time, particularly the sensor agility is a hot topic. It allows to rapidly acquire off-nadir targets, in order to sequence images of the same area in different observation angles leading to sampling of the directional reflectance factor of the canopy e.g., Pleiades-HR constellations (Lebegue et al., 2010). Although the hot spot region, corresponding to the bright area close to the exact backscattering direction, has been recognized as a potentially informative angular region, a majority of existing instruments with multi-angular capability do not currently measure in the hot spot direction. However, a considerable amount of images showing the hot spot region as well as its angular signature have been recorded in the last two decades. For instance, the airborne version of the Multi-angle Imaging SpectroRadiometer (AirMISR) (Gerstl et al., 1999), and the spaceborne Polarization and Directionality of the Earth’s Reflectances (POLDER) (Grant et al., 2003) instruments provided the Bidirectional Reflectance Distribution Function (BRDF) signatures that included the hot spot region. Recent studies have shown that a number of biophysical features can be retrieved from a sampled BRDF (just two or three parameters per inversion). For instance, one can cite the canopy architecture (Schleef & Atzberger, 2006) (i.e., the tree spatial distribution, canopy cover, leaf area index), the tree macro structure (Möttus et al., 2006) (e.g., tree height, the size and shape of the crowns and leaves), the understory reflectance (Canisius & Chen, 2007) and the clumping index (He et al., 2012). As the main aim of canopy remote sensing is to derive canopy properties (Comb et al., 2002) from the BRDF, it is important to adequately model it as a function of canopy features and scene geometry. For that a lot of theoretical works are trying to increase the accuracy of the BRDF modeling (Widlowski et al., 2006b), therefore the proposed approach complexities keep increasing and the models are becoming time consuming, particularly those based on Monte Carlo (MC) ray-tracing (for which the inversion is not
practical) (Widlowski et al., 2006b). Especially the hot spot, which needs simulating light paths in a canopy layer considering its 3D architecture, is known to provide more reliable results than 1D models, which approximate the architecture by considering vertical variations only. In particular, the traditional 1D models assume a random leaf distribution (Poisson distribution) neglecting any scales of leaf spatial organization (Widlowski et al., 2005). So, increasing the performance of 1D modeling of the hot spot signature can be a step towards better retrieval of canopy features (Liang et al., 2000).

Before being studied in the vegetation canopy, the hot spot effect was known one century ago. In fact, in 1887, Seeliger (1887) proposed the first hot spot model to interpret the phase curve of the Saturn rings showing a maximum brightness at the opposition direction and the phenomenon was known as the opposition effect. Seeliger assumed that the rings’ particles have a spherical shape. To be sunlit and observed, such particles should be contained in cylinders free from other particles having the same radius as one particle with cylinders’ axis respectively in sun and observation directions (cf. Fig. 1, in the particular case $r_s = r_o$). Thus, the hot spot problem was reduced to the estimation of the common volume of the two cylinders. After that a lot of studies were carried out to enhance this model (for more details, the reader is referred to Kuusk (1991)). In his study of the moon surface in 1963 and 1986, Hapke (1963, 1986) proposed a different method to model the hot spot effect. He assumed that the moon surface contained cylindrical pores. Afterward, the developed model was applied to investigate the BRDF of terrestrial bare soil surfaces (Pinty et al., 1989). However, in their study of the coherent backscattering calculation, Liang and Mishchenko (1997) and Mishchenko (1992) showed that this theory is not valid for such surfaces composed of fine particles that do not have well-defined shadows.

Canopy components (e.g., leaves or needles) can be assumed to be considerably larger than the wavelength and almost opaque, so they cast shadows on neighboring components. This way, the hot spot theory can be applied. As far as we know, Suits (1972) was the first to include such a phenomenon in his study of the canopy BRDF in 1972. He simply inserted an empirical correction term to the bidirectional gap probability expression. Then, a lot of theoretical works trying to model the physical phenomenon have been proposed. For example, Nilson (1977, 1980) has analytically described the bidirectional gap probabilities and the part of the hot spot effect caused by sunlit crowns and ground. In 1985, Kuusk proposed the first theoretical hot spot model adapted to a homogeneous vegetation canopy. He divided the vegetation layer into thin superposed sublayers, assumed independent in terms of leaf positions, and computed the bidirectional gap correlation in each sublayer. Then, by integration over the vegetation depth, the bidirectional gap probability was derived. Based on the Kuusk theory, Jupp and Strahler (1991) and Qin (1993) proposed to replace the joint gap correlation by a Boolean model and an overlap function, respectively. These canopy hot spot models and others are summarized and compared in Qin and Goel (1995). Such approaches suffer from the assumption of independence between sublayers. In fact, it is true that a horizontal leaf may belong only to one thin sublayer. However, the more inclined is the leaf, the larger number of successive sublayers it passes through and the assumption is violated. To overcome such a problem, Kallel (2010a, 2010b, 2012) proposed in his model, FDM/FDM2, the calibration of the hot spot factor (which is originally defined as the ratio between the leaf radius and the vegetation height) for each leaf distribution (e.g., for the planophile and eucryptophile distributions, the original parameter is multiplied by 1.5 and 3, respectively).

As first proposed by Seeliger, Verstraete et al. (1990) developed a hot spot model for semi-infinite canopies assuming that to be sunlit and observed, a given point should belong to cylinders free from leaves with horizontal circular bases in sun and observation directions (cf. Fig. 1). In this case, the radii of the two cylinders were assumed equal to the radius of an average equivalent horizontal sun fleck [the distribution of sunlit holes along a horizontal plane belonging to the vegetation layer (Miller & Norman, 1971)]. To extend it to the finite canopy case, Cobron et al. (1997) proposed some modifications to the approach. In particular, they derived a new formulation of the size of holes between leaves that are assumed disk-shaped.

The traditional hot spot model assumes leaves are randomly distributed (Poisson law) within a considered vegetation layer without any constraint about their spatial organization. However, different levels of foliage clustering could occur as already shown in a number of works dealing with gap frequencies within forest canopies (Nilson, 1999; Pisek et al., 2011; Wang et al., 2007). According to Nilson (1999), a stand is composed by a number of trees covering the area, the tree distribution could be Poisson as well as Binomial (i.e., regular or clumped). Moreover, inside a given crown, two kinds of foliage clustering could be considered: shoot-level and structures larger than shoot-level, e.g., crown level (Wang et al., 2007). In a number of works (e.g., Pisek et al. (2011)), all these structurations are taken into account in the derivation of the gap frequency, and allow inversion of LAI based on these frequency measures (Nilson, 1999). According to Pisek et al. (2011), such foliage organization has a huge impact on the gap fraction, which implies an important variation of the radiative regime and therefore the canopy reflectance in comparison with the Poisson case. The radiative transfer problem corresponding to media composed by big structures (e.g. stand or crown levels) is complex and can be solved using Monte Carlo 3-D models as they allow the simulation of heterogeneous medium reflectances (Widlowski et al., 2006b). However, when the clusters are too small (e.g. shoot level), the considered medium can be assumed homogeneous and radiative regime could be approximated using just 1-D models as it will be shown in this paper.

Our proposed approach is as follows. The assumption that an observed or sunlit point is belonging to a cylinder free from leaves is adopted. However, compared to Verstraete et al. (1990) or Gobron et al. (1997), where cylinder bases are equal to the sun fleck (the mean sunlit surface within the vegetation), the cylinders in our case are the smallest volumes free from leaf centers to ensure that a given point within the vegetation is sunlit or observed. Therefore,
the probabilities that the point is sunlit or observed (i.e., the gap fraction) are none other than the probabilities that the cylinders are free from leaf centers. Such probabilities depend on the cylinder volumes and the leaf spatial distribution. The latter could be totally random (Poisson) as assumed in traditional hot spot model (Gobron et al., 1997; Kuusk, 1985; Verstraete et al., 1990) or extended to the Binomial distribution (regular or clumped), as our model offers the possibility to deal with any spatial distribution. Following the same reasoning, it is possible to estimate the bidirectional gap probability as it corresponds to the probability that both cylinders (in sun and observation directions) are free from leaf centers. It is noted here that cylinders share a common volume which explains why the probability to be sunlit and observed is dependent. It is noted here that this link between gap and bidirectional gap probabilities was not done in the models of Verstraete et al. (1990) and Gobron et al. (1997). Indeed, they only propose a correction of the bidirectional gap probability computed assuming the independence between sun and sensor paths based on some normalization by the ratio between the two cylinder total volume and the sum of their volumes.

This paper is structured as follows. First, we present the theoretical background (Section 2). Then, we provide model experimental results (Section 3). Finally, we present our main conclusions and future perspectives (Section 4).

2. Theoretical background

In this section, we will present the theoretical basis for estimation of the gap fraction, the bidirectional gap distributions and the reflectance in the homogeneous vegetation canopy.

2.1. Estimation of gap probability

All along this section, a discrete vegetation layer located between the depth $-h$ and 0 with a given leaf area index (LAI) is considered. The layer is assumed homogeneous, so the spatial distribution of leaf centers, leaf angular and area distributions are assumed the same for each point within the vegetation layer. Leaves are assumed to be flat disks with a given size distribution. However, in this work, leaf size distribution is ignored and all leaves are supposed to be disks of identical size, the radius of the leaf of mean area is denoted by $r_l$. The leaf spatial distribution is described by the leaf center positions. The mean number of leaf centers per m$^2$ or the leaf number volume density, $n_l$, is given by

$$n_l = \frac{\text{LAI}}{h \pi r_l^2}. \quad (1)$$

The spatial leaf center distribution within the vegetation canopy could be simulated by a Poisson process, negative or positive Binomial distributions, as suggested by Nilson (1971). The Poisson distribution assumes an ‘entirely’ random distribution of leaf centers (i.e., no assumption about any systematic pattern in the leaf positions). For a given spatial region $V$ of volume $v$, the probability to find $m$ leaf centers is given by

$$p_{\text{poi},m}(m,v) = \frac{(n_l v)^m \exp(-n_l v)}{m!}. \quad (2)$$

Conversely, the Binomial distributions could be interpreted as having a vertical stratification of the foliage into independent spatial regions ($\Delta v_i$), but dependent leaf position inside each one. Two kinds of dependency can be assumed: (i) positive Binomial distribution ($p_+$) assuming regular foliage dispersion; and (ii) negative Binomial distribution ($p_-$) describing a clumped dispersion of foliage. They are modeled by Nilson (1971)

$$p_+(m,v) = C^m_N n_l \Delta v^m (1-n_l \Delta v)^N-m; \quad m = 0, ..., N,$$

$$p_-(m,v) = C^m_{N-1} n_l \Delta v^m (1+n_l \Delta v)^{-N-m}; \quad m = 0, ..., N, \quad (3)$$

where

$$N = \frac{v}{\Delta v}. \quad (4)$$

One could notice that, for $N$ real, Eq. (3) would not be valid. However, as we are interested in the probability of gap that it will be shown that it is none other than the probability that a given volume is empty and therefore can be estimated using Eq. (3). For $m=0$, the validity problem will not be encountered since for any value of $N$, $C^0_N = C^0_{N-1} = 1$, otherwise an extension to the real number case must be proposed.

Now, let $M$ be a point inside the vegetation at depth $z=[-h, 0]$ receiving sunlight from a direction $\Omega = (\theta, \phi)$. Let $V$ (of volume $v$) be a cylinder located from the depth $z$ to the top of the layer. The cylinder has its axis oriented at $\Omega_l$ and horizontal base in a form of a disk, $S$, with center $M$, area $s$ and radius $r=0$. Then, according to Eqs. (2) and (3), the probabilities that $V$ is free of leaf centers, $p_c(...z)$ ($\{\in pos, +, -\}$ corresponding to the Poisson, positive and negative Binomial distributions), are given by

$$p_{c,\text{poi}}(s,z) = \exp(-n_l v), \quad (5)$$

$$p_{c_+}(s,z) = (1-n_l \Delta v)^\frac{s}{\Delta v}, \quad (6)$$

$$p_{c_-}(s,z) = (1+n_l \Delta v)^\frac{s}{\Delta v}. \quad (7)$$

The volume $v$ of $V$ is a function of $r$ and $|z|$. It is given by

$$v = \pi r^2 |z|. \quad (8)$$

Note that, $p_{c_+}$ is different from the probability of a gap covering the whole area of $S$ (i.e. the probability that the entire $S$ be sunlit), $p_{\text{gap}_+}(s,z)$. Although no leaf centers are in the cylinder $V$, it remains possible that parts of leaves with their center outside of $V$ can block some of the radiation inside $V$. Thus, obviously

$$p_{\text{gap}_+} < p_{c_+}. \quad (9)$$

In the particular case, when the disk $S$ is reduced to only one point located at altitude $z$, $p_{c_-}(0,z) = 1$, whereas $p_{\text{gap}_+}(0,z)$ could not be equal to one, otherwise all points inside the vegetation would be sunlit. In the following, we will present a method to express $p_{\text{gap}_+}(0,z)$.

Let us consider a disk-shaped leaf inside the canopy of surface $S_l = \pi r^2$ ($r_l$ the leaf radius) and with normal orientation $\Omega_l(\theta_l, \phi_l)$. The leaf is illuminated from the direction $\Omega$. Its projection on the horizontal plane has the area $s_l$, such that

$$s_l = \frac{\Omega_l \Omega}{\mu_l} S_l. \quad (10)$$

Thus, the average of $s_l$ over all leaf normal orientations, $S_{\text{HIS}}$, which defines the mean projection area of a leaf on the horizontal plane, is given by Nilson (1971)

$$S_{\text{HIS}} = \frac{G}{\mu_l} s_l. \quad (11)$$
with $G_s$ the Ross–Nilson geometry function (G-function) (Nilson, 1971). It is clear that for each leaf orientation, $\Omega$, the leaf projection on the horizontal plane, $S_{H\Omega}(\Omega)$, is generally elliptically-shaped and taking into account all the possible leaf orientations, the mean surface, $S_{H\Omega}$, is a complex geometrical form. However, for the sake of simplicity, we assume that $S_{H\Omega}$ are disks, $S_{H\Omega}$, having all the same area ($S_{H\Omega}$). The corresponding disk radius $r_{H\Omega}$ is given by

$$r_{H\Omega} = \sqrt{\frac{G_s}{\mu_s}}. \quad (11)$$

Now, let us derive $p_{gap\,+(0,z)}$ for a point $M$ inside the vegetation at altitude $z$. To ensure that $M$ is observed from the direction $\Omega_s$, no leaves should cross the line $D$ of direction $\Omega_s$ linking $M$ to the top of the canopy. Therefore, it is necessary that for any altitude $t \in [0,0]$, the distance between the line $D$ and the centers of leaves be larger than $r_{gap}$. Then, to make sure that $M$ is in the gap when viewed from the direction $\Omega$, $M$ should be inside a cylinder free from leaf centers of axis $D$ and radius $r_{gap}$. The cylinder volume ($v_{gap}$) satisfies

$$v_{gap} = \pi r_{gap}^2 |z|. \quad (12)$$

So, the probabilities that $M$ is in a gap are given by

$$p_{gap\,+(0,z)} = \exp\left[-n_{+}v_{gap}\right],$$

$$p_{gap\,-(0,z)} = \exp\left[-n_{-}(\Delta\nu)v_{gap}\right], \quad (13)$$

$$p_{gap\,-(0,z)} = \exp\left[-n_{-}(\Delta\nu)v_{gap}\right].$$

with $n_{+}$ and $n_{-}$ the equivalent number of leaf centers per unit volume for the positive and negative Binomial distributions, respectively,

$$n_{+}(\Delta\nu) = -\frac{\log(1-n_{+}\Delta\nu)}{\Delta\nu},$$

$$n_{-}(\Delta\nu) = \frac{\log(1+n_{-}\Delta\nu)}{\Delta\nu}. \quad (14)$$

Considering the Poisson distribution and combining Eqs. (11)–(13) give

$$p_{gap\,+(0,z)} = \exp\left(n_{+}G_s \mu_s z\right) = \exp\left(n_{+}G_s \mu_s z \frac{\pi}{\mu_s} \right),$$

$$p_{gap\,-(0,z)} = \exp\left(n_{-}G_s \mu_s z \frac{\pi}{\mu_s} \right), \quad (15)$$

with $k$, the extinction coefficient, as defined by Verhoef (1984)

$$k = \frac{G_s \mu_s A_l}{\mu_s \pi}. \quad (16)$$

Similarly to the Poisson distribution, for the Binomial ones, the gap probabilities for a point are given by

$$p_{gap\,+}(0,z) = \exp(k_{+}z),$$

$$p_{gap\,-}(0,z) = \exp(k_{-}z). \quad (17)$$

with $k_{+}$ and $k_{-}$ are respectively the extinction in the positive and negative Binomial distribution cases and given by

$$k_{+} = \frac{n_{+}(\Delta\nu)}{n_{+} \frac{\pi}{\mu_s}},$$

$$k_{-} = \frac{n_{-}(\Delta\nu)}{n_{-} \frac{\pi}{\mu_s}}. \quad (18)$$

As we can see from Eqs. (15) to (17), the shape does not have any influence on the estimation of the gap probability. However, in the next section, the computation of the bidirectional gap probability needs the estimation of a common volume between two cylinders. It is noted finally that the gap probability for a given point ($M$) is just its probability to be sunlit. We showed here that it corresponds to the probability that $M$ belongs to a given cylinder free from leaf centers. Now, it is true that if $M$ is sunlit it belongs to an arbitrary shaped sun fleck of a certain area, however the estimation of the probability of gap for $M$ is not directly related to the sun fleck features. Particularly, the base area of the considered cylinder free from leaf centers is independent of the corresponding sun fleck area as it is claimed by Verstraete et al. (1990) and Gobron et al. (1997).

### 2.2. Bidirectional gap distribution

Fig. 1 shows a point $M(x,y,z)$ inside a vegetation layer sunlit and observed from the directions $\Omega_1$ and $\Omega_2 = (\theta_\phi, \phi_\theta)$, respectively. Two points $A$ and $B$ having the same $z$-coordinate $(t, t \geq z)$, such that $A$ and $B$ are belonging to the lines passing from $M$ with director vectors $\Omega_1$ and $\Omega_2$, respectively. According to Kuusk (1991), the distance $AB$, denoted by $d(t)$, can be written as

$$d(t) = (t-z)\Delta(\Omega_1, \Omega_2), \quad (19)$$

with

$$\Delta(\Omega_1, \Omega_2) = \sqrt{\tan^2(\theta_\phi) + \tan^2(\phi_\theta) - 2\cos(\phi_\theta)\tan(\theta_\phi)\tan(\phi_\theta)}. \quad (20)$$

To be sunlit and observed, respectively, from the directions $\Omega_1$ and $\Omega_2$, the point $M$ must belong to the cylinders $V_1$ and $V_2$ of radii $r_1 = \sqrt{\frac{G_s}{\mu_s}}$ and $r_2 = \sqrt{\frac{G_s}{\mu_s}}$ (with $G_s$ the G-function in the observation direction and $\mu_s = \cos(\theta_\phi)$) around the axis passing through $M$ in directions $\Omega_1$ and $\Omega_2$, respectively (cf. Fig. 1). The corresponding volumes $v_{gap} = V_1 \cup V_2$ must be free from leaf centers. Its volume, $v_{gap}(z)$, is given by

$$v_{gap}(z) = \int_2^0 s_{\Omega_1}(t) - s_{\Omega_2}(t) \, dt,$$

$$= \int_2^0 |s_{\Omega_1}(t) + s_{\Omega_2}(t) - 2s_{\Omega_2}(t) + s_{\Omega_2}(t) | \, dt,$$

$$= v_1 + v_2 - \int_2^0 s_{\Omega_2}(t) \, dt,$$

$$= \pi r_1^2 |z| + \pi r_2^2 |z| - \int_2^0 s_{\Omega_2}(t) \, dt,$$

where $s_{\Omega_1}(t)$ and $s_{\Omega_2}(t)$ are the area of the disks $S_{\Omega_1}(t)$ and $S_{\Omega_2}(t)$ at depth $t$ which correspond to the intersection between the horizontal plane.
\( z = t \) and the cylinders \( V_t \) and \( V_o \) respectively (cf. Fig. 1). \( S_{k(t)}(r) \) and \( S_{o(t)}(r) \) are the areas of \( S_{k}(t) = S_{o}(t) \) and \( S_{k}(t) - S_{o}(t) \) respectively.

To ensure that \( S_{k}(t) \cap S_{o}(t) \) is not an empty set, the distance between the disk centers \( (t-z)A(\Omega_o, \Omega_o) \) should be smaller than \( r_i + r_o \). Therefore, the altitude \( t \) must meet

\[
t \leq \frac{r_i + r_o}{\Delta(\Omega, \Omega)} + z = z_{\text{max}}.
\]

(22)

Moreover, it is possible to find \( S_{k}(t) \cap S_{o}(t) \) or \( S_{k}(t) \cap S_{o}(t) \) when \( r_i \leq r_o \) or \( r_o \leq r_i \) respectively. The distance \( d_z(t) \) meets the condition in this case

\[
d_z(t) \leq |r_i - r_o| + t \leq z = z_{\text{min}}.
\]

(23)

In this case, the area of intersection between the disks is the area of smallest disk between \( S_{k}(t) \) and \( S_{o}(t) \) (i.e., \( \min \{r_i, r_o\} \)). When, \( r_i = r_o \), the last case does not happen only for \( t = z \) (i.e., \( z_{\text{min}} = z \)). The integration of \( S_{k}(t) \cap S_{o}(t) \) in Eq. (21) is therefore decomposed into two terms (i.e., whether \( t < z_{\text{min}} \) or \( t > z_{\text{min}} \)).

According to Eqs. (22) and (23), Eq. (21) becomes

\[
v_o(z) = r_i r_o [z + \min \{r_i, r_o\}^2 - \min \{0, z_{\text{min}}\}^2] - \int_{\min \{z_{\text{min}}, 0\}}^{\min \{z_{\text{max}}, 0\}} S_{k}(r) \cap S_{o}(r) \, dt.
\]

When \( \leq \min \{z_{\text{max}}, 0\}, \min \{z_{\text{min}}, 0\} \], the intersection between disks \( \{S_{k}(t) \cap S_{o}(t) \} \) is a non-zero surface included both in \( S_{k}(t) \) and \( S_{o}(t) \) (cf. the gray area in Fig. 1). Therefore based on the geometric properties of the disks and the distance between their centers, it is possible to show that

\[
v_o(z) = r_i r_o \left[ 1 - \frac{r_i^2 - r_o^2}{d_z(t)} \right] + \frac{r_i^2}{d_z(t)} \left[ 1 - \frac{r_i^2 - r_o^2}{d_z(t)} \right] + \frac{r_i^2}{d_z(t)} \left[ 1 - \frac{r_i^2 - r_o^2}{d_z(t)} \right] \cdot (t-z)A(\Omega, \Omega).
\]

(25)

Eq. (24) becomes

\[
v_o(z) = r_i r_o [z + \min \{r_i, r_o\}^2 - \min \{0, z_{\text{min}}\}^2] - \int_{\min \{z_{\text{min}}, 0\}}^{\min \{z_{\text{max}}, 0\}} S_{k}(r) \cap S_{o}(r) \, dt.
\]

By substitution, \( u = (t-z)A(\Omega, \Omega) \),

\[
v_o(z) = r_i r_o [z + \min \{r_i, r_o\}^2 - \min \{0, z_{\text{min}}\}^2] - \int_{\min \{z_{\text{min}}, 0\}}^{\min \{z_{\text{max}}, 0\}} S_{k}(r) \cap S_{o}(r) \, dr.
\]

(26)

The estimation of the integrals in Eq. (27) is given in Appendix A.1. Finally, similar to the gap probability Eqs. (17) and (15), the bidirectional gap probabilities \( P_{g_{\text{gap}}} \) are given by

\[
\begin{align*}
P_{g_{\text{gap}}} &= \exp \left( -n_i v_o(z) \right), \\
P_{g_{\text{gap}}} &= \exp \left( -n_i v_o(z) \right), \\
P_{g_{\text{gap}}} &= \exp \left( -n_i v_o(z) \right).
\end{align*}
\]

(28)

Note that actually the base of the projection of a leaf of orientation, \( \Omega \), on a horizontal plane for the direction of sun and observation are ellipses \( S_{k}(\Omega) \) and \( S_{o}(\Omega) \), respectively (cf. Section 1). Therefore to be sung and observed, the point \( M \) must belong to elliptic-cylinders \( E(V_\Omega) \) and \( E(V_\Omega) \), respectively free from leaf centers of base \( S_{k}(\Omega) \) and \( S_{o}(\Omega) \), respectively. The bidirectional gap probability is therefore the probability that both ellipsic-cylinders are free from leaf centers. Therefore, the estimation of the total elliptic-cylinders volume is needed \( E(V_\Omega) \cup E(V_\Omega) \), then when integrating over all leaf orientations, \( \Omega \), an averaged total volume is obtained. The result replaces \( v_o(z) \) estimated in Eq. (27).

The drawback of this method is the estimation of the common area between two ellipses of both different centers and axes. A (semi)analytical expression of the global volume cannot be obtained in this case as proposed in this work. However a numerical solution could be addressed, but such a solution suffers from long running time as claimed by Verhoef (1998).

2.3. Reflectances due to single scattering

The reflectances due to the single scattering (of the solar irradiance) contribution from the foliage and the soil background are studied in this section.

In this section, first we present the derivation of the reflectances due to single scattering contribution in the general case, and second we present our model formulation.

2.3.1. Overview

This section deals only with the case of vegetation layer such that the leaf center spatial distribution is described by a Poisson distribution. Indeed, traditionally reflectance is calculated for a totally random leaf distribution without any condition about leaf spatial repartition which corresponds to the Poisson case (Ross, 1981).

When radiation strikes a leaf, the light is either absorbed or scattered (reflected or transmitted). The amount of each part is a function of the size, the orientation, the refractive index and the mesophyll structure of the leaf (Jacquemoud & Baret, 1990) as well as the wavelength and the incidence angle of the incident radiation. Considering the scattering phenomenon, each leaf having different inclination and azimuth angle has a different angular diagram of scattering. The scattering diagram averaged over all leaf orientations is called the bi-directional area scattering coefficient, \( w(Suits, 1972) \). It defines the infinitesimal radius times \( \pi, \frac{dE_o}{dz} \) in the view direction \( \Omega(\theta, \phi) \) caused by scattering of direct transmitted irradiance, \( E_o \), from the direction \( \Omega(\theta, \phi) \), within an infinitesimal layer of thickness \( dz \).

\[
\frac{dE_o}{dz} = wE_o.
\]

(29)

In general, leaves are assumed bi-Lambertian with hemispherical reflectance \( \rho \) and transmittance \( \tau \). Let us consider a leaf inside the vegetation with a normal orientation \( \Omega(\theta, \phi) \) receiving the direct transmitted radiation flux from an incident direction \( \Omega(\theta, \phi) \) and

The multiplication by \( \pi \) was introduced by Suits (1972) to define \( E_o \) as an equivalent irradiance corresponding to the scattered radiation in the direction \( \Omega(\theta, \phi) \) assuming that it is created by a horizontal Lambertian surface.
observed from direction $\Omega_i(\theta_i,\phi_i)$. The bi-directional area scattering coefficient, $w_i$, for this leaf normal orientation is derived as follows. If $E_i$ is the flux received by a unit area of the horizontal layer $dz$, the amount of intercepted flux by foliage of orientation $\Omega_i$ is given by $E_i \times w_i(\Omega_i, \Omega_o) / \mu_o \mu_i$ multiplied by the total area of leaves per unit area in the infinitesimal layer, $LA/h \times d z$. As the leaf is assumed bi-Lambertian, the amount of scattered radiance is given by $\rho$ depending on the leaf normal orientation relative to the incident and view directions. Finally the equivalent amount of “radiance” (\(\tau\)) are added to point out that we deal with radiance times \(\tau\) created by a horizontal layer is given multiplying the obtained radiance by $\Omega_o / \Omega_i$. It follows that $w_i(\Omega_i, \Omega_o) = LA(\Omega_i, \Omega_o) / \mu_o \mu_i \times \left\{ \begin{array}{ll} \rho & \text{if} \ (\Omega_i, \Omega_o) \geq 0, \\
 & \tau, \text{otherwise}. \end{array} \right.$ \(\text{(30)}\)

Assuming that the leaf orientation density function is given by a function $g(\theta, \phi)$ satisfying the normalizing condition

\[ \int_0^\pi \int_0^{2\pi} g(\theta, \phi) d\mu_i d\phi_i = 1, \]

with $\mu_i = \cos(\theta)_i$. \(\text{(31)}\)

The global bi-directional area scattering coefficient, $w$, taking into account the LIDF is therefore given by

\[ w(\Omega_i, \Omega_o) = \int_0^\pi \int_0^{2\pi} w_i(\Omega_i, \Omega_o) g(\theta, \phi) d\mu_i d\phi_i. \]

\(\text{(33)}\)

Next we will present the turbid case corresponding to a medium with null-size particles and for which the hot spot effect does not occur followed by the discrete case. Particularly, we present the modifications to be introduced when the finite-size leaves are considered.

2.2.1.1 Turbid case. In the following, the vegetation layer is assumed to be a turbid medium located between altitudes $-h$ and $0$. Let us consider a thin vegetation layer of depth $dz$ at altitude $z \in [-h, 0]$ receiving solar radiation from direction $\Omega_o$ and observed from direction $\Omega_i$. The “radiance” exiting the thin layer as a function of the incident radiation is therefore given by $w(\Omega_i, \Omega_o) dz$. Moreover, when traveling from the top of the canopy until reaching the considered thin layer, the incident radiation intensity decreases. Similarly, the created “radiance” decreases when traveling from the thin layer to the top. Both decreases of radiation/radiance in downward and upward directions are governed by the gap probabilities. They are respectively given by [cf. Eq. (15)]

\[ p_{gap}(\Omega_i, z) = \exp(kz), \]

\[ p_{gap}(\Omega_o, z) = \exp(Kz). \]

\(\text{(34)}\) where $k$ and $K$ are the extinction coefficients per unit length in the sun and sensor directions, respectively.

It follows that the elementary “radiance”, $d E_0(0)$, created at the top of the vegetation layer by scattering of the direct transmitted flux in the elementary layer, $dz$, is given by

\[ dE_0(0) = p_{gap}(\Omega_o, z) w(\Omega_i, \Omega_o) dE(z) \]

\[ = p_{gap}(\Omega_o, z) w(\Omega_i, \Omega_o) dz p_{gap}(\Omega_i, z) E_0(0). \]

\(\text{(35)}\)

By integration over the altitude $z$, one can find the single scattering “radiance” for the whole vegetation layer

\[ E_c(0) = E_0(0) w(\Omega_i, \Omega_o) \int_{-h}^{-h} p_{gap}(\Omega_o, z) p_{gap}(\Omega_i, z) dz \]

\[ = E_0(0) w(\Omega_i, \Omega_o) \frac{1 - \exp(-h[k + K])}{k + K}. \]

\(\text{(36)}\)

The reflectance due to single scattering contribution from foliage ($\rho_{fg}$) defined as the ratio between the top layer exiting radiance (without multiplication by $\tau$) and the radiance of a white Lambertian surface in the same illumination and observation conditions ($\rho_{fg} = E_0(0)$), is therefore given by

\[ \rho_{fg}^1 = w(\Omega_i, \Omega_o) \frac{1 - \exp(-h[k + K])}{k + K}. \]

\(\text{(37)}\)

The last reflectance expression corresponds to the reflectance due to single scattering contribution from foliage in the turbid case as shown in works of \(Suits(1972), Ross(1981)\) and \(Verhoef(1984)\), validated by \(Widlowski et al. (2006b)\) or \(Kallel et al. (2008)\).

The reflectance due to single scattering contribution from soil background, $\rho_{fs}$, corresponds to the ratio between the radiance created by radiation scattering by the soil and the radiance created by a white Lambertian soil for the incident irradiance and the same observation direction. In this case, the sun radiation is transmitted through the vegetation layer from the top to the bottom, scattered by the soil and transmitted to the top without any interaction with canopy elements. The soil scattering is described by the soil bidirectional reflectance $r_{soil}$, whereas the probabilities of upward and downward transmittance are given by the gap probabilities $p_{gap}(\Omega_o, -h)$ and $p_{gap}(\Omega_o, -h)$, respectively. Therefore,

\[ \rho_{fs}^0 = r_{soil}(\Omega_o \rightarrow -h) p_{gap}(\Omega_o, -h) \]

\[ \rho_{fs}^0 = r_{soil}(\Omega_o \rightarrow -h) p_{gap}(\Omega_o, -h). \]

\(\text{(38)}\)

2.3.1.2. Discrete case. When the medium is assumed discrete, the reflectances should be increased since the downward and upward transmittance probabilities are partly correlated as already explained in Section 2. The bidirectional gap probability ($p_{gap}(\Omega_o, z)$) should replace the product $p_{gap}(\Omega_o, z) p_{gap}(\Omega_o, z)$ in reflectance expressions [cf. Eqs. (38) and (39)] (\(Kuusk, 1985\)). It yields to

\[ \rho_{fg}^1 = w(\Omega_i, \Omega_o) \int_{-h}^{-h} p_{gap}(\Omega_o, z) \]

\[ \rho_{fs}^0 = r_{soil}(\Omega_o \rightarrow -h) p_{gap}(\Omega_o, -h). \]

\(\text{(39)}\)

To model the bidirectional gap probability, Kuusk (1985) proposes the decomposition of the vegetation layer into thin sub-layers assuming that gaps are independent from a sub-layer to another. Correlation between gap probabilities in sun and observation directions inside a thin layer is modeled as a negative exponential function of the distance between downward and upward paths. In this case, the bidirectional gap probability ($p_{gap,hv}$) is given by

\[ p_{gap,hv}(\Omega_o, \Omega_i, z) = p_{gap}(\Omega_o, z) p_{gap}(\Omega_i, z) C_{ifs}(\Omega_i, \Omega_o, z), \]

\(\text{(40)}\)

with $C_{ifs}$ a correction factor,

\[ C_{ifs}(\Omega_i, \Omega_o, z) = \exp \left[ \frac{\sqrt{K}}{b} (1 - \exp(bz)) \right]. \]

\(\text{(41)}\)

where $b$ is a function of the vegetation features, the different solid angles and the hot spot factor $b$ defined as the ratio between the leaf diameter and the layer height (Kuusk, 1991).

In his dissertation, Verhoef (1998) shows that the term $\sqrt{K}$ can be interpreted as a sort of overlap between leaf projections on a horizontal plane for the direction of sun and observation. He proposes to estimate numerically the exact overlap parameter. Then compared to the original Kuusk's model, a difference of about 2% was found for moderate canopy parameters. Although this method allows a theoretical derivation of the overlap function, it does not overcome the assumption of vertical independency between foliage which leads to reflectance underestimation.
Verstraete et al. (1990) and Gobron et al. (1997) propose a different hot spot model. Their model is currently called the semi-discrete model. In contrast to our approach, the cylinder volume is not used directly to derive the gap probability, but a corrective term is added to the turbid bidirectional gap probability. The obtained bidirectional gap probability \( P_{\text{gap,Discrete}}(\Omega_s, \Omega_o, z) \) is given by

\[
P_{\text{gap,Discrete}}(\Omega_s, \Omega_o, z) = \exp[(\lambda(z)(k + K)]z,
\]

where \( \lambda \) is the correction term

\[
\lambda(z) = 1 - \frac{\partial_1(z) \cap \partial_2(z)}{\partial_1(z)} \quad (42)
\]

where \( \partial_1(z) \) and \( \partial_2(z) \) are cylinders free of scatterers in the directions of sun and observation, respectively. These cylinders have the same radius which is the mean sun fleck radius. The bidirectional gap probability presented in Eq. (43) could be the same as the one in our model, if \( \partial_1(z) = V_{l}(z) \) and \( \partial_2(z) = V_{o}(z) \). However, the cylinder definitions in these two approaches are completely different. In our case, the cylinders are defined as the minimal volumes not containing leaf centers and allowing the given point to be sunlit and observed. Therefore the probability that both cylinders are free from leaf centers is no other than the bidirectional gap probability. Conversely, as defined by Verstraete et al. (1990) and Gobron et al. (1997), the cylinder bases correspond to the mean sun fleck area within the vegetation average over depth and orientation.

In terms of complexity, their cylinder bases do not vary as a function of the sun or sensor directions, therefore \( \partial_1(z) \) and \( \partial_2(z) \) have the same radii and their intersection computation is easier than in our case. Moreover in physical point of view, sun fleck size and probability of gap do not have the same meaning and the expression given by Verstraete et al. (1990) and Gobron et al. (1997) can be just viewed as an approximation of the bidirectional gap probability.

The two hot spot models (i.e., Kusuk’s model and the Semi-Discrete model) presented in this section will be compared to our model by numerical simulations in Section 3. For an extensive large overview about the different vegetation hot spot models, the readers are referred to Qin and Goel (1995).

2.3.2. Our model reflectance

In the Poisson case, our model reflectance fits with the expressions in Eq. (40). Therefore, the reflectances due to single scattering contribution from foliage and soil background, denoted respectively by \( \rho_{\text{pois}, \text{f}} \) and \( \rho_{\text{pois}, \text{so}} \) are given by [cf. Eq. (28)]

\[
\rho_{\text{pois}, \text{f}}^+ = w_+(\Omega_s, \Omega_o) \int_{-\Delta h}^{\Delta h} P_{\text{gap,pois}}(\Omega_s, \Omega_o, z) dz,
\]

\[
\rho_{\text{pois}, \text{f}}^- = w_-(\Omega_s, \Omega_o) \int_{-\Delta h}^{\Delta h} P_{\text{gap,pois}}(\Omega_s, \Omega_o, z) dz,
\]

\[
\rho_{\text{pois}, \text{so}} = r_{\text{soil}}(\Omega_s \rightarrow \Omega_o) \int_{-\Delta h}^{\Delta h} P_{\text{gap,pois}}(\Omega_s, \Omega_o, -h) dz.
\]

Let us derive the reflectance for the case of the Binomial distributions. According to Eq. (18), for \( \Delta v > 0 \), \( k_- < k_+ \), which means that the probabilities of radiation interception and therefore scattering for positive (respectively negative) Binomial distribution are higher (respectively lower) than these probabilities in the Poisson case. It implies that the positive (respectively negative) Binomial distribution creates more (respectively less) diffuse flux than the Poisson one. It follows that \( w_- = w_w = w_w \), where \( w_w \) and \( w_\_ \) are the bi-directional area scattering coefficient relative to the positive and negative Binomial distributions, respectively. Moreover, as the scattering intensity is linearly linked to the intercepted radiation (Kallel et al., 2008), the bi-directional area scattering coefficient is proportional to the extinction coefficient. It follows that the increase (respectively, decrease) of the extinction in the regular (respectively, clumped) case by the factor \( \frac{w_-}{w_+} \) (respectively, \( \frac{w_+}{w_-} \) [cf. Eq. (18)], leads to an increase (respectively, decrease) of the bi-directional scattering coefficient by the same factor, therefore

\[
w_- = \frac{n_-}{n_+} (\Delta v)_w,
\]

\[
w_- = \frac{n_-}{n_+} (\Delta v)_w.
\]

The reflectances due to single scattering contribution from foliage in the Binomial cases can be therefore deduced from the Poisson one Eq. (28)

\[
\rho_{\text{pois}, \text{f}}^+ = w_+(\Omega_s, \Omega_o) \int_{-\Delta h}^{\Delta h} P_{\text{gap,pois}}(\Omega_s, \Omega_o, z) dz,
\]

\[
\rho_{\text{pois}, \text{f}}^- = w_-(\Omega_s, \Omega_o) \int_{-\Delta h}^{\Delta h} P_{\text{gap,pois}}(\Omega_s, \Omega_o, -h) dz.
\]

The reflectances due to single scattering contribution from soil background in the Binomial cases are given by

\[
\rho_{\text{pois}, \text{so}}^+ = r_{\text{soil}}(\Omega_s \rightarrow \Omega_o) P_{\text{gap,pois}}(\Omega_s, \Omega_o, -h),
\]

\[
\rho_{\text{pois}, \text{so}}^- = r_{\text{soil}}(\Omega_s \rightarrow \Omega_o) P_{\text{gap,pois}}(\Omega_s, \Omega_o, h).
\]

To estimate the integrals in the single scattering from foliage reflectance expression [cf. Eqs. (45) and (47)], we adopt the same numerical solution as proposed by Verhoef (1998); details are given in Appendix A.2. Such processing is complex since it includes an iterative algorithm as it divides the vegetation layer into 60 sublayers to approximate the integration over the depth \([−h, 0]\). In addition, in each iteration a numerical estimation of the volume \( v_{so} \) [cf. Eq. (27)] based on elliptic integral codes (Abramowitz & Stegun, 1964) is needed. However, despite these complex computations, the running time is very short: few milliseconds (Intel Centrino Laptop).

Note that when the leaf radius \( r_l \) tends to zero, all the estimated reflectance tends to the turbid case as proved in Appendix B.

Note finally, as for regular (respectively, clumped) distribution \( k_- = \frac{n_-}{n_+} k \) and \( w_+ = \frac{n_-}{n_+} w \), for Binomial leaf spatial distribution, the estimation of the reflectance due to single scattering contribution boils down to the estimation of the reflectance in the Poisson case considering an apparent LAI value, LAI\_\_ and LAI\_\_ for regular and clumped distribution, respectively, they are given by:

\[
\text{LAI}_+ = \frac{n_+}{n_-} (\Delta v)_w \text{LAI},
\]

\[
\text{LAI}_- = \frac{n_-}{n_+} (\Delta v)_w \text{LAI}.
\]

3. Results of numerical experiments

Below our model is denoted by LSDM (acronym of Leaf Spatial Distribution based Model). In this section, the canopy reflectances and albedo simulated by different versions of the LSDM, corresponding to the different leaf spatial distributions are inter-compared and the impacts of regularity and clumping of the leaf distribution are studied. Also, the LSDM simulation results are compared to those provided by the Kuusk and Semi-Discrete models. Then the validation of the model is done using the ROMC web-based tool (Widlowski et al., 2008).

In our experiments, the leaf orientation density, results of numerical experiments, is assumed uniformly distributed on azimuth angle. For the sake of simplicity, results of numerical experiments are integrated over azimuth angle and replaced by the leaf inclination distribution function (LIDF), noted results of numerical experiments (Verhoef, 1984). The Bunnik parametrization is used to
model the LIDF. So, the distribution \( f_i \) as a function of the leaf normal’s zenith angle \( \theta_i \) is given by

\[
f_i(\theta_i) = \frac{2}{\pi} (a_i + b_i \cos(2c_i \theta_i)) + d_i \sin(\theta_i),
\]

with \( a_i = 1, b_i = 1, c_i = 1 \) and \( d_i = 0 \) for a planophile distribution, \( a_i = -1, b_i = 1, c_i = 1 \) and \( d_i = 0 \) for an erectophile distribution, and \( a_i = 0, b_i = 0, c_i = 0 \) and \( d_i = 1 \) for a uniform distribution. It is noted that the term \( d_i \sin(\theta_i) \) was not introduced by Bunnik (1978) but added recently by researchers from the Earth Observation Science team of the Joint Research Center (JRC) institute to model the uniform distribution (i.e., \( f_i(\theta_i) = \sin(\theta_i) \)). However, such distribution was named spherical by a lot of other researchers in the community [e.g., Verhoef (1985); Wang et al. (2007)] as the frequency of leaf inclination is the same for surface elements of a sphere. Moreover, the uniform distribution as reported by Verhoef (1984), is given by \( f_i(\theta_i) = \frac{1}{2\pi} \) because in this case all the leaf inclinations have the same frequency. Anyway as we will use the JRC ROMC database (Widlowski et al., 2008) for validation, we will adopt their notation.

3.1. Model comparison

In addition to be compared to the Kuusk model and NADIM code (i.e., Gobron et al.’s semi-discrete code), the simulations by our model with different leaf spatial distributions are inter-compared.

3.1.1. Comparison with Kuusk and Semi-Discrete

Fig. 2 shows the results of simulation of the reflectance due to scattering contribution from foliage. Its angular variations in the principal and cross planes are shown for planophile and erectophile LIDF, using the LSDM, Kuusk as well as NADIM models. The considered LSDM leaf distribution in this case is the Poisson one because both the Kuusk and NADIM models assume totally random foliage distribution.

The comparison between the two LIDF shows that the reflectance is higher in the planophile case than in the erectophile one. Indeed, the more foliage is horizontal, the more it scatters light when the sun is high above the horizon. The hot spot peak is shown in the principal plane for \( \theta_o \) values close to \( \theta_s \). Moreover, the hot spot peak is larger in the planophile case than the erectophile one since the mutual shadowing between leaves is higher for more horizontal leaves (Kallel, 2010b; Kallel et al., 2008).

The Kuusk model reflectances are lower than the LSDM ones. The simulation in the principal plane shows that the hot spot peak is the narrowest for the Kuusk model mainly in the erectophile case. Indeed, the Kuusk model takes into account the light path (downward before scattering and upward after scattering) correlation only in horizontal thin sublayers. This assumption is adequate only when leaves are horizontal. However, leaves are not necessarily horizontal. Vertical leaves cause path dependency in non-horizontal directions, too, which means that Kuusk model underestimates the path correlation and so the single scattering from foliage reflectance. This problem is more pronounced in the erectophile case since foliage normal...
orientations are closer to the horizontal (i.e., more vertical leaves) than in the LSDM case. NADIM-simulated curves are different from the LSDM curves, too. In particular, NADIM-simulated curves are the highest, particularly in the planophile case, and are very close to the LSDM with regular distribution with $\Delta \nu = 5 \times 10^{-4}$. In fact, NADIM uses a different foliage parametrization, namely the cylinder radius is taken equal to the mean sun fleck radius (Gobron et al., 1997). In general, the sun fleck radius is larger than the radii of the cylinders in direction of sun and observation $[r_s$ and $r_o$, respectively, cf. Eq. (11)] as considered by the LSDM model, since for this model the radii are defined as the minimal distances between a given point inside the vegetation and leaf centers allowing to this point to be sunlit and observed, respectively. The relative contribution of the cylinders’ common volume considered by the NADIM code is therefore larger than in the LSDM one which implies that NADIM bidirectional gap probability is higher than the LSDM one.

Fig. 3 shows the variation of the albedo due to scattering contribution from foliage, $\rho_{sd}^1$, as a function of LAI for the compared models for two LIDF, erectophile and planophile, and for two hot spot sizes 0.05 and 0.1. Remember that,

$$\rho_{sd}^1(\Omega_e) = \frac{1}{4\pi} \int_{0}^{\pi} \int_{-\pi}^{\pi} \rho_{sd}^1(\Omega_{rs} \rightarrow \Omega_e) \cos(\theta_o) d\mu_o d\varphi_o. \quad (50)$$

Similarly to the bidirectional reflectance, the simulated albedo for the erectophile leaves is smaller than the one for planophile leaves. LSDM with Poisson distribution, Kuusk and NADIM model-simulated albedo increase as a function of LAI, since the increase in LAI is an increase in the number of scatterers. For these models, the albedo increases as a function of the leaf size, too. Indeed, remember that to be sunlit and observed, each point $M$ should belong to the cylinders in sun and observation directions (cf. Fig. 1). Along with an increase in leaf size, the cylinder diameters and the percentage of the cylinder common volume $[\text{cf. Eq. (21)}]$ increase. Now, the hot spot correction term in the bidirectional gap probability, $\lambda(z)$ as defined by Eq. (43) (also valid for LSDM), is a decreasing function of the common volume, which means that the reflectance is an increasing function of the common volume and therefore it is an increasing function of $r_s$. Besides, as in Fig. 2, the comparison between these models shows that the Kuusk model-simulated curves are the lowest and those by NADIM are the highest. These differences increase together with increasing LAI and $n$. Indeed, for low LAI values, the reflectance depends mainly on the well-exposed leaves for which the bidirectional gap probability is almost equal to one for all the models, which explains the relatively close reflectance simulations in this case. Conversely, at high LAI values an important proportion of the reflectance is provided from photon collisions within deep layers (i.e., collisions with leaves located under a large number of leaves), thus the differences in the simulation of the bidirectional gap probability become more evident, explaining the simulation divergence between the models. Another reason of divergence between NADIM and LSDM for high LAI values could be the coarse discretization of the semi-discrete equations using the NADIM code decreasing therefore its accuracy for deep layers ($\text{LAI}>5$). Indeed, this code was developed at a time when computing constraints were an issue. In terms of variation as a function of the hot spot size, plots show that for small $n$ values all the model reflectances converge to the turbid medium reflectance,
large $r_i$ values cause larger differences in the simulated bidirectional gap probabilities and larger differences in simulated albedo.

3.1.2. Leaf distribution comparison

In this section the impact of the regularity and the clumping on a vegetation layer reflectance is studied. As in the last section, the BRDF and the directional–hemispherical are studied. Now, as the curve shapes remains the same when the Binomial distribution are considered (just a reflectance increasing and decreasing for the regular and clumped distribution, respectively), we will focus here on the impact of regularity/clumping on reflectance. We propose to study a normalized difference parameter, named $\Delta x$, with $x \in \{+, -\}$ for positive and negative Binomial distributions, respectively and $x \in \rho_{so}^1 \rho_{sd}$ when the BRDF and the directional–hemispherical reflectances are considered, respectively. $\Delta x$ is given by

$$\Delta x = \frac{x - x_{pol}}{x_{pol}}. \tag{51}$$

Fig. 4 shows the same simulation experiments as presented in Fig. 4 except the leaf distribution that is assumed Binomial, $\Delta \rho_{so}$ is plotted in this case for the different leaf distribution parameter $\Delta v$. In comparison between different LSDM versions (Poisson, regular and clumped foliage distributions), Fig. 4 shows that the reflectance is the lowest ($\Delta \rho_{so}^1 < 0$) for the clumped distribution and the highest for the regular one ($\Delta \rho_{so}^1 > 0$). Moreover, the more regular the distribution is ($\Delta v$ increases), the higher the reflectance is. Conversely, the more clumped is the distribution ($\Delta v$ decreases), the lower the reflectance is. This variation can be explained as follows. Compared to the Poisson case, when a clumped (respectively, regular) leaf distribution is considered, the sun radiation penetrates deeper (respectively, shallower) into the canopy which implies that the probability of photon to be scattered back is less (respectively, higher).

Before interpreting the angular variation, let us remember that in the turbid case and for positive and negative Binomial leaf distributions, the BRDF ($\rho_{so}^1$ and $\rho_{so}^1$, respectively) are given by [cf. Eqs. (38) and (47)]

$$\rho_{so}^1 = \frac{w(\Omega, \Omega_w)}{\Omega_k} \exp[-h(\Omega_1 + \ldots + \Omega_k)] \times (1 - \exp[-h(\Omega_{so} + \ldots + \Omega_k)])$$

$$\rho_{so}^1 = \frac{w(\Omega, \Omega_w)}{\Omega_k} \exp[-h(\Omega_1 + \ldots + \Omega_k)] \times (1 - \exp[-h(\Omega_{so} + \ldots + \Omega_k)]) \tag{52}.$$  

where $x \in \{+, -\}$.

Note that, for Eq. (52) the first part of the formula does not depend on the leaf distribution.

Fig. 4 shows that for high sensor inclination angle ($\theta_s \approx 90^\circ$), all $\Delta \rho_{so}^1$ values are close to zero which means that the dependency on leaf distribution is low. Remember that in this case the observation direction is far from the sun direction, which means that it is possible to approximate the BRDF using Eq. (52). Moreover, for one side, it is clear that the difference between the different formula is given by $\{1 - \exp[-h(\Omega_{so} + \ldots + \Omega_k)]\}$, from the other side for such high inclination $\mu_o$ approaches 0 and therefore $K \ldots$ approaches infinity [cf. Eq. (16)], therefore all the formula differences approach 0 that explains the convergence of the different reflectances to the same value and $\Delta \rho_{so}^1 \to 0$. After that, when the inclination decreases, the hot spot contribution becomes non-negligible and different from a leaf distribution to another, i.e. wide peak for planophile and narrow peak for erectophile, and the corresponding reflectance diverge accordingly. Indeed, remember that for given directions of sun and observation, the hot spot effect contribution depends on the percentage of common volume between cylinders free from leaf centers [cf. Eq. (28)]. Moreover, the wider the cylinders, the more important is the percentage of common volume. Now, as the cylinder bases are linearly linked to the extinction [cf. Eq. (16)] and $k_s < k_s < k_s$, it implies narrow hot spot peak for the clumped distribution (low reflectance) and wide hot spot peak for the regular distribution (high reflectance) which explains such absolute value

![Fig. 4](attachment:image.png)
of $\Delta \rho_{sd}^{1/2}$ increasing. This norm is larger for planophile than erectophile leaf distribution because beam interception (for high sun elevation) is more important for more horizontal leaves (planophile) which implies higher extinction for planophile distribution and therefore wider hot spot peak. Close to the direction of backscattering, $\Delta \rho_{sd}^{1/2}$ decreases again mainly in the planophile case. Again, this phenomenon could be explained by the width of the hot spot region, indeed close to the backscattering region the cylinders free from leaf centers in direction of sun and sensor become close and the percentage of common volume increases accordingly to reach 100% in the exact backscattering direction independently from the cylinder sizes. It implies that the reflectance difference due to the hot spot contribution to the reflectance is reduced close to the backscattering direction which explains the norm of $\Delta \rho_{sd}^{1/2}$ decreasing. This effect is more pronounced in the planophile case because the extinction is the largest and therefore the cylinders free from leaf centers are the widest in this case, therefore the common volume percentage is the most important in this case which implies a fast convergence to 100%.

Note that if the hot spot effect is not taken into account $\Delta \rho_{sd}^{1/2}$ remains close to zero in both principle and cross planes.

Fig. 5 plots the variation of $\Delta \rho_{sd}^{1/2}$ as a function of LAI for different LIDF and leaf sizes. To separate the contribution of the leaf spatial distribution from the hot spot effect, plots on the left...
size present the same experiments as those on the right side but without taking into account the hot spot effect. Thus, the simulations without hot spot effect are first studied and the conclusions about the impact of foliage clumping/regularity are given, then the hot spot effect is added and its impact on reflectance is analyzed.

For low LAI values, all \( \Delta \rho_{sd} \) values approach zero which means that the reflectances are the same for the different leaf distributions. Indeed, for low LAI values, \( n_t \) is low [cf. Eq. (1)], \( n_{+} \) and \( n_{-} \) approach \( n_t \) [cf. Eq. (14)] and thus \( P_{gap,+} \) and \( P_{gap,-} \) (respectively, \( w_+ \) and \( w_- \)) approach \( P_{gap,pw} \) [cf. Eq. (18)] [respectively, \( w \) [cf. Eq. (46)]]. Therefore, \( \rho_{so,+} \) and \( \rho_{so,-} \) approach \( \rho_{so} \).

Table 1

<table>
<thead>
<tr>
<th>Experiment</th>
<th>( \theta_s )</th>
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<th>LIDF</th>
<th>Wavelength</th>
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<td>ERE</td>
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<td>0°, 30°, 60°</td>
<td>TUR</td>
<td>5</td>
<td>ERE</td>
<td>PLA NR1</td>
</tr>
<tr>
<td>HOM11</td>
<td>0°, 30°, 60°</td>
<td>DIS</td>
<td>1</td>
<td>ERE</td>
<td>PLA NR1</td>
</tr>
<tr>
<td>HOM12</td>
<td>0°, 30°, 60°</td>
<td>DIS</td>
<td>2</td>
<td>ERE</td>
<td>PLA NR1</td>
</tr>
<tr>
<td>HOM15</td>
<td>0°, 30°, 60°</td>
<td>DIS</td>
<td>5</td>
<td>ERE</td>
<td>PLA NR1</td>
</tr>
</tbody>
</table>
Then, when $\text{LAI}$ increases the plots show that first scattering albedo is the lowest for the clumped distribution ($\Delta \rho_{s1}^{1} < 0$) and the highest for the regular one ($\Delta \rho_{s1}^{1} > 0$). Also, the albedo decreases (respectively increases) along with an increase in $\Delta v$ for the clumped (respectively the regular) distribution. The comparison between the leaf sizes shows that for small leaf size, the norm of $\Delta \rho_{1}$ is the highest, which mean that for small leaves the reflectance corresponding to the different foliage spatial distributions is far from each other. In fact, on one hand, the regular (respectively clumped) distribution depends on the number of leaves per m$^2$ ($n_l$, i.e., more leaves imply more regular (respectively clumped) spatial pattern. On the other hand, at a fixed LAI, the number of leaves is inversely proportional to leaf size. So, the dependence of LSDM-simulated albedo on $\Delta v$ is higher for small leaves ($n_l = 0.05$) than for big leaves ($n_l = 0.1$). Indeed, the increase in LAI implies an increase in $n_l$, particularly when $n_l$ approaches $\Delta v^{-1}$, $n_l$ approaches infinity ($n_{l,\text{in}}$ is not defined for $n_l \geq \Delta v^{-1}$) and when $n_l$ approaches infinity, $n_{l,\text{in}}/n_l$ approaches zero. Therefore, the increase of LAI leads to an increase (respectively decrease) of the apparent LAI, (respectively, LAI$_{\text{L}}$) [cf. Eq. (49)] which explains such a huge increase of the norm of $\Delta \rho_{1}$ for a low $n_l$ value. After this increase, the norm of $\Delta \rho_{s1}^{1}$ decreases again and approaches zero. In fact, when LAI increases, $K_\text{r}$ and $K_\text{c}$ approach infinity and all the reflectance $\rho_{s1}^{1}$ converges towards $\frac{n_l K_{\text{c}}}{n_l + K_{\text{c}}}$ (that is independent from both leaf spatial distribution and LAI).

When the hot spot is taken into account, we see that the curves are almost similar to those without hot spot for low LAI values ($\text{LAI} \leq 2$), but when LAI increases and when the hot spot effect is taken into account the norm of $\Delta \rho_{s1}^{1}$ increases again. Such effect is similar to the shown phenomenon when the BRDF is studied at the beginning of this subsection. In fact, this huge difference of reflectance simulations between the different spatial distributions is explained by the increase of the hot spot effect contribution to reflectance and the difference between the peak width, i.e. huge (respectively, narrow) peak for regular (respectively, clumped) distribution which leads to an increase (respectively, decrease) of reflectance in comparison with the Poisson case.

### 3.2. Validation

The validation of the proposed model is performed using the RAMI On-line Model Checker (ROMC) exercise (Widlowski et al., 2006) which is a web-based tool allowing the evaluation of the canopy RT models. The evaluation consists of comparing the tested model reflectances to the results of simulation by reference models provided by model benchmarking within the frames of the third Radiation transfer Model Intercomparison (RAMI) (Widlowski et al., 2006b). The reference is estimated based on the surrogate truth 3D Monte Carlo models such as DART (Gastellu-Etchegorry et al., 1996), FLIGHT (North, 1996) and Rayspread (Widlowski et al., 2006a) which show generally very close results to each other.

In the ROMC present version, three canopy structure cases are presented: homogeneous (HOM), heterogeneous (HET) and a combination between them (HETHOM). For each case, two vegetation kinds are assumed: turbid (TUR) and discrete (DIS). The leaf radius is assumed to be zero for the turbid case and equal to 0.05 m for the discrete one. Two wavelengths (red and near-infrared) and a purist corner cases are considered, they are noted RED, NIR and NR1, respectively. For these cases, the leaf reflectance and transmittance ($\rho$, $\tau$) values (0.0546, 0.0149), (0.4957, 0.4409) and (0.5, 0.5) are assumed, respectively. For these three cases, the soil reflectance is assumed to be equal to 0.127, 0.159 and 1, respectively. Three LIDFs are considered: planophile (PLA), uniform (UNI) and erectophile (ERE). Table 1 shows the nine related simulation experiments tested by ROMC. Moreover, ROMC proposes twelve measures such as the single scattering contribution from foliage, from soil background, multiple-scattered and total BRDF in principal and cross planes as well as the spectral albedo of the canopy, i.e., the directional-hemispherical reflectance and foliage absorption, etc. Moreover, a lot of new measures were recently added in the RAMI-IV exercise.

As the canopy heterogeneity is not modeled in our algorithm, the present study deals with homogeneous vegetation only. Moreover, since we are interested in the validation of a hot spot model, only measures dealing with the single scattering BRDFs are presented. Although the hot spot appears only in the discrete medium case, simulations for the turbid case are presented to study the impact of the hot spot effect.

In the following, some detailed results concerning the purist corner case are presented and then global results about all numerical experiments are shown.

As only the single scattering reflectances are considered, the results of purist corner, near-infrared or red domains are almost equivalent. In fact, for all of them the bidirectional gap probability is the same (since it does not depend on leaf albedo) and the difference between them concerns the bi-directional area scattering coefficient, $\omega$ [cf. Eqs. (47) and (48)]. We test the six possible experiments, corresponding to three...
values of LAI: 1, 2 and 5 and two leaf distributions: planophile and erectophile. They correspond to the experiments HOM11, HOM12 and HOM15 as presented in Table 1. To be short, only one sun zenith angle ($\theta_s = 30^\circ$).

Fig. 6 shows the single scattering from foliage reflectance variation in the principal plane for LSDM, NADIM and Kuusk as well as the ROMC reference (ROMCREF) model. Due to the discretization step of the observation zenith angle ($\theta_o$) in ROMC (2°), the reflectance in the exact backscattering direction is not shown. Indeed, ($\theta_o$) could not be equal to ($\theta_s = 30^\circ$). By comparing Fig. 6a, c, e or b, d, f, we can see that the reflectance increases along with an increase in LAI. Moreover, the reflectance in the planophile case is higher than the erectophile case as already explained in Section 1. The hot spot peak is the highest for the largest LAI values and also in the planophile case it is wider than the erectophile one. In fact, for such a high sun elevation ($\theta_s = 30^\circ$), the effective sunlit leaf area (i.e. the area of the projected leaf in a plane normal to the sun direction) is the highest in the planophile case which yields to the largest mutual shadowing (cf. Section 1).

In order to make a deeper comparison study, we test the global performance of the models in both the turbid and the discrete cases. Fig. 7 shows the single scattering from foliage reflectance simulated by the LSDM, NADIM and Kuusk models plotted against the reference ROMCREF reflectances for the six experiments in the discrete medium case presented in Fig. 6 (cf. Fig. 7.b) and the corresponding simulations in the turbid case (cf. Fig. 7.a). The closer to the 1:1 line the results are, the closer to the reference ROMCREF reflectances the simulations are and therefore could be considered better. The difference between Fig. 7a and b is the hot spot effect. Fig. 7.a shows that all the models give rather similar results. Particularly, LSDM and Kuusk are equivalent and provide very small error ($RMS = 4.3 \times 10^{-3}$). When the hot spot is taken into account, the model errors increase in general. The best results are by LSDM with $RMS = 4.3 \times 10^{-3}$, while for NADIM $RMS$ increases slightly to reach $5.9 \times 10^{-3}$. It is noted here that such a performance difference could not be extremely significant if taking into account that ROMCREF simulations are not totally free from error. Due to the hot spot peak underestimation (cf. Section 1), Kuusk model performances are largely decreased from the turbid to the discrete case with $RMS = \ldots$
0.0119. In terms of angular variation in the principal plane, Fig. 6 shows that our model is always inside the ROMCREF gray accuracy region (error lower than 5%). It can be also noticed that as the hot spot peak is narrower in the erectophile case, the error is lower in this case. Besides, compared to Kuusk’s model, we see that, in general, our model performs better. The hot spot peak width is generally underestimated by the Kuusk’s model. This phenomenon is more pronounced in the erectophile case as already explained in Section 1. NADIM code provides better results than Kuusk’s model, its curves are slightly higher than the LSDM ones but in general within the ROMCREF gray region. However, NADIM simulation is just above the gray region close to the hot spot area for experiment HOM15_DIS_PLA_NR1 (cf. Fig. 6.e).

Fig. 8 shows the variation of the LSDM, NADIM and Kuusk model single scattering reflectances versus those simulated by ROMCREF in the principal and cross planes for all the discrete medium experiments presented in Table 1. In general, we see that LSDM is the closest to the 1:1 line. In terms of global performance, LSDM RMS is always lower than for the others. NADIM and Kuusk model results are generally above and below the 1:1 line, respectively, leading that they overestimate and underestimate the hot spot peak, respectively. NADIM and Kuusk model RMS comparison shows that NADIM (respectively Kuusk) provides better estimation of the single scattering from foliage (respectively from soil background) reflectance.

When a bias occurs between the actual reflectance and simulation, the hot spot parameter \( r_i \) can be adjusted in order to minimize the bias. In particular, the Kuusk’s model has been used by Kallel (2010a,b) and Kallel (2012) to model the hot spot effect and \( r_i \) was taken equal to 1.5 and 3 times the leaf radius for planophile and erectophile leaf distributions, respectively. Using, LSDM such calibration is not needed. However, for actual vegetation canopies, leaves are not necessarily disk-shaped and do not have the same size. In this case, and for all the models, one has to calibrate the radius of the assumed disk-shaped leaf in order to represent a certain reality. However, even if the hot spot parameter adjustment is needed for all the models, less parameter adjustment is needed for better physically based models. For instance, the adjustment for LSDM does not depend on the leaf orientation distribution.

4. Conclusion

A physical hot spot model called LSDM was presented in this paper. A point within the vegetation canopy is sunlit or observed, if it belongs to a certain cylinder free from leaf centers. The base of the cylinder is centered on the considered point and axis oriented to the sun and observation directions, respectively. It was shown that the bidirectional gap probability for the point to be sunlit and observed is none other than the probability that both cylinders are free from leaf centers at the same time. To a great extent, the hot spot feature is determined by the common volume of these two cylinders. A new algorithm to calculate the common volume of the cylinders via elliptic integrals was proposed. In addition to the Poisson leaf spatial distribution as assumed by classical hot spot models, regular (positive Binomial distribution) or clumped (negative Binomial distribution) distributions were modeled using LSDM. Different model versions corresponding to each distribution were compared. Particularly, it was shown that compared to the Poisson distribution, the regular (respectively the clumped) distribution increases (respectively decreases) the single scattering from foliage reflectance, since it increases (respectively decreases) the apparent LAI value.

The model validation was done based on the ROMC web-based tool. The two single scattering reflectances as simulated by the LSDM model with Poisson distribution, NADIM code (corresponding to the Semi-Discrete model) and the Kuusk model were compared with the ROMC reference simulations. Almost for all the experiments, our model showed the best agreement with the reference. In general, Kuusk model underestimated the hot spot peak whereas NADIM code slightly overestimated the reflectance.

Future research will deal with the integration of our approach in the FDM radiative transfer model (Kallel, 2010a, 2010b, 2012) in order to enhance its performances. In this case, a decomposition of the LSDM fluxes into a number of virtual sub-fluxes with constant extinctions is needed. Also, the development of new RTM for Binomial leaf spatial distribution is among our perspectives.

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Appendix A. Integral numerical computation

Appendix A.1. \( v_{so} \) computation

Assume in the following that \( r_s \leq r_o \).

To estimate \( v_{so} \), all the integrals in Eq. (27) should be computed. As solution, we propose here to transform them (integration by substitution) to elliptic integrals (Abramowitz & Stegun, 1964). Two kinds will be used,

\[
F(\phi|m) = \int_{0}^{\phi} \frac{d\phi}{\sqrt{1-m^2 \sin^2(\phi)}},
\]

\[
E(\phi|m) = \int_{0}^{\phi} \frac{d\phi}{\sqrt{1-m^2 \sin^2(\phi)}},
\]

with \( F \) and \( E \) the first and second kinds of the incomplete elliptic integral, respectively (Abramowitz & Stegun, 1964). Optimal numerical codes to estimate these integrals are proposed in Abramowitz and Stegun (1964).

The three integrals in Eq. (27) are separately transformed based on integration by substitution. Let \( \phi = \arccos \left( \frac{r_s}{r_o} \frac{r_s + r_o}{r_o^2 + u} \right) \). Therefore,

\[
u_s = r_s \cos(\phi) + r_o \sqrt{1 - \left( \frac{r_s}{r_o} \right)^2 \sin^2(\phi)}.
\]

By substitution \( \phi \) for \( u \) in the first integral of Eq. (27),

\[
\int_{0}^{\phi s} \arccos \left( \frac{r_s^2 - r_o^2}{u} + u \right) du = \int_{0}^{\phi} \left( \frac{r_s}{r_o} \cos(\phi) + \sqrt{r_o^2 - r_s^2 \sin^2(\phi)} \right) \phi(\phi) d\phi,
\]

Similarly, for the last integral of Eq. (27),

\[
\int_{0}^{\phi s} \sqrt{1 - \left( \frac{r_s^2 - r_o^2}{u} + u \right)} du = \int_{0}^{\phi} \frac{r_s}{r_o} \cos(\phi) \phi(\phi) d\phi + \sqrt{r_o^2 - r_s^2 \sin^2(\phi)} + \frac{r_o}{r_s} \left( r_o^2 - r_s^2 \right) \int_{0}^{\phi} \frac{1}{\sqrt{\phi(\phi)}} d\phi.
\]

Let us deal with the second integral of Eq. (27).
Let $\phi = \arccos \left[ \frac{1}{\varepsilon_4} \left( \frac{r_s^2 - r_l^2 + u}{a} \right) \right]$. Therefore,

$$u = \begin{cases} r_s \cos (\phi) - r_s \sqrt{1 - \left( \frac{r_s^2}{r_l^2} \right) \sin^2 (\phi)}, & \text{if } u \in [a, u^*] \\ r_s \cos (\phi) - r_s \sqrt{1 - \left( \frac{r_s^2}{r_l^2} \right) \sin^2 (\phi)}, & \text{if } u \in [u^*, b] \end{cases} \quad (A.5)$$

with

$$u^* = \sqrt{r_s^2 - r_l^2}. \quad (A.6)$$

By substitution $\phi^*$ for $u$ in the second integral of Eq. (27),

$$J_0^r \arcsin \left[ \frac{\sin (\phi^*)}{\varepsilon_4} \right] \sin (\phi^*) \, d\phi = \int \phi \left( r_s \cos (\phi) - \sqrt{r_s^2 - r_l^2 \sin^2 (\phi)} - r_s \sin (\phi) \right) d\phi + r_s \int_{\phi^*}^{\pi} \sin (\phi) \, d\phi,$$

and,

$$J_0^r \arcsin \left[ \frac{\sin (\phi^*)}{\varepsilon_4} \right] \sin (\phi^*) \, d\phi = \int \phi \left( r_s \cos (\phi) + \sqrt{r_s^2 - r_l^2 \sin^2 (\phi)} - r_s \sin (\phi) \right) d\phi - r_s \int_{\phi^*}^{\pi} \sin (\phi) \, d\phi.$$

Let $\phi^*$ such that $\left( \frac{\phi^*}{\pi} \right)_s \sin (\phi^*)$. By substitution $\phi^*$ for $\phi$ in X expression,

$$X = \frac{r_s^2 - r_l^2}{r_s} \sin (\phi^*) + \frac{r_s \sin (\phi^*)}{r_s} \left( r_l \sin (\phi^*) \right) \frac{\phi^*}{\phi(a)} \quad (A.8)$$

**Appendix A.2. Single scattering from foliage reflection estimation**

As shown in Eq. (47), the single scattering reflectance from foliage reflectances derivation needs the estimation of a complex integral where an analytical expression is not trivial. An approximation of the exact solution is therefore needed.

To do it, the solution proposed in Kuusk’s model code is adopted. The interval $[a, b, 0]$ is divided into $N$ intervals $[a_i, a_{i+1}]$, such that $a_1 = -b, a_{N+1} = 0$. The length of the intervals follows a logarithmic increase law from the top to the bottom. Then, it is possible to write the integral in $\rho_{\text{f,poisson}}$ expression [cf. Eq. (46)] as

$$\int_{z=-h}^{z=0} \exp \left[ - n_i \nu_{\text{v,ao}} (z) \right] d\zeta = \sum_{i=1}^{N} \left( \int_{z=a_i}^{z=a_{i+1}} \exp \left[ - n_i \nu_{\text{v,ao}} (z) \right] d\zeta \right). \quad (A.10)$$

For each integral $[a_i, a_{i+1}]$, the integral is approximated as

$$\int_{z=a_i}^{z=a_{i+1}} \exp \left[ - n_i \nu_{\text{v,ao}} (z) \right] d\zeta \approx \left( a_{i+1} - a_i \right) \exp \left[ - n_i \nu_{\text{v,ao}} (a_i) \right] - \exp \left[ - n_i \nu_{\text{v,ao}} (a_{i+1}) \right] \left( \frac{a_{i+1} - a_i}{\nu_{\text{v,ao}} (a_{i+1}) - \nu_{\text{v,ao}} (a_i)} \right) \quad (A.11)$$

In our case, $N$ is taken equal to 60.

Similarly, it is possible to estimate the values of $\rho_{\text{f,poisson}}$ and $\rho_{\text{f,poisson}}$.

**Appendix B. Model limit ($r_l \ll 1$)**

In this appendix, we will prove that the LIDAR reflectance converges to the turbid reflectance as given in Verhoef (1984) when $r_l$ tends to zero.

According to Eq. (11)

$$\lim_{r_l \to 0} r_l = 0 \quad \text{and} \quad \lim_{r_l \to 0} r_l = 0. \quad (B.1)$$

Next, let us prove that

$$\lim_{r_l \to 0} \text{Diff}_w(r_l) = 0, \quad (B.2)$$

with

$$\text{Diff}_w(r_l) = \frac{\nu_{\text{v,ao}} (z) - n_i \nu_{\text{ai}} (z) - n_i \nu_{\text{ai}} (z)}{r_l^2} \geq 0. \quad (B.3)$$

According to Eq. (27)

$$\lim_{r_l \to 0} \text{Diff}_w(r_l) = \lim_{r_l \to 0} \min \left[ \frac{\rho_{\text{f,poisson}}}{\rho_{\text{f,poisson}}} \right] = \frac{1}{2 \pi} \left[ \frac{\nu_{\text{v,ao}} (z) - \nu_{\text{ai}} (z)}{r_l^2} \right] \geq 0.$$

It follows that for $r_l \ll 1$,

$$\nu_{\text{v,ao}} (z) \approx \nu_{\text{ai}} (z) + n_i \nu_{\text{ai}} (z). \quad (B.5)$$

Then, the reflectance due to single scattering contribution from foliage in the Poisson case [cf. Eq. (47)] is

$$\rho_{\text{f,poisson}} = \nu_{\text{v,ao}} (z) \int_{z=-h}^{z=0} \exp \left[ - n_i \nu_{\text{v,ao}} (z) \right] d\zeta,$$

$$\approx \nu_{\text{v,ao}} (z) \int_{z=-h}^{z=0} \exp \left[ - n_i \left( n_i \nu_{\text{ai}} (z) + n_i \nu_{\text{ai}} (z) \right) \right] d\zeta,$$

$$\approx \nu_{\text{v,ao}} (z) \int_{z=-h}^{z=0} \nu_{\text{ai}} (z) \frac{1}{n_i^2 \nu_{\text{ai}} (z)} \nu_{\text{ai}} (z) \left[ \frac{r_l^2}{r_l^2 + n_i^2 \nu_{\text{ai}} (z)} \right] d\zeta. \quad (B.6)$$

The last reflectance expression corresponds to the turbid case as shown in Section 1. Similarly, it is possible to prove that the other single scattering reflectances for the different leaf distributions given by Eqs. (47) and (48) converge to the turbid case when $r_l$ tends to zero. These results prove the validity of our model for small hot spot parameter.
References


