A comparative study of 1D and 2D approaches for simulating flows at right angled dividing junctions

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1. Introduction

The division of a channel flow into two (or more) flows is a common occurrence within many fluvial systems. These fluvial forms are commonly called bifurcations, or diffluences, and they are fundamental forms within braided and distributary systems such as alluvial fans and river deltas.

The past 50 years have mostly witnessed substantial efforts to understand flow processes at intersections between channels. This focus has reflected the importance of river channel junctions as key components of both dendritic drainage networks and braided river systems.

The detailed hydrodynamics of junction flows is complex [1] and there are a number of parameters that characterise the flow’s physics. These include the size, shape, slope, and angle [2] between the two combining/dividing [3–6] channels. Hence, scientific interest has increased, mostly toward theoretical considerations [1,3,7–10], experiments conducted in laboratories [2,4,10–12] as well as recent 3D numerical modelling [4,10,13].

Confluence and bifurcation units are key elements within many river networks, having a major impact upon the routing of flow. Although much progress has been made in understanding river confluences, little recognition (compared to the
confluence issue) has been given to bifurcations and dividing systems [3,5,6,9,14]. Focusing on the 1D bifurcation problem [15–18], compared to the extensive research performed on confluences, there has been much less research on what happens in water flow separations, moreover, with a 90° T-junction type [3,5].

Having an inflow divided into two parts – side and downstream – the lateral contribution is established by the use of a mathematical model of the side weir [19]. This flow diversion device is different from a junction because it has a lateral crest, and is usually used to divert water from the main waterway into irrigation systems, drainage networks or water distribution projects. The side weir is by definition an overflow and metering diversion structure installed on one or both sides of a main channel with the purpose of allowing part of the liquid to spill over the side and into another channel situated above the weir crest.

Following the idea of Rajaratnam and Pattabiraman [17], we aim to calculate the flow separation at an intersection by considering a side weir without a weir crest (zero crest length). By including the Hager [19] side weir model into the conservative form of the shallow water equations, a new model is obtained allowing the calculation of the water level and the discharge in the downstream branches while taking into account the effect of the lateral outflow discharge. The main advantage of this model is that it allows the lateral overflow without prior knowledge of the flow regime, because it is introduced within an unsteady hyperbolic conservative model and thus it can handle the presence of flow discontinuities, such as hydraulic jumps, and transcritical flows with shocks [20].

A flow over a side weir can be accurately modelled [21] by incorporating the lateral overflow parameters within the source terms of the conservative form of the unsteady shallow water equations. By this means, if one considers a zero weir crest height, the coupling of the intersection model and the canal model becomes natural and the lateral outflow effects act routinely within the Saint-Venant model [22].

The aim of this paper is to compare the 1D and 2D approaches when simulating flows at a right angled dividing junction and to show the advantage of the 2D analysis of the junction. The numerical approximation of the two approaches is performed by a second order Runge–Kutta Discontinuous Galerkin (RKDG) scheme [23]. The flow bifurcation cases are defined considering super-, trans- and subcritical flows at the separation point. The numerical results are evaluated through a set of experiments that are performed at the INSA (Lyon). We are interested in this comparison of the upstream discharge distribution in the downstream branches.

2. 1D mathematical model

Flow over a side weir is one of the most complex flows to simulate in a 1D analysis. A complete analytical solution for the equations governing flows over a side weir [24,25] is very complicated. In real-life applications, the hydraulic behaviour of lateral overflows often encounters a discontinuous evolution of the depth line through the occurrence of a hydraulic jump (super- to subcritical flow regime passage), in addition to several cases of transcritical flows (sub- to supercritical flow) caused by the fast evolution of the flow regime. We therefore believe that using the 1D unsteady conservative form of the Saint-Venant hyperbolic system built explicitly with the side weir model [19] provides an efficient tool for the water flow simulation through the T-dividing system confronted by the transient changes and discontinuous behaviour of the water flow. It is generally acknowledged that a conservative form for hyperbolic conservation laws is essential for the development of robust water resource software codes. Accordingly, the proposed model takes the following form:

\[ U_t + F_x = G, \]

with,

\[ U = \begin{pmatrix} A \\ Q \end{pmatrix}, \quad F = \begin{pmatrix} Q \\ Q^2 / A + g s_f^2 \end{pmatrix} \quad \text{and} \quad G = \begin{pmatrix} Q_t \\ gA(S_0 - S_f) + QQ_L / A \end{pmatrix}, \]

where \( A \) is the cross section, \( Q \) is the flow discharge, \( g \) is the gravitational acceleration, \( B \) is the width of the channel, \( Q_L \) is the discharge overflow, \( Q_t \) is the bottom slope and \( E_{Q_t} \) is the friction slope. If one considers the side weir as a succession of sharp crested weirs of length \( dx \), the work of Hager [25] made it possible to take into account the lateral discharge involving a series of coefficients that allow including the effects of the side weir’s bottom slope, the lateral velocity, the direction of the lateral velocity, and the effect of a possible width contraction in the side weir. By a De Marchi [26] type formula and by adding the coefficient that takes into account the lateral velocity, its direction, and width contraction effect, the discharge overflow relationship is given as follows:

\[ Q_L = -0.6n \cdot C_w \sqrt{gH^2 (y - W)^{3/2} \times \sqrt{(1 - w)/(3 - 2y - W)} \times [1 - \theta_1(3(1 - y)/(y - W))^{1/2}]}, \]

where \( y = h/H \) is the dimensionless variable of the water depth, \( W = w/H \) the dimensionless variable of the crest height (equal to zero in this paper), \( \theta_1 \) the contraction angle of the side weir (\( \theta_1 = 0 \) for this work), \( H \) the energy head expressed in terms of water depth, \( C_w \) the shape coefficient of the weir (\( C_w = 1 \) in the case of a sharp crested weir) and \( n^* \) the number of overflow walls (\( n^* = 1 \) in this work).

The numerical approximation is performed by a second order Runge–Kutta Discontinuous Galerkin (RKDG) scheme [23].
3. 2D mathematical model

Assuming that the flow is homogenous, incompressible, two-dimensional, viscous with a hydrostatic pressure distribution and with the absence of Coriolis and wind forces, the non-linear partial differential equation system used to describe the 2D, depth-averaged, free-surface flow is as follows:

\[
\frac{\partial U}{\partial t} + \nabla F = S, \tag{3}
\]

in which \( F = (E, G) \) and

\[
U = \begin{pmatrix} h \\ hu \\ hv \end{pmatrix}, \quad E = \begin{pmatrix} hu \\ hu^2 + gh^2/2 \\ huv \end{pmatrix}, \quad G = \begin{pmatrix} hv \\ huv \\ hv^2 + gh^2/2 \end{pmatrix}
\]

and

\[
S = S_0 + S_f = \begin{pmatrix} 0 \\ ghS_{ox} \\ ghS_{oy} \end{pmatrix} + \begin{pmatrix} 0 \\ -ghS_{fx} \\ -ghS_{fy} \end{pmatrix}
\]

is the source term.

In the above relations \( h \) represents the water depth, \( u, v \) are the depth-averaged velocities along the longitudinal (x) and transverse (y) directions respectively. \((S_{ox}, S_{oy})\) are the bottom slopes along the x and y direction respectively and \((S_{fx}, S_{fy})\) are the friction slopes in the x and y directions, which are defined as:

\[
S_{fx} = \frac{n_M^2 u \sqrt{u^2 + v^2}}{h^{1/3}}, \quad S_{fy} = \frac{n_M^2 v \sqrt{u^2 + v^2}}{h^{1/3}},
\]

where \( n_M \) is the Manning’s roughness coefficient.

3.1. Spatial discretization

The DG finite element formulation is written for each element \( K \). Eq. (3) is first multiplied by the test function \( \psi_h(x, y) \), and the flux integrals are further integrated by parts. The whole process yields:

\[
\begin{aligned}
\frac{d}{dt} \int_K U_h(x, y, t) \psi_h(x, y) dK &= \int_K F(U_h(x, y, t)) \cdot \nabla \psi_h(x, y) dK - \int_{\partial K} F(U_h(x, y, t)) \cdot n_k \psi_h(x, y) d\Gamma \\
&+ \int_K S(U_h(x, y, t)) \psi_h(x, y) dK,
\end{aligned} \tag{4}
\]

where \( n_k = (n_x, n_y) \) is the outward unit normal of the element’s boundary surface.

In this paper, linear triangular elements are used to discretize the 2D spatial domain. Hence, the solution at any point in each element can be represented in terms of the midpoint solutions \( U_i \) and the shape functions \( \varphi_i \) at each midpoint:

\[
U_h(x, y) = \sum_i U_i \varphi_i(x, y). \tag{5}
\]

Using the standard Galerkin approach, shape functions and test functions are identical. Rewriting Eq. (4), the discontinuous Galerkin space discretization can be summarized by a system of ordinary differential equations as:

\[
\frac{d}{dt} M U_h = L_h U(U_h), \tag{6}
\]

where \( M \) is the mass matrix and \( L_h \) is an operator describing the spatial discretization.

By using orthogonal shape functions, the mass matrix \( M \) in the linear Eq. (6) becomes diagonal. For triangular elements, a linear shape function can be obtained by choosing a function that takes the value of 1 at the midpoint \( m \) of the \( i \)th edge of a triangle, and the value of 0 at the midpoints of the other two edges. Thus, the diagonal mass matrix can be expressed as follows:

\[
M = |K| \text{diag} \left[ \frac{1}{3} \frac{1}{3} \frac{1}{3} \right]. \tag{7}
\]

The integrals are approximated by a three-midpoint rule for the triangles and two-point Gauss integration for the line integrals [27].
3.2. Numerical flux

The approximation of the numerical flux in Eq. (4) for the discontinuous Galerkin method is identical to the approximated solution of a Riemann problem according to finite volume methods. By using linear or higher subcell resolution for higher-order discontinuous Galerkin methods, the scheme gets less sensitivity to the choice of the numerical flux than the Godunov type methods. In that case, a simple Lax–Friedrich flux gives good results

\[ F(U_L, U_R) \cdot n = \frac{1}{2} [(F(U_L) + F(U_R)) \cdot n - a_{\text{max}}(U_R - U_L)], \]

where \( a_{\text{max}} = \max(|a^1|, |a^2|, |a^3|) \), and \( a^k \) \( (k = 1, \ldots, 3) \) are the eigenvalues of the Jacobian matrix:

\[
\tilde{A} = \frac{\partial (F \cdot n)}{\partial U} = \begin{bmatrix}
0 & n_x & n_y \\
(c^2 - u^2)n_x - u vn_y & 2un_x + vn_y & un_y \\
(c^2 - u^2)n_y - v un_x & vun_x & un_x + 2vn_y
\end{bmatrix}
\]

and \( c = \sqrt{gh} \) is the wave celerity.

3.3. Temporal discretization

The second order accurate two-stage TVD Runge–Kutta schemes of Cockburn and Shu [28] is employed in this work. According to [28], the optimal two-stage, second order accurate TVD Runge–Kutta scheme is expressed as:

\[
\begin{align*}
U^1_h &= U^n_h + \Delta t \cdot L_{b, U}(U^n_h), \\
U^{n+1}_h &= \frac{1}{2} (U^n_h + U^1_h + \Delta t \cdot L_{b, U}(U^1_h)).
\end{align*}
\]

(9)

The time step for a two-dimensional element \( i \) is determined according to the [RKDG formulation], namely:

\[
\Delta t = CFL \frac{|K_i|}{\sum_{j=1}^3 |l_j| \min(u \cdot n_x + v \cdot n_y - c, 0)}.
\]

(10)

Here \( u, v \) and \( c \) are evaluated at the barycenter of the element, \( j \) is the index of the sides surrounding the element, \( l_j \) is the length of the side and \( 0 < CFL \leq 1 \) is the Courant number.

3.4. Slope limiter

The following slope limiter is represented by a scalar quantity \( u_h \). Its extension to systems is outlined. For further information, see Cockburn and Shu [28]. For an arbitrary triangle \( K_0 \) and its surrounding neighbours \( K_i, i = 1, \ldots, 3 \), the notations \( b_i, i = 0, \ldots, 3 \) and \( m_i, i = 1, \ldots, 3 \) refer respectively to the barycentres of the triangles and the midpoints of the edges within \( K_0 \) (Fig. 1).

Choosing any edge midpoint \( m_1 \), we get:

\[
m_1 - b_0 = x_1(b_1 - b_0) + x_2(b_2 - b_0)
\]

for \( x_1, x_2 \in \mathbb{R}^2 \).

For any linear function \( u_h \), the mean gradient can be expressed as:

\[
\Delta \Pi(m_1, K_0) = x_1(u_h(b_1) - u_h(b_0)) + x_2(u_h(b_2) - u_h(b_0)).
\]

(12)

By using the shape functions \( \varphi_i \) \( (i = 1, \ldots, 3) \), \( u_h \) can be expressed over \( K_0 \) as follows:

![Fig. 1. One-dimensional junction problem.](image)
\[ u_h(x, y) = \sum_{i=1}^{3} u_h(m_i) \varphi_i(x, y) = u_h(b_0) + \sum_{i=1}^{3} (u_h(m_i) - u_h(b_0)) \varphi_i(x, y). \] (13)

First, we calculate the quantities:
\[ \Delta_i = \bar{m}(u_h(m_i) - u_h(b_0), v\Delta I(m_i, K_0)) \] (14)
for \( \nu > 1 \). Where \( \nu \) is a positive constant number equal to 1.5 and \( \bar{m} \) is the TVB minmod function defined as follows:
\[ \bar{m}(a_1, a_2) = \begin{cases} a_1 & \text{if } |a_1| \leq M(\Delta x)^2, \\ m(a_1, a_2) & \text{otherwise}, \end{cases} \] (15)
where \( M \) is a given positive constant; and \( m \) is the minmod function, defined as follows:
\[ m(a_1, a_2) = \begin{cases} s \min_{i=1,2} |a_i| & \text{if } s = \text{sign}(a_1) = \text{sign}(a_2), \\ 0 & \text{otherwise}. \end{cases} \] (16)

Consequently, reconstruction is carried out according to the following two cases:

1. If \( \sum_{i=1}^{3} \Delta_i = 0 \), the new midpoint value is given by:
\[ u_h(m_i) = u_h(b_0) + \sum_{i=1}^{3} \Delta_i \varphi_i(x, y) \] (17)

2. If \( \sum_{i=1}^{3} \Delta_i \neq 0 \), we compute:
\[ \text{pos} = \sum_{i=1}^{3} \max(0, \Delta_i), \quad \text{neg} = \sum_{i=1}^{3} \max(0, -\Delta_i) \] (18)

and define:
\[ \theta^+ = \min \left( \frac{\text{neg}}{\text{pos}} \right), \quad \theta^- = \min \left( \frac{\text{pos}}{\text{neg}} \right). \] (19)

Finally, the new midpoint value is given by:
\[ u_h(m_i) = u_h(b_0) + \sum_{i=1}^{3} \hat{\Delta}_i \varphi_i(x, y), \] (20)

where
\[ \hat{\Delta}_i = \theta^+ \max(0, \Delta_i) - \theta^- \max(0, -\Delta_i). \] (21)

Since the shallow water equations are a system of equations, the slope limiting must be performed in the local characteristic variables in the direction of the vector \( m_i - b_0 \). The variables are therefore transformed by \( T^{-1} \) into the characteristic space, where \( T \) is the matrix of right eigenvectors of the following Jacobian matrix:
\[ \partial_U(E(U(b_0)), G(U(b_0))) \frac{m_i - b_0}{|m_i - b_0|}. \] (22)

### 3.5. Boundary conditions

It is well known that the Riemann invariants of the 1D shallow-water equations [29] are:
\[ R^+ = u + 2c \quad \text{and} \quad R^- = u - 2c. \] (23)

Such that \( u \pm 2c \) is constant along \( u \pm c \), respectively. \( R^+ \) represents the state to the left, \( R^- \) that to the right. At a boundary, the right side is outside the domain that is given by the \( R^- \) relationship and is replaced by the boundary conditions itself. The \( R^+ \) relationship can be written as:
\[ (u, v)_L \cdot n + 2 \sqrt{gh_L} = (u, v)_R \cdot n + 2 \sqrt{gh_R}, \] (24)
in which the subscripts \( L \) and \( R \) indicate the left and solution variables, respectively. This can be combined with the boundary condition to obtain a solution for \( h, u, \) and \( v \). In general, the normal flux may then be calculated at the boundary face, namely:
\[ F \cdot n = \begin{pmatrix} h, (u, v), \cdot n \\ h, u, (u, v), \cdot n + \frac{1}{2} gh^2 n_x \\ h, v, (u, v), \cdot n + \frac{1}{2} gh^2 n_y \end{pmatrix}. \tag{25} \]

At an inflow boundary condition, at any given time, the following discharge condition holds:

\[ q = h, (u, v), \cdot n. \tag{26} \]

which, when combined with the relationship \( c^2 = gh \) and Eq. (24), gives:

\[ 2c^3 - (u + 2c_0)c^2 + gq_0 = 0. \tag{27} \]

which can be solved iteratively for \( c_0 \).

In the case of a depth boundary condition, \( h_0 \) is given and \( (u, v), n \) is calculated directly from Eq. (24).

In the case of a velocity boundary condition, \( (u, v), n \) is given and \( h_0 \) is calculated by modifying Eq. (24).

Finally, the normal flux in Eq. (25) can be calculated by using \( h_0, u, \) and \( v \). In the case of a supercritical inflow, three boundary conditions are needed, so that \( h_0, u, \) and \( v \) are directly given and the normal flux is easily obtained.

In the case of free outfall condition, which makes waves pass the boundary without reflection, all physical variables \( h_0, u, \) and \( v \) at the boundary face are the same as the internal variables.

At the solid boundary walls: the no-slip condition is assumed to hold, meaning that the normal and tangential velocity components are equal to zero there.

\[ (u, v), \cdot n = 0. \tag{28} \]

4. Evaluations and discussions

In this section, the two approaches 1D and 2D are used for handling open-channel flow bifurcations. For the 1D simulation, we consider a zero crest height, while for the 2D simulation, we simply use the 2D Saint-Venant equations. The results depicted from the two approaches are validated with the experimental results that are performed at the INSA (Lyon). All the channels are rectangular, 2.54 m long and 0.3 m wide. A main inflow division, to lateral and downstream outflows, is investigated; the subscripts “\( u \)”, “\( L \)” and “\( d \)” indicate the flow parameters at the junction relative, respectively, to the upstream, lateral and downstream flows. Assuming flow continuity through the T-junction, the following equality holds:

\[ Q_u = Q_L + Q_d. \tag{29} \]

The experimental pilot used to depict the experimental data is illustrated in Fig. 2. The discharges are measured by electromagnetic sensors with a measurement error of less than 1%. For technical reasons, the crossroads is not a square of 300 x 300 mm, but a cross of 370 x 370 mm as displayed in Fig. 3. There are also changes in the channels’ bottom slope in the regions of intersection as seen in Fig. 3. Three sets of measurements are performed for the purpose of testing the
proposed numerical technique by considering super-, trans- and subcritical flow bifurcations. In the following subsection the two approaches are evaluated and the numerically predicted results are weighted against the experimental results in terms of lateral discharge and downstream discharge.

In all the cases, the channel is discretized into 31 nodes of calculation for the 1D approach and 1607 triangular cells for the 2D approach. The walls are out of glass and the flow is hydraulically smooth. However, we considered a Strickler coefficient adjusted for the experimental data to \(K_s = 120\) (\(n_M = 1/K_s\)).

### 4.1. Supercritical flow

The supercritical inflow is maintained at the upstream, downstream and the lateral outflow. Three basic cases are considered corresponding to three various bed slope selections equal, respectively, to 1%, 3% and 5.75%. Since the flow is supercritical, the water depth \(h_u\) and the flow discharge \(Q_u\) must both be supplied at the inflow. For each case of bed slope, a selection of flow variable inputs, which are listed in Table 1, are considered. Accordingly, the upstream Froude number ranged between 1.70 \(\leq Fr_u \leq 2.00\); 3.17 \(\leq Fr_u \leq 3.53\); and 4.19 \(\leq Fr_u \leq 4.92\) for, respectively, in the 1%, 3% and 5.75% bed slopes cases. All the other missing boundary conditions are depicted numerically. For an informative understanding of the flow pattern, Fig. 4 displays the evolution of the supercritical jet through the intersection. Table 1 summarizes the numerically predicted and the experimental outflow discharges. Figs. 5–7 show the distribution of flow in the downstream branches for the three configurations of slope, respectively. These figures confirm that the distribution of flow is correctly predicted by the 2D

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Fig. 3. View from above of the experimental set up.
approach. However, the 1D approach does not always give satisfactory results. It can be seen that the flows are sometimes badly estimated by this approach.

For a quantitative comparison, we introduce an estimator of quality $E_{QT}$ (estimator of quality for the prediction of the flow’s distribution) defined as follow:

$$E_{QT} = 100 \times \max (\text{abs}(Q_c - Q_m)/Q_T),$$

where $Q_c$ is the calculated flow, $Q_m$ is the measured flow and $Q_T$ is the total flow injected at the inflow ($Q_T = Q_u$). Table 2 lists the errors between the simulated and the experimental values according to the Eq. (30). The mean error generated by the 1D approach for the different configurations of slope is of 4.35% with a minimum of 2.56% and a maximum of 6.14%, whereas the mean error of the 2D approach is negligible (0.66%) with a minimum of 0.04% and a maximum of 1.28%. The errors obtained by the 2D approach are always very small.

4.2. Transcritical flow

The flow is subcritical at the upstream and the downstream boundaries. At the lateral branch, the regime switches to supercritical (Fig. 8). For this experiment, the upstream and downstream channels’ slopes are equal to zero as well as the
side weir slope. To allow a better control of the downstream end boundary, an adjustable frontal weir with a length of $w_d$ is placed 2.54 m from the intersection of the branches. Owing to the subcritical nature of the flow, a single physical condition ($Q_u$) must be specified at the upstream inflow and the downstream outflow ($h_d$). Several test cases are considered, that deal
with a subcritical inflow Froude number that varies between 0.49 < Fr < 0.92. The inflow physical boundary condition \( Q_u \) is set according to the value listed in Table 3; whereas the outflow physical boundary condition \( h_d \) is set according to the following formula (corresponding to a submerged weir):

\[
h_d = W_d + \left( \frac{Q_d}{b_d} \right)^{1/3} \left[ 1 + 0.793 \left( \frac{W_d}{b_d} \right)^{0.731} \right] .
\]  

(31)

\( Q_d \) is the outflow discharge that is updated numerically and \( b_d \) designates the bottom width of the downstream channel. Table 3 summarizes all the considered test cases with the experimental discharges (\( Q_u \) and \( Q_d \)) versus the numerically predicted values.

Fig. 9 shows the distribution of flow in the downstream branches by the two approaches in the case of transcritical flow. The results presented show that, in general, the distribution of flow is reasonably predicted by the 2D approach. Conversely, a significant variation is observed between the results of the 1D approach and the experimental measures. Table 4 indicates the values of the indicator of quality for the distribution of the flow discharge by the two approaches 1D and 2D. The mean error of the 1D approach is 6.55%, whereas for the 2D approach, the mean error is 2.52%.

### 4.3. Subcritical flow

The flow is subcritical through all the boundaries (Fig. 10). For this experimental investigation, all slopes are equal to zero in addition to the side weir slope. Two frontal weirs are involved at the end of the lateral and downstream reaches allowing.
the flow control of the main flow and lateral outflow. The weirs are set 2.54 m from the intersection with adjustable lengths \( w_d \) and \( w_L \). The boundary conditions of the main flow are similar to the previous case. The lateral outflow is deduced numerically. However, since this case deals with a submerged flow over the lateral weir, the numerical lateral discharge \( Q_L \) has to be adjusted by the relationship of Brater and King [30]:

The flow pattern of the subcritical bifurcation case is shown in Fig. 10.

### Table 4
Values of the estimators of quality for the two approaches 1D and 2D (transcritical flow).

<table>
<thead>
<tr>
<th>Approach</th>
<th>( E_{Q_L} )</th>
<th>( E_{Q_d} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1D</td>
<td>7.32</td>
<td>5.77</td>
</tr>
<tr>
<td>2D</td>
<td>3.00</td>
<td>2.04</td>
</tr>
</tbody>
</table>

Fig. 9. Comparison of the flow discharges by the two approaches 1D and 2D for the transcritical case. (a) Lateral flow and (b) downstream flow.
where, \( h_L \) stands for the water depth downstream of the lateral branch and is exactly calculated as in Eq. (32).

Table 5 lists the different cases considered for the numerical simulations and the experiments.

The predicted downstream flows \( Q_L \) and \( Q_d \) are compared with the measured downstream flows in Fig. 11. The results show that the distribution of the discharges in the downstream branches is correctly predicted by the two approaches 1D and 2D in the case of subcritical flow. Table 6 indicates the values of the indicators of quality for the two approaches 1D and 2D (calculated according to Eq. (30)). The mean error of the 1D approach is 3.63\%, whereas for the 2D approach, the mean error is 1.29\%. It can be seen that the errors are small with a small advantage for the 2D approach. The error of the 1D approach is less significant than those obtained for the two other types of flow (supercritical and transcritical).

![Fig. 11. Comparison of the flow discharges by the two approaches 1D and 2D for the subcritical case. (a) Lateral flow and (b) downstream flow.](image)

\[
Q_L^{\text{sub}} = Q_L \left(1 - \left(\frac{y \cdot H}{h_L}\right)^{1.5}\right)^{0.385},
\]  
(32)
5. Conclusion

Over the past few years, understanding of river junctions has improved. Compared to the confluence issue, dividing flow models and the bifurcation problem have been given less recognition. Attention has mostly focused on 1D theoretical 90° dividing models, empirical relationships, experimental designs and very recently 3D numerical simulations. Typically, the flow at a bifurcation point may be predicted by empirical formulas that, unfortunately, depend on the flow regime and are hence not practical. On the other hand, adopting 1D theoretical dividing model, within the 1D Saint-Venant characteristic form, for managing internal boundary conditions may also be used. However, most of the dividing models, constructed for a 90° T-junction, are restricted to subcritical flows. Furthermore, an iterative procedure for solving nonlinear equations is required.

In this paper, we present 1D and 2D numerical simulations for estimating flow distributions at a T-junction. For the 1D simulation, the mathematical model is based on the conservative form of the shallow water equations unified explicitly with Hager’s side weir model, which was considered with a zero side crest height as a tool to take into account the lateral outflow. The advantage of this method stems from the fact that it enables the direct numerical calculation of flow distributions at a bifurcation despite the transient change in the flow regime.

Conversely, the two-dimensional modelling is carried out simply by the 2D Saint-Venant equations which do not require the use of additional equations or the concept of the side weir. The discretization of the two approaches was carried out with a second-order Runge–Kutta Discontinuous Galerkin (RKDG) scheme.

A set of experiments, conducted at the INSA of Lyon, was described considering super-, trans- and subcritical lateral flow bifurcations. The results of the two approaches 1D and 2D are validated by the experimental data concerning the distribution of the upstream flow in the downstream branches.

The results presented confirm the advantage of the 2D approach which gives results similar to the experimental data for all the types of flow. The maximal error of this approach is 3%, whereas the error of the 1D approach is satisfactory for the subcritical case and becomes increasingly significant for the two types of flow (transcritical and supercritical), reaching 7.32%.

References