Receive Antenna Selection in Diversely Polarized MIMO Transmissions with Convex Optimization

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Abstract
In this paper, we present a low complexity approach to receive antenna selection for capacity maximization, based on the theory of convex optimization. By relaxing the antenna selection variables from discrete to continuous, we arrive at a convex optimization problem. We show via extensive Monte-Carlo simulations that the proposed algorithm provides performance very close to that of optimal selection based on exhaustive search. We consecutively optimize not only the selection of the best antennas but also the angular orientation of individual antenna elements in the array for a so-called true polarization diversity system. Dual and triple polarized antenna structures are a very good solution for realizing compact devices and also robust against many imperfections as compared to spatially separated antenna structures. Effectively we extend our work from two dimensional antenna structures to three dimensions. We model such polarized antenna systems and then apply convex optimization theory for selecting the best possible antennas in terms of capacity maximization. Channel parameters like transmit and receive correlations, cross-polarization discrimination (XPD) are taken into consideration while modeling polarized systems. We compare our results with the Spatially Separated (SP) MIMO with and without selection by performing extensive Monte-Carlo simulations. We found that by using convex optimization algorithm, the performance of multiple polarized systems can be significantly enhanced. For certain channel conditions we observe that triple polarized systems increase the performance significantly compared to dual-polarized and spatially separated systems. We observed that applying selection at the receiver only boosts the performance in Non Line of Sight (NLOS) channels compared to Line of Sight (LOS) channels.

Keywords: Antenna Selection, True polarization diversity, dual polarization, triple polarization, spatial correlation, angular correlation, cross polarization discrimination (XPD), convex optimization.
1. Introduction

Multiple-Input Multiple-Output (MIMO) systems have received increased attention because they significantly improve wireless link performance through capacity and diversity gains [24]. A major limiting factor in the deployment of MIMO systems is the cost of multiple analog chains (such as low noise amplifiers, mixers and analog-to-digital converters) at the receiver end. Antenna selection at the transmitter/receiver is a powerful technique that reduces the number of analog chains required, yet preserving the diversity benefits obtained from the full MIMO system. With antenna selection, a limited number of transmit/receive chains are dynamically multiplexed between several transmit/receive antennas. MIMO antenna selection techniques have thus been extensively studied, and there are several antenna selection criteria. For full-diversity space-time codes, a subset of available antennas can be selected to maximize the channel norm [10]. For spatial-multiplexing systems, antennas can be selected to minimize the error ratios [17]. A useful tutorial paper on antenna selection can be found in [21]. Various selection algorithms applied to MIMO OFDM systems can be found in [14]. Exhaustive search based on maximum output SNR is proposed in [18], when the system uses linear receivers. Since exhaustive search is computationally expensive for large MIMO systems, several sub-optimal algorithms with lower complexity are derived at the expense of performance. A selection algorithm based on accurate approximation of the conditional error probability of quasi-static MIMO systems is derived in [19]. In [28], the authors formulate the receive antenna selection problem as a combinatorial optimization problem and relax it to a convex optimization problem. They employ an interior point algorithm, i.e., a barrier method, to solve a relaxed convex problem. However, they treat only the case of capacity maximization. An alternative approach to receive antenna selection for capacity maximization that offers near optimal performance at a complexity, significantly lower than the schemes in [11] but marginally greater than the schemes in [32], is described in [7]. In [25, 26] a new approach to antenna selection is proposed, based on the minimization of the union bound, which is the sum of the all Pairwise Error Probabilities (PEPs). Our approach is based on formulating the selection problem as a combinatorial optimization problem and relaxing it to obtain a problem with a concave objective function and convex constraints. We follow the lines of [28][7], and extend it to systems with both spatial and angular correlation, so-called True Polarization Diversity (TPD) [31, 30, 29] arrays. We optimize the performance of systems with such arrays of antennas which are both spatially separated and also inclined at a certain angle. A model for combined spatial and angular correlation functions is also given in [6], but we adhere to the work from Valenzuela [31, 30, 29]. We apply a simple norm based antenna selection method to polarization diverse array. Application of receive antenna selection on polarized array can be found in [16][15]. We proceed to apply optimization now to the system of arrays with Dual-Polarized (DP) and Triple-Polarized (TP) antenna structures. Application of receive antenna selection on polarized array can be found in [16][15]. We first model the DP and TP systems with respect to
many channel characteristics, e.g., K-factor, channel correlations and cross-polarization discrimination (XPD). A good investigation on the modeling of DP MIMO channels in [5]. In [13] the author models TP systems and presents the performance in terms of outage probabilities. We then compare the results with the Spatially Separated-Single Polarized (SS-SP) systems with the same channel characteristics. We extend our DP and TP systems to Spatially-Separated Dual-Polarized (SS-DP) and Triple Polarized (SS-TP) systems. These systems are a combination of both spatial and polarization domain. The remainder of this paper is organized as follows: Section 2 details the generic model and the structure of polarization diverse arrays. We also describe the correlation models for both spatial and angular arrays in this section. We proceed forward to describe channel model for three dimensional antenna structures in 3. In Section 4, the performance metric to optimize with receive antenna selection, is outlined with details. Description of the performance as an optimization problem is presented in 5 for 2D and 3D structures. Important simulation results for polarization diverse systems and triple-polarized systems and comparison with the performance of Uniform Linear Arrays (ULA) are discussed in Section 6. We conclude our work in Section 7.

2. Channel Model for 2D Antenna Structures

We consider a MIMO system with $N_T$ transmit and $M_R$ receive antennas. The channel is assumed to have frequency-flat Rayleigh fading with additive white Gaussian noise (AWGN) at the receiver. The received signal can thus be represented as

$$x(k) = \sqrt{E_s}Hs(k) + n(k),$$

where $M_R \times 1$ vector $x(k) = [x_1(k), \ldots, x_{M_R}(k)]^T$ represents the $k^{th}$ sample of the signals collected at the $M_R$ receive antennas, sampled at symbol rate. The $N_T \times 1$ vector $s(k) = [s_1(k), \ldots, s_{N_T}(k)]^T$ is the $k^{th}$ sample of the signal transmitted from the $N_T$ transmit antennas. The symbol $E_s$ denotes the average energy per receive antenna and per channel use, $n(k) = [n_1(k), \ldots, n_{M_R}(k)]^T$ describes the noise of an AWGN channel with energy $N_0/2$ per complex dimension and $H$ is the $M_R \times N_T$ channel matrix, where $H_{p,q}(p = 1, \ldots, M_R, q = 1, \ldots, N_T)$ is a scalar channel between the $p^{th}$ receive antenna and $q^{th}$ transmit antenna. The entries of $H$ are assumed to be Zero-Mean Circularly Symmetric Complex Gaussian (ZMCS CG), such that the covariance matrix of any two columns of $H$ is a scaled identity matrix. Perfect Channel State Information (CSI) is assumed at the receiver while performing antenna subset selection. No CSI is available at the transmitter. The correlation models are taken from the work of [31, 30, 29, 6]. The array is with an aperture size of $L_r = \lambda/2$, the antennas in the array are randomly oriented in space and also separated by the spatial separation of $d_r$. Thus, we have $d_r = L_r/(M_R - 1)$. The inter element distance in ULA configuration depends on the radius. This limits the total number of antennas that can be stacked in a given area constraint. From [8] and [20], a practical measure for $r$ is given to be $0.025\lambda$. Thus, a maximum of nine
antenna elements can be stacked in such configurations. The angles are represented by $\theta_r$. The radiation patterns of all the elements in a ULA configuration are constant. But in an array of polarized antenna elements, different patterns exist due to the slant angles, hence introducing both, pattern and polarization diversity. Here, for the sake of simplicity we assume only polarization diversity and discard the effects produced by pattern diversity. The investigations of [6, 30, 4] describe the correlation models for structures with both angular as well as spatial diversity. We work on the modified model given in [6], which also is in agreement to the model presented in [30]. The spatial correlation between two consecutive identical antennas can be found in [31], given as

$$\varsigma_r = \sin(q_r)/q_r,$$  \hfill (2)

and is illustrated in Fig. 1, where $q_r = 2\pi d_r/\lambda$ and $d_r$ is the inter-element distance as defined earlier. The correlation function between antenna elements, separated by an angular displacement is established by an equivalence between angular and spatial separation. This is called true polarization diversity [29] and shown below as

$$\varsigma_a = \sin(q_a)/q_a,$$  \hfill (3)

where $q_a = 2\pi \theta_r$. For a small number of receiving antennas and under Rayleigh fading scenarios the angular separation $\theta_r$ can be made equivalent to a spatial separation by

$$\theta_r = \varphi_{i-j}/180^\circ,$$  \hfill (4)

where $\varphi_{i-j} = \varphi_i - \varphi_j$ is the angular difference between two dipoles, and $\varphi_i$ and $\varphi_j$ are the orientation angles of dipoles, $i$ and $j$ with respect to vertical axis. The angular correlation function is shown in Fig. 2. The combined spatial-polarization correlation function as given in [6] is a separable function of space $d_r$ and angle $\theta_r$ variables, shown below

$$\varsigma(d_r, \theta_r) = \text{sinc}(kd_r) \cos \theta_r$$  \hfill (5)

If we have a ULA configuration, $\varsigma_r = \text{sinc}(kd_r)$ and $\varsigma_a = \cos \theta_r$ for the angular separated configuration. We use these simple models in order to describe correlation values. It should be noted that effects of mutual coupling are ignored here for the sake of simplicity. We have shown a six element true polarization diversity antenna array in Fig. 3.

3. Channel Model for 3D Antenna Structures

The channel is modeled as a Ricean fading channel, i.e, the channel matrix can be composed of a fixed (possibly line-of-sight) part and a random (fast fading) part according to

$$H = \sqrt{\frac{K}{K+1}} \bar{H} + \sqrt{\frac{1}{K+1}} \tilde{H}$$  \hfill (6)
Figure 1: Spatial correlation function.

Figure 2: Angular correlation function.

Figure 3: True polarization diversity antenna array with $M_R = 6$ antenna elements.
where $K$ is the Ricean $K$-factor, $\tilde{H}$ is a deterministic matrix and $H$ is a random matrix. The random matrix $H$ consists of complex Gaussian entries which are independent from one channel realization to the next. In other words, if $K = 0$ then $H$ models a pure Rayleigh fading channel and if $K = \infty$ then it models a static channel. For a DP system, the channel matrix is described in V and H polarizations, i.e., its elements represent the input-output relation from V to V, V to H, H to H, and H to V polarized waves,

$$H_{DP} = \begin{bmatrix} \tilde{h}_{VV} & \tilde{h}_{VH} & \tilde{h}_{HV} & \tilde{h}_{HH} \end{bmatrix},$$

and that for $3 \times 3$ triple-polarized channels represented as,

$$H_{TP} = \begin{bmatrix} \tilde{h}_{VV} & \tilde{h}_{VH} & \tilde{h}_{VZ} \\ \tilde{h}_{HV} & \tilde{h}_{HH} & \tilde{h}_{HZ} \\ \tilde{h}_{VZ} & \tilde{h}_{HZ} & \tilde{h}_{ZZ} \end{bmatrix},$$

A $4 \times 4$ MIMO channel with two spatially separated DP antennas on each side can for example be written as,

$$H = \begin{bmatrix} h_{1V,1V} & h_{1V,1H} & h_{1V,2V} & h_{1V,2H} \\ h_{1H,1V} & h_{1H,1H} & h_{1H,2V} & h_{1H,2H} \\ h_{2V,1V} & h_{2V,1H} & h_{2V,2V} & h_{2V,2H} \\ h_{2H,1V} & h_{2H,1H} & h_{2H,2V} & h_{2H,2H} \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix},$$

where the scalar channel between the $i^{th}$ transmit antenna and the $j^{th}$ receive antenna is denoted by $h_{jV,iV}$ for the vertical component and $h_{jH,iH}$ for the horizontal component. The cross-components are denoted by $h_{jV,iH}$ and $h_{jH,iV}$, respectively.

3.1. Channel XPD

An electromagnetic wave can change its polarization state due to reflections, as it propagates through a channel. A common way of describing a channel’s...
Figure 5: Configuration of triple-polarized system.

ability to separate V and H polarizations is the so called channel XPD factor which is for the fast fading part of the channel and is defined as,

\[ X_V = E\left\{ |\tilde{h}_{VV}|^2 \right\} / E\left\{ |\tilde{h}_{HV}|^2 \right\}, \]

\[ X_H = E\left\{ |\tilde{h}_{HH}|^2 \right\} / E\left\{ |\tilde{h}_{VH}|^2 \right\}. \]

Similarly for TP channels we have the following XPD definitions as,

\[ X_{VH} = E\left\{ |\tilde{h}_{VV}|^2 \right\} / E\left\{ |\tilde{h}_{HV}|^2 \right\}, \]

\[ X_{HV} = E\left\{ |\tilde{h}_{HH}|^2 \right\} / E\left\{ |\tilde{h}_{VH}|^2 \right\}, \]

\[ X_{ZV} = E\left\{ |\tilde{h}_{ZZ}|^2 \right\} / E\left\{ |\tilde{h}_{VZ}|^2 \right\}, \]

\[ X_{VZ} = E\left\{ |\tilde{h}_{VV}|^2 \right\} / E\left\{ |\tilde{h}_{ZV}|^2 \right\}, \]

\[ X_{HZ} = E\left\{ |\tilde{h}_{HH}|^2 \right\} / E\left\{ |\tilde{h}_{ZH}|^2 \right\}, \]

\[ X_{ZH} = E\left\{ |\tilde{h}_{ZZ}|^2 \right\} / E\left\{ |\tilde{h}_{HZ}|^2 \right\}, \]

where a symmetric leakage is assumed. The same “symmetry” assumption was made for V/H polarized waves in [23] and it is also motivated by the measurements reported in [1] where the leakage from V to H and H to V have the same power on average. We also define the variable \( \alpha, 0 < \alpha \leq 1 \), which corresponds to the part of the radiated power that is coupled from V to H and vice versa. There is perfect discrimination between the V and H polarized components as \( \alpha \to 0 \), and a “leakage“ between polarizations when \( 0 < \alpha \leq 1 \). The relation between the channel XPD and \( \alpha \) is, thus, given by,

\[ XPD = \frac{1 - \alpha}{\alpha}, \quad 0 < \alpha \leq 1, \]
where we have used the following normalizations,

\[ E\left\{ |\tilde{h}_{VV}|^2 \right\} = E\left\{ |\tilde{h}_{HH}|^2 \right\} = 1 - \alpha \quad (13) \]

\[ E\left\{ |\tilde{h}_{HV}|^2 \right\} = E\left\{ |\tilde{h}_{VH}|^2 \right\} = \alpha. \quad (14) \]

Similarly for TP array we have some additional normalizations as follows,

\[ E\left\{ |\tilde{h}_{VV}|^2 \right\} = E\left\{ |\tilde{h}_{HH}|^2 \right\} = E\left\{ |\tilde{h}_{ZZ}|^2 \right\} = 1 - (\alpha_1 + \alpha_2). \quad (15) \]

\[ E\left\{ |\tilde{h}_{HV}|^2 \right\} = E\left\{ |\tilde{h}_{VZ}|^2 \right\} = E\left\{ |\tilde{h}_{ZH}|^2 \right\} = \alpha_1. \quad (16) \]

\[ E\left\{ |\tilde{h}_{HV}|^2 \right\} = E\left\{ |\tilde{h}_{VZ}|^2 \right\} = E\left\{ |\tilde{h}_{ZH}|^2 \right\} = \alpha_2. \quad (17) \]

The above normalizations are motivated by power or energy conservation arguments. That is, the channel cannot introduce more energy to the transmitted signal and with this normalization the power is conserved by subtracting from the co-polarized component the corresponding amount of power \( \alpha \) that has leaked into the cross-polarized component. This normalization is of great importance when comparing DP to SP systems. The XPD for TP channel can then be represented by,

\[ \text{XPD} = \frac{1 - (\alpha_1 + \alpha_2)}{\alpha_1 + \alpha_2}, \quad 0 < (\alpha_1 + \alpha_2) \leq 1, \quad (18) \]

Similarly, we define the channel XPD for the fixed part of the DP channel as,

\[ \text{XPD}_f = \frac{1 - \alpha_f}{\alpha_f}, \quad 0 < \alpha_f \leq 1. \quad (19) \]

with the following normalizations

\[ |\tilde{h}_{VV}|^2 = |\tilde{h}_{HH}|^2 = 1 - \alpha_f. \quad (20) \]

\[ |\tilde{h}_{HV}|^2 = |\tilde{h}_{VZ}|^2 = \alpha_f. \quad (21) \]

Similarly for triple polarized array we have some additional normalizations as follows,

\[ |\tilde{h}_{VV}|^2 = |\tilde{h}_{HH}|^2 = |\tilde{h}_{ZZ}|^2 = 1 - (\alpha_{1f} + \alpha_{2f}). \quad (22) \]

\[ |\tilde{h}_{HV}|^2 = |\tilde{h}_{ZV}|^2 = |\tilde{h}_{ZH}|^2 = \alpha_{1f}. \quad (23) \]

\[ |\tilde{h}_{HV}|^2 = |\tilde{h}_{VZ}|^2 = |\tilde{h}_{ZH}|^2 = \alpha_{2f}. \quad (24) \]

The channel XPD for the fixed part of the TP channel is given by,

\[ \text{XPD}_f = \frac{1 - (\alpha_{1f} + \alpha_{2f})}{\alpha_{1f} + \alpha_{2f}}, \quad 0 < (\alpha_{1f} + \alpha_{2f}) \leq 1. \quad (25) \]
3.2. Channel Correlation

The elements of the Spatially Separated-Single Polarized (SS-SP) MIMO channel matrix will be correlated, when the channel is not rich enough, i.e., when there is not enough scattering to decorrelate the elements of the channel matrix and/or when the antenna spacing is too small. We define the transmit, $t_s$, and receive, $r_s$, spatial and co-polarized correlation coefficients as,

$$t_s = \frac{E\{\tilde{h}_{iV,iV}\tilde{h}_{iV,jV}^*\}}{1-\alpha}, i \neq j,$$

$$r_s = \frac{E\{\tilde{h}_{iV,iH}\tilde{h}_{iH,iH}^*\}}{1-\alpha}, i \neq j, \tag{26}$$

Similarly, we define the transmit, $t_p$, and receive, $r_p$, polarization correlation coefficients as,

$$t_p = \frac{E\{\tilde{h}_{iV,iV}\tilde{h}_{iV,iH}^*\}}{\sqrt{\alpha(1-\alpha)}}, \tag{28}$$

$$r_p = \frac{E\{\tilde{h}_{iV,iV}\tilde{h}_{iV,iH}^*\}}{\sqrt{\alpha(1-\alpha)}}, \tag{29}$$

For example, the measurements reported in [9] showed that the average envelope correlations (worst case) were all less than 0.2, and, in fact, all of the reported measurements in [1] showed that $t_p \approx r_p \approx 0$. The correlations between elements of TP structures can be shown in a straight forward manner as above.

3.3. Total Channel

A $2 \times 2$ dual-polarized MIMO channel is expressed as follows,

$$\tilde{H}_{DP} = \Sigma_{DP} \odot \left( C_{r_p}^{1/2} W_{2 \times 2} C_{t_p}^{1/2} \right), \tag{30}$$

$$\Sigma_{DP} = \begin{bmatrix} \sqrt{1-\alpha} & \sqrt{\alpha} \\ \sqrt{\alpha} & \sqrt{1-\alpha} \end{bmatrix}, \tag{31}$$

$$C_{r_p} = \begin{bmatrix} 1 & 0 \\ r_p & 1 \end{bmatrix}; \quad C_{t_p} = \begin{bmatrix} 1 & 0 \\ t_p & 1 \end{bmatrix}, \tag{32}$$

are the polarization leakage, receive and transmit correlation matrices. A $3 \times 3$ triple-polarized MIMO channel is expressed as follows,

$$\tilde{H}_{TP} = \Sigma_{TP} \odot \left( C_{r_p}^{1/2} W_{3 \times 3} C_{t_p}^{1/2} \right), \tag{33}$$

where
\[ \Sigma_{TP} = \begin{bmatrix} \sqrt{1-\beta} & \sqrt{\alpha_1} & \sqrt{\alpha_2} \\ \sqrt{\alpha_2} & \sqrt{1-\beta} & \sqrt{\alpha_1} \\ \sqrt{\alpha_1} & \sqrt{\alpha_2} & \sqrt{1-\beta} \end{bmatrix}, \] (34)

where \( \beta = (\alpha_1 + \alpha_2) \) and the condition for “symmetry” is that \( 0 \leq \beta \leq 1 \).

\[ C_{rp} = \begin{bmatrix} 1 & r_p & r_p \\ r_p^* & 1 & r_p \\ r_p^* & r_p & 1 \end{bmatrix}; \quad C_{tp} = \begin{bmatrix} 1 & t_p & t_p \\ t_p^* & 1 & t_p \\ t_p^* & t_p & 1 \end{bmatrix}, \] (35)

are the polarization leakage, receive and transmit correlation matrices. Here we assume that the correlation values for each pair of polarization, in a TP structure are equal. Extension to arrays of multiple SS-DP and SS-TP antenna arrays are straightforward and shown below as,

\[ \tilde{\mathbf{H}}_{DP} = \mathbf{1}_{M_R/2 \times N_T/2} \otimes \Sigma_{DP} \otimes \left( \mathbf{C}_{r,DP}^{1/2} \mathbf{W}_{M_R \times N_T} \mathbf{C}_{t,DP}^{1/2} \right), \] (36)

where \( M_R \) and \( N_T \) are the number of receive and transmit antennas respectively. They should always be multiples of two for the DP case.

\[ \tilde{\mathbf{H}}_{TP} = \mathbf{1}_{M_R/3 \times N_T/3} \otimes \Sigma_{TP} \otimes \left( \mathbf{C}_{r,TP}^{1/2} \mathbf{W}_{M_R \times N_T} \mathbf{C}_{t,TP}^{1/2} \right), \] (37)

where \( M_R \) and \( N_T \) should always be multiples of three for the TP case. The \( \mathbf{C}_{r,\cdot} = \mathbf{C}_{r,s} \otimes \mathbf{C}_{r,p} \) and \( \mathbf{C}_{t,\cdot} = \mathbf{C}_{t,s} \otimes \mathbf{C}_{t,p} \) are the receive correlation and transmit correlation matrices of the \( M_R \times M_R \) MIMO channel with \( M_R \) spatially separated dual-polarized and triple-polarized antennas on each side. The matrix \( \mathbf{1}_{M_R/2 \times N_T/2} \) and \( \mathbf{1}_{M_R/3 \times N_T/3} \) are representing matrices of all elements to be one respectively. The spatial correlation matrices are given, for example for a \( 2 \times 2 \) SS system, as follows,

\[ C_{rs} = \begin{bmatrix} 1 & r_s \\ r_s^* & 1 \end{bmatrix}; \quad C_{ts} = \begin{bmatrix} 1 & t_s \\ t_s^* & 1 \end{bmatrix}. \] (38)

4. Receive Antenna Selection in MIMO

The earliest works on antenna selection have been in the context of Single-Input Multiple-Output (SIMO) systems. For example, selection diversity, where the receiver only selects the strongest antenna signal has long been used in SIMO systems [27]. Receive antenna selection in MIMO systems offer more degrees of freedom than in SIMO systems. We focus here on receive antenna selection for capacity maximization. The capacity of the MIMO system described in Section II is given by the well known formula

\[ C(\mathbf{H}) = \log_2 \det \left( \mathbf{I}_{N_T} + \frac{\gamma}{N_T} \mathbf{R}_{ss} \mathbf{H}^H \mathbf{H} \right), \] (39)

where \( \gamma = E_s/N_0 \), \( \mathbf{R}_{ss} = E \left( \mathbf{s}(k) \mathbf{s}(k)^H \right) \) is the covariance matrix of the transmitted signals with trace\( (\mathbf{R}_{ss}) = 1 \). The determinant is denoted by \( \det(\cdot) \) and
\( I_{N_T} \) represents the \( N_T \times N_T \) identity matrix. However, when only \( M'_R < M_R \) receive antennas are used, the capacity becomes a function of the antennas chosen. If we represent the indices of the selected antennas by \( r = [r_1, \ldots, r_{M'_R}] \), the effective channel matrix is \( H \) with those rows only corresponding to these indices. Denoting the resulting \( M'_R \times N_T \) matrix by \( H_r \), the channel capacity with antenna selection is given by

\[
C_r(H_r) = \log_2 \det \left( I_{N_T} + \frac{\gamma}{N_T} R_{ss} H_r^H H_r \right). \tag{40}
\]

In the absence of CSI at the transmitter, \( R_{ss} \) is chosen as \( I_{N_T} \). Our goal is to chose the index set \( r \) such that the capacity in (40) is maximized. A closed form characterization of the optimal solution is difficult. We propose a possible selection scheme in the next section.

5. Optimization algorithm for Antenna Selection

We formulate the problem of receive antenna selection as a constrained convex optimization problem [3] that can be solved efficiently using numerical methods such as interior-point algorithms [22]. Similar to [7], the \( \Delta_i(i = 1, \ldots, M_R) \) is defined such that,

\[
\Delta_i = \begin{cases} 
1, & \text{\( i \)th receive antenna selected} \\
0, & \text{otherwise.}
\end{cases} \tag{41}
\]

By definition, \( \Delta_i = 1 \) if \( r_i \in r \), and 0 else. Now, consider an \( M_R \times M_R \) diagonal matrix \( \Delta \) that has \( \Delta_i \) as its diagonal entries. Thus, the MIMO channel capacity with antenna selection can be re-written as

\[
C_r(\Delta) = \log_2 \det \left( I_{M_R} + \frac{\gamma}{N_T} H^H \Delta H \right) = \log_2 \det \left( I_{M_R} + \frac{\gamma}{N_T} \Delta HH^H \right). \tag{42}
\]

The second Equality (42) follows from the matrix identity

\[
\det(\mathbf{I}_m + \mathbf{A} \mathbf{B}) = \det(\mathbf{I}_n + \mathbf{B} \mathbf{A}).
\]

The capacity expression given by \( C_r(\Delta) \) is concave in \( \Delta \). The proof follows from the following facts: The function \( f(\mathbf{X}) = \log_2 \det(\mathbf{X}) \) is concave in the entries of \( \mathbf{X} \) if \( \mathbf{X} \) is a positive definite matrix, and the concavity of a function is preserved under an affine transformation [3]. We transform (42) into another form that includes the correlation matrices,

\[
C_r(\Delta) = \log_2 \det \left( I_{M_R} + \frac{\gamma}{N_T} \Delta R_{R}^{1/2} H R_{T}^{1/2} H^H R_{R}^{1/2} \right), \tag{43}
\]
where \( \mathbf{R}_T^{1/2} \) and \( \mathbf{R}_R^{1/2} \) are the normalized correlation matrices at the transmit and receive side. We assume that antennas at the transmit side are well separated to avoid any correlation. The matrix \( \mathbf{R}_T^{1/2} \) would then be an identity matrix and can be ignored in the above equation. After applying rotation and simplification, (43) can be written as,

\[
C_r(\Delta) = \log_2 \det \left( \mathbf{I}_{M_R} + \frac{\gamma}{N_T} \mathbf{R}_S^{H/2} \mathbf{R}_P^{H/2} \mathbf{R}_T^{1/2} \mathbf{R}_S^{IH} \right).
\]  

(44)

We split the correlation matrix \( \mathbf{R}_T^{1/2} \) into two parts: the spatial separation and the polarization of individual antenna elements and obtain,

\[
C_r(\Delta) = \log_2 \det \left( \mathbf{I}_{M_R} + \frac{\gamma}{N_T} \mathbf{R}_S^{H/2} \cdot \mathbf{R}_P^{H/2} \cdot \mathbf{R}_T^{1/2} \mathbf{R}_S^{IH} \right),
\]  

(45)

where \( \mathbf{R}_S^{1/2} \) is the normalized correlation matrix due to the spatial separation and \( \mathbf{R}_P^{1/2} \) is the additional correlation matrix due the polarization of antenna elements. The elements of these matrices are found from (2) and (3), respectively.

The variables \( \Delta_i \) are binary valued (0 or 1) integer variables, thereby rendering the selection problem NP-hard. We seek a simplification by relaxing the binary integer constraints and allowing \( \Delta_i \in [0, 1] \). To make things easily tractable we divide the optimization problem into two parts. We first find the optimum \( \mathbf{R}_P^{1/2} \) and then find the optimum \( \Delta \) as a separate optimization problem. Thus, the problem of receive antenna subset selection for capacity maximization is approximated by the constrained convex relaxation plus rounding schemes:

\[
\text{maximize } \log_2 \det \left( \mathbf{I}_{M_R} + \frac{\gamma}{N_T} \mathbf{R}_S^{H/2} \cdot \mathbf{R}_P^{H/2} \cdot \mathbf{R}_T^{1/2} \mathbf{R}_S^{IH} \right)
\]  

\[
\text{subject to } \quad r_p(m, m) = 1, \quad m = 1, \ldots, M_R \quad \text{(46b)}
\]

\[
|r_p(m, n)| \leq 1, \quad m, n = 1, \ldots, M_R; m \neq n \quad \text{(46c)}
\]

\[
\mathbf{R}_S^{1/2} \cdot \mathbf{R}_P^{1/2} \leq [1]_{M_R \times M_R} \quad \text{(46d)}
\]

where \([1]_{M_R \times M_R}\) is a matrix of all the elements equal to one. We now suppose that \( \mathbf{R}_P^{1/2} = \mathbf{R}_S^{1/2} \cdot \mathbf{R}_P^{1/2} \), where \( \mathbf{R}_P^{1/2} \) is the optimum correlation matrix. We use this matrix \( \mathbf{R}_P^{1/2} \) obtained from (46d), to obtain the optimum \( \Delta \),

\[
\text{maximize } \log_2 \det \left( \mathbf{I}_{M_R} + \frac{\gamma}{N_T} \mathbf{R}_S^{H/2} \mathbf{R}_P^{H/2} \mathbf{R}_T^{1/2} \mathbf{R}_S^{IH} \right)
\]  

\[
\text{subject to } \quad 0 \leq \Delta_i \leq 1, \quad i = 1, \ldots, M_R
\]  

\[
\text{trace}(\Delta) = \sum_{i=1}^{M_R} \Delta_i = M'_R.
\]  

(47c)
The objective function in (46a) is concave because the correlation matrices defined by $R^{1/2}_{P}$ and $R^{1/2}_{S}$ are positive definite and hermitian. Since the constraints (46b)-(46d) are linear and affine, the whole optimization algorithm (46) is concave and can be solved efficiently using disciplined convex programming [12]. Similarly, the constraints (47b)-(47c) are linear and affine, so the optimization problem (47) is concave and can be solved using disciplined convex programming [12]. Also the diagonal matrix $\Delta$ is positive semi-definite. From the optimum values of $R^{1/2}_{P}$ found, we can proceed to find the optimum angles of polarization or orientation. From the (possibly) fractional solution obtained by solving the above problem, the $M'_R$ largest $\Delta_i$'s are chosen and the corresponding indices represent the receive antennas to be selected. The optimum capacity (40) is then calculated by using only the selected subset $r$, which is found through (46) and (47). The ergodic capacity after selection now reads,

$$C(\tilde{\Delta}) = \log_2 \det \left( I_{M'_R} + \frac{\gamma}{N_T} \tilde{R}^{H/2}_{PD} \tilde{\Delta} \tilde{R}^{1/2}_{PD} \tilde{H} \tilde{H}^H \right),$$

(48)

where $(\tilde{\cdot})$ denotes a matrix, composed only of the selected $r$. In summary we try to find the optimum angles $\theta_r$'s, which optimize the ergodic capacity with receive antenna selection. Practically this system is only realizable, if all the antenna elements in an array can be independently rotated around their axes. Physically realizing such system is not easy, but methods to emulate the rotating effect through the use of parasitic elements has been investigated in [2]. The problem of receive antenna subset selection for capacity maximization in dual and triple polarized systems is approximated by the constrained convex relaxation plus rounding schemes as follows.

$$\max \log_2 \det \left( I_{M'_R} + \frac{\gamma}{N_T} \Delta \tilde{H} \tilde{H}^H \right)$$

(49a)

subject to

$$0 \leq \Delta_i \leq 1, \quad i = 1, \ldots,$$

(49b)

$$\text{trace}(\Delta) = \sum_{i=1}^{M_R} \Delta_i = M'_R.$$  

(49c)

where $H = \tilde{H}_{DP}$ is given by (30) for DP systems and $H = \tilde{H}_{TP}$ is given by (33) for TP antenna systems.

6. Results

In this section, we evaluate the performance of the proposed antenna selection algorithm via Monte-Carlo simulations [12]. We solve the optimization algorithm using the MATLAB based tool for convex optimization called CVX [12]. We use ergodic capacity as a metric for performance evaluation, which is obtained by averaging over results, obtained from 1000 independent realizations.
of the channel matrix $H$. For each realization, the entries of the channel matrix are uncorrelated ZMCSCG random variables. We take the example of real valued correlation matrices calculated from (5). In Fig. 6 we show the results for $M'_R/6$ selection. In Fig. 7 we show the results for capacity against $M'_R$ for values of $N_T$. In Fig. 6 and 7 we also show the simulation results for systems with only vertical oriented antenna elements i.e, only separated spatially (ULA). We see clearly that the performance of these systems is substantially less than the systems which contain both spatial and angular separation. The optimization problem similar to (50) for only spatially separated systems is given by,

$$\text{maximize} \quad \log_2 \det \left( I_{M_R} + \frac{\gamma}{N_T} R_{S}^{H/2} \Delta R_{S}^{1/2} HH^H \right) \quad (50a)$$

subject to

$$0 \leq \Delta_i \leq 1, \quad i = 1, \ldots, M_R \quad (50b)$$

$$\text{trace}(\Delta) = \sum_{i=1}^{M_R} \Delta_i = M'_R. \quad (50c)$$

The above stated optimization problem is now simpler because of only one matrix $\Delta$ to be optimized with two constraints. As an example for a polarization diverse system, we show in (51), the diagonal matrix $\tilde{\Delta}$ for a $2/6$ selection. We see that $\text{trace}(\Delta) = \sum_{i=1}^{M_R} \Delta_i = 2$. We take the two largest elements of the vector $\text{trace}(\Delta)$ and find the ergodic capacity with the respective indices $(r = 2, 3)$ of the rows of the channel matrix $H$. Now we show an optimum correlation matrix in (53) $R_{P}^{1/2}$ for a given $R_{S}^{1/2}$, calculated for the optimum $\Delta$, as an example. The $\tilde{\Delta}$ matrix formed after selection, is given in (52). We use the same indices $(r = 2, 3)$ again to select the rows and columns of correlation matrix $R_{P}^{1/2}$. The selected correlation matrix is shown in (54). From this matrix the corresponding angles are $\theta_r = 0, 71^\circ$. We show more examples of selection systems with the corresponding optimum angles in Table 1 at 20dB SNR.

$$\Delta = \begin{bmatrix}
0.3957 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.3847 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.2196 \\
\end{bmatrix}. \quad (51)$$

$$\tilde{\Delta} = \begin{bmatrix}
1 & 0 \\
0 & 0.3957 \\
\end{bmatrix}. \quad (52)$$

$$R_{P}^{1/2} = \begin{bmatrix}
1.000 & 0.189 & 0.174 & 0.033 & 0.000 & 0.229 \\
0.189 & 1.000 & 0.000 & 0.139 & 0.297 & 0.951 \\
0.174 & 0.000 & 1.000 & 0.081 & 0.050 & 0.210 \\
0.033 & 0.139 & 0.081 & 1.000 & 0.000 & 0.000 \\
0.000 & 0.297 & 0.050 & 0.000 & 1.000 & 0.143 \\
0.229 & 0.951 & 0.210 & 0.000 & 0.143 & 1.000 \\
\end{bmatrix}. \quad (53)$$
Figure 6: Ergodic capacity v/s SNR, $M_R = 6$, $N_T = 1,2,3,4$, $M'_R = N_T$, for Polarization Diverse (PD) and Uniform Linear Array (ULA) systems.

We now include the results for 3D antenna structures. We evaluate the capacity

$$\bar{R}^{1/2} = \begin{bmatrix} 1.000 & 0.189 \\ 0.189 & 1.000 \end{bmatrix}.$$  \hspace{1cm} (54)

We now include the results for 3D antenna structures. We evaluate the capacity

Table 1: Optimum Angles with $M'_R/9$ Selection at 20dB SNR for $M_R = 1, \cdots, 5$

<table>
<thead>
<tr>
<th>$M'_R$</th>
<th>Indices ($r$)</th>
<th>Angles ($\theta^r_f$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>2,6</td>
<td>0.62</td>
</tr>
<tr>
<td>3</td>
<td>3,7,9</td>
<td>0.56, 76</td>
</tr>
<tr>
<td>4</td>
<td>2,5,7,9</td>
<td>0.73, 78,90</td>
</tr>
<tr>
<td>5</td>
<td>1,4,5,6,7</td>
<td>0.55, 90,65,70</td>
</tr>
</tbody>
</table>

for different channel scenarios depending on parameters like correlation, XPD and K-factor. For all Ricean fading examples the fixed $2 \times 2$ channel components are given by to,

$$\bar{\mathbf{H}}_{DP} = \begin{bmatrix} \sqrt{1-\alpha_f} & \sqrt{\alpha_f} \\ \sqrt{\alpha_f} & \sqrt{1-\alpha_f} \end{bmatrix}.$$  \hspace{1cm} (55)

Similarly for the triple-polarized case we have,

$$\bar{\mathbf{H}}_{TP} = \begin{bmatrix} \sqrt{1-\beta_f} & \sqrt{\alpha_1 \beta_f} & \sqrt{\alpha_2 \beta_f} \\ \sqrt{\alpha_1 \beta_f} & \sqrt{1-\beta_f} & \sqrt{\alpha_1} \\ \sqrt{\alpha_2 \beta_f} & \sqrt{\alpha_1} & \sqrt{1-\alpha_2 \beta_f} \end{bmatrix},$$  \hspace{1cm} (56)

where $\beta_f = (\alpha_1 + \alpha_2)$. A nominal value of XPD$_f = \frac{1-\beta_f}{\beta_f}$ = 15dB is chosen for simulations [5]. For the SS-SP systems, we have the following matrices with fixed channel (see Eq. (6)).

$$\bar{\mathbf{H}}_{2SS-SP} = \begin{bmatrix} \sqrt{1-\alpha_f} & \sqrt{1-\alpha_f} \\ \sqrt{1-\alpha_f} & \sqrt{1-\alpha_f} \end{bmatrix}.$$  \hspace{1cm} (57)
Similarly for three SS-SP case we have,

$$\mathbf{H}_{3SS-SP} = \begin{bmatrix} \sqrt{1-\alpha_f} & \sqrt{1-\alpha_f} & \sqrt{1-\alpha_f} \\ \sqrt{1-\alpha_f} & \sqrt{1-\alpha_f} & \sqrt{1-\alpha_f} \\ \sqrt{1-\alpha_f} & \sqrt{1-\alpha_f} & \sqrt{1-\alpha_f} \end{bmatrix}. \quad (58)$$

Throughout all simulations we used typical correlation values $t_p = r_p = 0.3$ and $t_s = r_s = 0.5$ [1][9]. We compute ergodic capacity by averaging over 100 instantaneous capacity values, varying the matrix $\mathbf{W} \in \mathcal{M}(0,1)$ as i.i.d complex-valued Gaussian. We compare Antenna Selection (AS) methods by selecting $M'_R$ out of $M_R$ antennas against Non Antenna Selection (NAS) by utilizing all $M_R$ antennas. As selection method we apply (47).

6.1. Effect of SNR on the capacity in Rayleigh channels

From Figure 8 we observe that with the given channel parameters, the performance of 2SS-DP and 3SS-SP systems is almost the same for all $M'_R$. The 3SS-TP systems has a better performance with selection for values of $M'_R > 4$ as compared to 2SS-DP. We also observe in the figure that all the systems with antenna selection perform better compared to non Antenna Selection (NAS) systems. The 3SS-TP system has the best overall performance.

6.2. Effect of XPD on the capacity in Rayleigh channels

In Figure 9, we show the impact of the XPD parameter on the ergodic capacity of the polarized systems with and without selection. We use the case of $M'_R = 6$ as an example for both DP and TP systems. For a fair comparison of DP and TP systems we used XPD$_f = 15$dB for both systems. In the simulations we used $\beta_f = (\alpha_{1f} + \alpha_{2f})$. We assumed $\alpha_{1f} = \alpha_{2f}$ in our simulations (see Eq. (18)). The same condition is applied for the varying XPD from (34) and the condition of symmetry is taken as $\beta = (\alpha_1 + \alpha_2)$. Again $\alpha_1 = \alpha_2$ is assumed for simulations (see Eq. (25)). We observe that with the given channel parameters, SS-SP systems are not effected by the $\alpha$ values. We observe that 3SS-TP without selection has the best performance. A selection within 3SS-TP systems is far better than a selection within 2SS-DP.
6.3. Effect of K-factor on the capacity

Figure 10 shows the performance in terms of Ricean K-factor. We observe that the performance gets worse when the LOS component K increases. We also observe that DP systems with or without antenna selection are affected more by the K-factor compared to TP systems. We also extract from the figure that TP systems with selection perform a lot better than all other systems except full complexity TP systems.

6.4. Comparison between CO and CM selection methods

In Figure 11 we compared the CO antenna selection method to the well known Capacity Maximization (CM) method based on exhaustive search for the maximum norm of the selected sub-channels [21][18]. For CM the average was taken over $10^5$ channel realizations for the matrix $W$. We observe that the CO method performs better compared to the CM method for DP systems specially for $M'_R = 2, ..., 5$. The gain acquired is almost 3.2 bit/s/Hz at $M'_R = 5$. For TP systems we also see a significant gain achieved by using this method. A gain of 2.5 bit/s/Hz can be observed at $M'_R = 3$. We also observe a gain of 0.5 bit/s/Hz at $M'_R = 5$ for 3SS-SP systems while comparing the CO to the CM method.

7. Conclusions

In this work we investigated an polarization diverse antenna array with compact 2D aperture, to optimize the performance in terms of ergodic capacity. We used the relaxation of a binary integer constraint to have a convex optimization algorithm and solved it, using disciplined convex programming. The optimization algorithm not only finds the best antennas for selection but also finds the optimum orientation angles for antenna elements within an array. We also compared the results with an array consisting of linear uniform elements. We found
Figure 9: Capacity vs XPD for dual-polarized $N_T = 2$, $M_R = 10$ and triple-polarized $N_T = 3$, $M_R = 9$ with $M_R' = 6$, SNR=10dB, $t_p = r_p = 0.3$, $t_s = r_s = 0.5$, Rayleigh fading $K = 0$.

Figure 10: Capacity vs K-factor for dual-polarized $N_T = 2$, $M_R = 10$ and triple-polarized $N_T = 3$, $M_R = 9$ with $M_R' = 6$, SNR=10dB, $t_p = r_p = 0.3$, $t_s = r_s = 0.5$, and XPD = 10dB, XPD$f = 15$dB.

Figure 11: Capacity vs antennas selected for dual-polarized $N_T = 2$, $M_R = 10$ and triple-polarized $N_T = 3$, $M_R = 9$ with $M_R' = 6$, SNR=10dB, $t_p = r_p = 0.3$, $t_s = r_s = 0.5$, Rayleigh fading $K = 0$ and XPD = 10dB. Comparison between Convex Optimization (CO) and Capacity Maximization (CM) based selection).
that by using an optimization algorithm, the performance of polarization diverse systems can be significantly enhanced. We extended our models to dual and triple polarized MIMO channels. We used the same convex optimization techniques to optimize the performance of such systems for maximizing ergodic capacity. We used the relaxation of a binary integer constraint to have a convex optimization algorithm and solved it, using disciplined convex programming. The optimization algorithm finds the best antennas for selection. We also compared the results with an array consisting of spatially separated single polarized array of linear elements. We found that by using an optimization algorithm, the performance of multiple polarized systems can be significantly enhanced. For certain channel conditions we see that triple polarized systems increase the performance significantly compared to spatially separated systems. We also observe that applying selection at the receiver only boosts the performance in NLOS channels compared to LOS channels. A comparison with the exhaustive search method of capacity maximization for selection shows that convex optimization based search method performs better for polarized MIMO systems with antenna selection.

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