Abstract—Sufficient conditions are derived under which $M$-ary partial- and full-response continuous phase modulation (CPM) space–time (ST) codes will attain both full spatial diversity and optimal coding gain. General code construction rules are desirable due to the nonlinearity and inherent memory of the CPM signals which makes manual design or computer search difficult. Using a linear decomposition of CPM signals with tilted phase, we identify a rank criterion for $M$-ary partial- and full-response CPM that specifies the set of allowable modulation indices. We also propose a coding gain design criterion. Optimization of the coding gain for CPM ST codes is shown to depend on the CPM frequency/phase shaping pulse, modulation index, and codewords. The modulation indices and phase shaping functions that improve the coding gain are specified. Finally, optimization of coding gain for ST-CPM and orthogonal ST-CPM codewords is discussed.

Index Terms—Coding gain, continuous phase modulation (CPM), determinant criterion, linear decomposition, rank criterion, Rayleigh fading, space-time (ST) coding, spatial diversity, tilted phase.

I. INTRODUCTION

SPACE–TIME (ST) coding transmits coded waveforms from multiple antennas to maximize link performance. Full spatial diversity is one design objective for ST codes, being upper-bounded by the product $L_t \times L_r$, where $L_t$ and $L_r$ are the number of transmit and receive antennas, respectively. Coding gain optimization is another design objective for ST codes. Many different ST codes have been developed for static- and quasi-static Rayleigh-fading channels [1]–[19]. Tarokh et al. [1] devised the rank and determinant criteria for spatial diversity that optimizes the worst case pairwise error probability (PWEP) and presented some simple design rules that guarantee full spatial diversity for linear modulation schemes. Several ST code designs based on the rank and determinant criteria are proposed in [2]–[4]. In [5], the determinant criterion is strengthened by showing that the Euclidean distance must be optimized in order to optimize the product distance. Finally, in [6] it is shown that different design criteria apply depending on the diversity order. For sufficient degrees of freedom ($L_t \times L_r \geq 4$), the code performance is governed by the minimum squared Euclidean distance of the code. For small $L_t \times L_r$, the rank and determinant criteria will govern the code performance.

One of main difficulties encountered when deriving general design rules for ST codes is that the diversity and coding design criteria apply to the complex domain of baseband-modulated waveforms, whereas block code designs are usually conducted over finite fields. Hammons and Gamal [7] proposed rank criteria that ensure full diversity for ST codes with binary phase-shift keying (BPSK) and quaternary phase-shift keying (QPSK). Liu et al. [8] generalized these rank criteria for higher order quadrature amplitude modulation (QAM) constellations. ST coding can also be applied to continuous phase modulated signals. Zhang and Fitz [9], [10] derived design criteria for ST-coded continuous phase modulation (CPM) (ST-CPM) on quasi-static fading channels and identified a rank criterion, but only for particular CPM schemes: full-response 2$^n$-ary CPM with $h = 1/2$, full-response 4$^n$-ary CPM with $h = 1/4, h = 3/4$, partial-response binary CPM with $h = 1/2$, and partial-response 4-ary CPM with $h = 1/4, h = 3/4$.

Some attempts to optimize coding gain of ST-CPM have been made in [11]–[13]. In [11], [12], an orthogonal ST-CPM system is proposed with $L_t = 2$, where different CPM schemes are used on the two transmit antennas. This method attains full diversity and good coding gain, but requires bandwidth expansion due to a large CPM modulation index. In [13], an ST-CPM scheme is proposed in $L_t = 2$ that uses different mapping rules on the two antennas to achieve full diversity and optimal coding gain. However, the extension to $L_t > 2$ has not been considered. In conclusion, a general framework for ST-CPM is lacking both in terms of diversity order and coding gain.

This paper derives sufficient conditions under which any $M$-ary partial- and full-response ST-CPM arrangement will attain full spatial diversity for any $L_t$. Design rules are specified for coding gain optimization. Parallelizing the work of Mengali and Morelli [20], we first derive a linear decomposition of CPM signals with tilted phase. The tilted phase is time-invariant and simplifies receiver processing [21]. However, we must stress that this paper is not about minimizing complexity, but rather to provide a general framework for ST-CPM. Based on our linear decomposition, we propose a rank criterion for $M$-ary partial- and full-response CPM that eventually defines a set of optimal modulation indices for any candidate CPM scheme (excluding...
multi-h CPM). We also propose a coding gain design criterion for ST-CPM. Maximization of the coding gain for ST-CPM depends not only on the codewords as in linear modulation, but also on the frequency-phase shaping function. Since the computer search for the codes and phase shaping functions that maximize the coding gain is a computationally challenging problem, a coding gain optimization criterion that improves (or maximizes) the coding gain is developed. Finally, we discuss the optimization of ST-CPM and orthogonal ST-CPM as special cases.

The remainder of the paper is as follows. Section II describes ST-CPM on a quasi-static fading channel. Section III derives the linear decomposition of CPM signals with tilted phase. Section IV presents our design criteria for M-ary partial- and full-response ST-CPM. Section V presents several examples and simulation results verifying the developed ST-CPM rank and coding gain design optimization criteria. Section VI concludes the paper.

II. ST-CPM SYSTEM MODEL

This section describes ST-CPM on a quasi-static fading channel. We consider an ST-CPM system with $L_t$ transmit antennas and $L_r$ receive antennas. As shown in Fig. 1, the ST encoder uses a block code $C$ to encode blocks of $K_b$ information symbols into length-$N = N_t L_t$ codeword vectors $\mathbf{u} \in C$ that are mapped onto an $L_t \times N_c$ matrix $\mathbf{U}$ in the following manner: codeword

$$\mathbf{u} = \left( u_0^{(1)}, \ldots, u_0^{(L_t)}, \ldots, u_{N_c-1}^{(1)}, \ldots, u_{N_c-1}^{(L_t)} \right)$$

is mapped to the $L_t \times N_c$ matrix

$$\mathbf{U} = \begin{bmatrix}
  u_0^{(1)} & \cdots & u_0^{(L_t)} \\
  u_1^{(1)} & \cdots & u_1^{(L_t)} \\
  \vdots & \ddots & \vdots \\
  u_{N_c-1}^{(1)} & \cdots & u_{N_c-1}^{(L_t)}
\end{bmatrix}$$

where $u_k^{(i)}$ is the code symbol assigned to $i$th transmit antenna at time epoch $k$.

The outputs of the ST encoder are $L_t$ streams of symbols, that are input to separate tilted-phase CPM modulators that drive the antennas. Due to the tilted phase representation, the ST encoder outputs can be directly input to the CPM modulators without the need for additional modulation mapping as is the case if an excess-phase CPM modulator is used. The $L_t$-modulated signals are simultaneously transmitted from $L_t$ transmit antennas.

The signal at each receiver antenna is a noisy superposition of the $L_t$ transmitted signals, each affected by quasi-static flat Rayleigh fading, and independent zero-mean complex additive white Gaussian noise (AWGN). The received signal can be represented in the convenient vector form

$$\mathbf{r}(t) = \sqrt{E_s} \mathbf{H}^T \mathbf{s}(t, \mathbf{U}) + \mathbf{n}(t)$$

where $\mathbf{s}(t, \mathbf{U}) = [s_1(t, \mathbf{u}^{(1)}), \ldots, s_{L_r}(t, \mathbf{u}^{(L_r)})]^T$ is the vector of transmitted signals, $\mathbf{u}^{(i)} = [u_0^{(i)}, u_1^{(i)}, \ldots, u_{N_c-1}^{(i)}]$ is the vector of the code symbols assigned to $i$th transmit antenna, $\mathbf{r}(t) = [r_1(t), \ldots, r_{L_r}(t)]^T$ is the vector of received signals, $\mathbf{n}(t) = [n_1(t), \ldots, n_{L_r}(t)]^T$ contains the noise samples that are independent zero-mean complex Gaussian random variables with variance $N_0/2$ per dimension, $\mathbf{H} = [h_{ij}]_{L_r \times L_t}$ is the matrix of complex channel fading gains, $E_s$ is the symbol energy, and $(\cdot)^T$ denotes the matrix transpose operation.

In some embodiments, the ST-CPM receiver employs maximum-likelihood sequence detection (MLSD) as implemented with the Viterbi algorithm. The bit-error rate performance of MLSD is typically evaluated by upper-bounding the pairwise error probability for any two ST codewords $\mathbf{U}$ and $\mathbf{U}$, defined as in (2). Let $\mathbf{s}(t, \mathbf{U})$ and $\mathbf{s}(t, \mathbf{U})$ denote the CPM vectors corresponding to matrices $\mathbf{U}$ and $\mathbf{U}$, respectively. Finally, let $\mathbf{U}_s$ be the matrix of correlation functions of the differential CPM signals received at the different antennas [9]

$$\mathbf{U}_s = \begin{bmatrix}
  \int_0^{N_t T_c} \left| \Delta s_1(t) \right|^2 dt & \cdots & \int_0^{N_t T_c} \Delta s_1(t) \Delta s_{L_t}(t) dt \\
  \cdots & \cdots & \cdots \\
  \int_0^{N_t T_c} \Delta s_{L_t}(t) \Delta s_1(t) dt & \cdots & \int_0^{N_t T_c} \left| \Delta s_{L_t}(t) \right|^2 dt
\end{bmatrix}$$

where differential CPM signals are defined as $\Delta s_i(t) \triangleq s_i(t, \mathbf{u}^{(i)}) - s_i(t, \mathbf{u}^{(i)})$ for $1 \leq i \leq L_t$ and $T_c$ denotes the symbol duration. For a quasi-static flat Rayleigh-fading channel, the pairwise error probability has the following, asymptotically tight, upper bound [1]:

$$P_e(\mathbf{U}, \mathbf{U} \mid \mathbf{H}) \leq \left( \frac{1}{\prod_{i=1}^{r} \left( \frac{1 + \lambda_i E_s}{4 N_0} \right)} \right)^{L_r}$$

where $r$ is the rank of matrix $\mathbf{U}_s$, $\{\lambda_1, \ldots, \lambda_r\}$ are the nonzero eigenvalues of $\mathbf{U}_s$, and $\eta = (\lambda_1 \lambda_2 \cdots \lambda_r)^{1/r}$ is their geometric mean. From (5), we can conclude that ST-CPM has a direct analogy to ST-coded linear modulation. The rank criterion and the determinant criterion used for ST linear modulation are directly applicable to ST-CPM, the only difference being that CPM has the “signal” matrix as in (4).

III. LINEAR DECOMPOSITION OF CPM WITH TILTED PHASE

Laurent [22] showed that a binary CPM signal can be decomposed into a linear combination of pulse amplitude modulated (PAM) waveforms. Mengali and Morelli [20] extended Laurent’s CPM signal decomposition to M-ary CPM signals. These linear decomposition approaches [22, 20] use the CPM excess phase, but as mentioned previously, a decomposition based on the time-invariant CPM tilted phase is desirable. Paralleling the
work of Mengali and Morelli [20], we derive a linear decomposition of CPM signals with tilted phase.

The CPM tilted-phase baseband complex envelope is [21]

\[ s(t, u) = \sqrt{2T_c} e^{j\psi(t, u)} \]  

(6)

where \( nT_c \leq t \leq (n + 1)T_c \), and

\[ \psi(t, u) = 2\pi h \sum_{k=0}^{n-L} u_k + 4\pi h \sum_{t=0}^{L-1} u_{n-i} \beta(t - (n-i)T_c) + \pi h W(t - nT_c) \]  

(7)

is the tilted phase, \( h \) is the modulation index, \( u = (u_0, \ldots, u_{N_c-1}) \) is the information sequence with elements chosen from the \( M \)-ary alphabet \( \{0, 1, \ldots, M-1\} \), and \( T_c \) is the symbol duration. The term \( W(t) \) in (7) is

\[ W(t) = (M-1) \frac{t}{T_c} + (M-1)(1-L) \]  

(8)

The phase shaping pulse \( \beta(t) \) is defined by \( \beta(t) = \int_0^t \varphi(t') dt' \), where \( \varphi(t) \) is the frequency shaping pulse of length \( LT_c \) such that \( \beta(LT_c) = 1/2 \) for \( t \geq LT_c \). The CPM waveform has full response when \( L = 1 \) and partial response when \( L > 1 \).

To derive the CPM tilted-phase decomposition, first note that some integer \( F \) exists such that \( 2^{F-1} < M \leq 2^F \). Since \( u_k \) varies in the range \( 0 \leq u_k \leq (M-1) \leq 2^F - 1 \), \( u_k \) has the radix-2 representation \( u_k = \sum_{k=0}^{L-1} \gamma_{k, l} 2^l \), where \( \gamma_{k, l} \in \{0, 1\} \). Hence, the tilted phase in (7) can be rewritten as

\[ \psi(t, u) = \pi h W(t - nT_c) + \sum_{k=0}^{L-1} 2\pi h 2^k \left( \sum_{l=0}^{L-1} \gamma_{k, l} t + 2 \sum_{l=0}^{L-1} \gamma_{n-l, i} \beta(t - (n-i)T_c) \right). \]  

(9)

Then, the CPM tilted phase complex envelope has the form shown in (10) at the bottom of the page.

The next step replaces the partial-response term associated with the \((n-i)\)th bit in (10) by an equivalent sum of two terms, such that only the second term depends upon the \((n-i)\)th bit. The partial-response term in (10) associated with the \(i\)th bit of symbol \( u_k \) satisfies

\[ e^{jk\pi h 2^i \gamma_{k, i} \beta(t)} = e^{2\pi h 2^k \beta(t)} \left[ \frac{\sin(\pi h 2^l - 2\pi h 2^l \beta(t))}{\sin(\pi h 2^l)} + e^{2j\pi h 2^l \gamma_{k, l} \sin(2\pi h 2^l \beta(t))} \sin(\pi h 2^l) e^{-j\pi h 2^l} \right]. \]  

(11)

The equality in (11) can be shown by applying the Euler transformation on sine functions and by replacing \( \gamma_{k, i} \) with 0 and 1, respectively.

Define \( s^{(i)}(t) \) as shown in (12), also shown at the bottom of the page. Therefore, when \( 0 \leq t \leq LT_c \),

\[ e^{jk\pi h 2^i \gamma_{k, i} \beta(t)} = s^{(0)}(t + LT_c) + e^{2\pi h 2^i \gamma_{k, i} \beta(t)} s^{(0)}(t). \]  

(13)

From (10) and (13), the CPM tilted-phase complex envelope on the interval \( nT_c \leq t \leq (n+1)T_c \) becomes

\[ s(t, u) = \frac{2}{T_c} e^{j\pi h W(t - nT_c)} \sum_{0}^{F-1} e^{2\pi h 2^k \sum_{l=0}^{L-1} \gamma_{k, l} t + \sum_{l=0}^{L-1} \gamma_{n-l, i} \beta(t - (n-i)T_c) + \pi h W(t - nT_c)} \]  

\[ \cdot \sum_{0}^{F-1} \sum_{0}^{N-1} B_{k,i} \gamma_{k,i} \left[ s^{(0)}(t + \gamma_{k,i} \beta(t)) \right]. \]  

(14)

Closely following the algorithm in [20], we obtain the complex envelope of \( M \)-ary partial-response CPM signals with tilted phase as

\[ s(t, u) = \sum_{k=0}^{F-1} \sum_{i=0}^{N-1} B_{k,i} \gamma_{k,i} \left[ s^{(0)}(t + \gamma_{k,i} \beta(t)) \right]. \]  

(15)

where \( R = (2^F - 1)2^{F(L-1)} \) and the Laurent functions \( g_{k}(t) \) and symbols \( B_{k,i} \) are defined below. The Laurent functions \( g_{k}(t) \) are

\[ g_{k}(t) = e^{j\pi h W(t)} \]  

\[ \cdot \sum_{0}^{F-1} \sum_{0}^{N-1} B_{k,i} \exp \left[ j2\pi h 2^l \left( \sum_{l=0}^{L-1} \gamma_{l, j} \beta(t) + \sum_{l=0}^{L-1} \gamma_{n-l, i} \beta(t - (n-i)T_c) \right) \right]. \]  

(16)

where \( d_{i,l,j} \in \{0, 1\} \) are coefficients in the binary representation of the index \( d_{i,l,j} = \sum_{l=0}^{L-1} a_{i,l,j} \cdot 2^{l-1}, 0 \leq d_{i,l,j} \leq 2^{L-1}-1 \), and functions \( s^{(0)}(t) \) are defined as in (12).

The symbols \( B_{k,i} \) are

\[ B_{k,i} = \prod_{l=0}^{L-1} \exp \left[ j2\pi h 2^l \left( \sum_{l=0}^{L-1} \gamma_{l, j} \beta(t) + \sum_{l=0}^{L-1} \gamma_{n-l, i} \beta(t - (n-i)T_c) \right) \right]. \]  

(17)
where $\gamma_{j,t} \in \{0,1\}$ are coefficients in the binary representation of the information symbol $u_{t,i}$. The integer $j$, used in (15)–(17), is chosen from the set $j \in \{0, \ldots, 2^{F(L-1)}-1\}$ and satisfies $j = \sum_{d_{0,t}}^{F-1} (2^{F(L-1)}d_{0,t})$, where $0 \leq d_{0,t} \leq 2L-1$. Finally, integers $e_{j,t}^{(k-w_j)}$, used in (15)–(17), are chosen to satisfy $0 \leq e_{j,t}^{(k-w_j)} \leq T_{j,t} - 1$ and $\prod_{d=0}^{L-1} e_{j,t}^{(k-w_j)} = 0$, where $T_{j,t}$ are durations of the functions $\prod_{d=0}^{L-1} s(t)^{(j+i+Ld_{0,t}+e_{j,t}^{(k-w_j)})T_{c}}$ and

$$w_{j} = \sum_{n=0}^{j-1} \left( \prod_{t=0}^{F-1} T_{n,t} - 1 \right).$$

An outline for the derivation of (15)–(17) is presented in Appendix A.

### IV. Design Criteria for ST-CPM

This section first derives sufficient conditions under which any $M$-ary partial- and full-response ST-CPM arrangement will attain full spatial diversity for any $L_4$. Then, design rules are specified for coding gain optimization. Optimization of the coding gain for ST-CPM is shown to depend on the CPM frequency/phase shaping pulse, modulation index, and codewords. The modulation indices and phase shaping functions that optimize the coding gain are specified. Finally, optimization of coding gain for ST-CPM and orthogonal ST-CPM codewords is discussed.

The CPM modulator inputs in Fig. 1 are elements from the $L_4 \times N_c$ matrix $U$ defined in (2), while the outputs are elements from the vector $s(t, U) = [s_1(t, U^{(1)}), \ldots, s_{L_4}(t, U^{(L_4)})]^T$, where the signals $s_i(t, U^{(i)})$ are defined in (15) and $1 \leq i \leq L_4$. Assume that $h = K / P$, where $K$ and $P$ are relatively prime integers. For $M$-ary, partial/full-response, ST-CPM codes, we define

$$v^{(i)}_{n,k} \triangleq \left[ \sum_{r=0}^{\frac{n_{c}-1}{2}} u^{(i)}_{r} \right] \mod P,$$

where

$$X^{(i)} = \sum_{t=0}^{L-1} \sum_{r=1}^{L-1} \gamma_{r,w_j}^{(i)} a_{d_{0,t}+r}^{(i)} \in \{0,1\}$$

are coefficients in the binary representation of the symbols $u_{t,i}^{(i)}$, and $a_{d_{0,t}+r}^{(i)} \in \{0,1\}$ are coefficients in the binary representation of the index $d_{0,t}+r \leq 2L-1$. Note that $v^{(i)}_{n,k}$ can only assume values from the set $\{0,1, \ldots, P-1\}$.

From (15)–(18), the $L_4 \times 1$ ST-CPM vector $s(t, U)$ can be written as given in (19) at the bottom of the page, where $g_{k}(t) \triangleq \left[ g_{0,k}(t), \ldots, g_{N_c-1,k}(t) \right]^T$ and $g_{n,k}(t) \triangleq g_{k}(t-nT_c)$. Then, the $L_4 \times 1$ differential ST-CPM vector $\Delta s(t) \triangleq s(t, U) - s(t, \hat{U})$ for two ST codewords, $U$ and $\hat{U}$, is given in the matrix shown in (20), also at the bottom of the page. To simplify further notations, we define the $L_4 \times N_c$ matrix of accumulative values as $V_{k} \triangleq [v^{(i)}_{n,k}]$, where $1 \leq i \leq L_4, 0 \leq n \leq N_c - 1$, and $0 \leq k \leq R-1$. Furthermore, we denote the $L_4 \times N_c$ matrices in (19) and (20) as $Z_{k} \triangleq [e^{2\pi jh v^{(i)}_{n,k}}]$ and $\Delta Z_{k} \triangleq Z_{k} - \hat{Z}_{k}$, respectively. Now, the ST-CPM vector $s(t, U)$ can be rewritten as

$$s(t, U) = \sum_{k=0}^{R-1} Z_{k} g_{k}(t),$$

whereas the differential ST-CPM vector can be rewritten as

$$\Delta s(t) = \sum_{k=0}^{R-1} \Delta Z_{k} g_{k}(t).$$

Using (4) and (22), the matrix of correlation functions of the differential ST-CPM signals, $U_s$, can be written as

$$U_s = \int_{0}^{N_cT_c} \Delta s(t) \Delta s^H(t) dt = \sum_{k=0}^{R-1} \sum_{m=0}^{N_c-1} \Delta Z_{k} G_{k,m} \Delta Z_{m}^H$$

where $G_{k,m} = \int_{0}^{N_cT_c} g_{k}(t) g_{m}^H(t)dt$ and $(\cdot)^H$ denotes the Hermitian operation.

#### A. Full-Diversity Design Criteria

The ST-CPM signals achieve full diversity if the matrix $U_s$ in (23) has full rank for any two ST codewords. In [9], it is shown that the matrix $U_s$ has full rank if the elements of the differential ST-CPM vector $\Delta s(t)$ in (22) are linearly independent. Here, we show that a sufficient condition for achieving full diversity is that the matrices $\Delta Z_{k}$ have full rank $L_4$ for any two
ST codewords $\mathbf{U}$, and $\tilde{\mathbf{U}}$. To show this result, we first introduce the following lemma.

**Lemma 1:** Suppose that all the functions $g_{n,k}(t)$, for $0 \leq n \leq N_c - 1$ and $0 \leq k \leq R - 1$, are collected to form the vector

$$
\mathbf{g}(t) = [g_{0,0}(t), \ldots, g_{N_c-1,0}(t), \ldots, g_{0,R-1}(t), \ldots, g_{N_c-1,R-1}(t)]^T.
$$

The components of the vector $\mathbf{g}(t)$ are linearly independent.

The proof is shown in Appendix B.

Using the results from Lemma 1, we show the following.

**Lemma 2:** If the complex matrices $\Delta \mathbf{Z}_k = (\mathbf{Z}_k - \tilde{\mathbf{Z}}_k)$ have full rank, then the differential ST-CPM vector $\Delta \mathbf{s}(t)$ has linearly independent elements, i.e., the ST-CPM system achieves full spatial diversity $L_k$.

The proof is shown in Appendix C. Note that the Laurent functions $g_{n,k}(t)$ do not affect the full-diversity design criterion. However, they affect the coding gain, as discussed in the next subsection.

Next we determine conditions under which the matrices $\Delta \mathbf{Z}_k$ have full rank. Observe that each $2^F$-ary codeword $\mathbf{U}$, defined in (2), has the form

$$
\mathbf{U} = \sum_{t=0}^{F-1} 2^t \Phi_t(\mathbf{U})
$$

(24)

where $\Phi_t(\cdot)$ denotes the operation $(\mathbf{U}/2^t)_{\text{mod}2}$.

**Theorem 3 (Rank Design Criterion):** Denote $\mathcal{C}$ as a linear $L_t \times N_c$ ST code over the commutative ring of integers modulo-$2^F$, $\mathbb{Z}_{2^F}$, with $L_t \leq N_c$. Suppose that all nonzero codewords $\mathbf{U} \in \mathcal{C}$ have different nonzero modulo-$2^F$ projections $\Phi_0(\mathbf{U})$ and that matrices $\Phi_0(\mathbf{U})$ have full rank over the binary field $\mathbb{F}$. Then for any partial- or full-response $2^F$-CPM scheme with $h = K/2^F$ (where $1 \leq x \leq F$, and $2^F$ and $K$ are relatively prime integers), the ST code $\mathcal{C}$ achieves full spatial diversity $L_k$.

**Proof:** Earlier, we have defined the accumulative matrices $\mathbf{V}_k$ as

$$
\mathbf{V}_k = \left[ \gamma_{i,k}^{(i)} \right] = \left[ \left( \sum_{i=0}^{n-1} u_r^{(i)} - X^{(i)} \right) \mod 2^F \right]
$$

(25)

where $1 \leq i \leq L_t$, $0 \leq n \leq N_c - 1$, $0 \leq k \leq R - 1$,

$$
X^{(i)} = \sum_{l=0}^{F-1} 2^l \sum_{r=1}^{L_t-1} \gamma_{r,l}^{(i)} u_r^{(i)} a_{l,d_{r,l}}, \gamma_{r,l}^{(i)} \in \{0,1\}
$$

are coefficients in the binary representation of the symbols $u_r^{(i)}$, $a_{l,d_{r,l}} \in \{0,1\}$ are coefficients in the binary representation of the index $0 \leq d_{r,l} \leq 2^{L_t} - 1$, and $1 \leq x \leq F$. The modulo-$2^F$ projection of the matrix $\mathbf{V}_k$ is

$$
\Phi_0(\mathbf{V}_k) = \left[ \left( \sum_{i=0}^{n-1} e_{i,k}^{(i)} \phi_0 \left( u_r^{(i)} \right) \right) \mod 2^F \right] \Phi_0 \left( X^{(i)} \right)
$$

(26)

where $\oplus$ denotes modulo-$2^F$ summation. Since rows in the accumulative matrices $\Phi_0(\mathbf{V}_k)$ are obtained using linear nonsingular transformations of the rows in $\Phi_0(\mathbf{U})$, which do not change the rank property, the accumulative matrices $\Phi_0(\mathbf{V}_k)$ also have full rank. From the injection property of these linear transformations, if $\Phi_0(\mathbf{U}) \neq \Phi_0(\mathbf{U})$ then $\Phi_0(\mathbf{V}_k) \neq \Phi_0(\mathbf{V}_k)$.

Next, we show that the matrices $\Delta \mathbf{Z}_k$ have full rank for any two different accumulative matrices $\mathbf{V}_k$ and $\tilde{\mathbf{V}}_k$. First, we note that $\Delta \mathbf{Z}_k$ are matrices over the cyclotomic number field $\mathbb{Q}[\beta]$, where $\beta = \exp(j2\pi K/2^F)$. Let $\mathbb{Z}[\beta]$ denote the ring of algebraic integers in $\mathbb{Q}[\beta]$. The matrices $\Delta \mathbf{Z}_k$ have full rank if it is impossible to find a nonzero vector $\mathbf{K} = [k_1, \ldots, k_L]^T$ such that

$$
\mathbf{K}^T \Delta \mathbf{Z}_k = 0,
$$

(27)

elements of the vector $\mathbf{K}$ are drawn from $\mathbb{Z}[\beta]$, and not all of them are divisible by $(1 - \beta)$ [15]. We will prove this by contradiction. Assume that it is possible to find such a vector $\mathbf{K}$. Since $\Phi_0(\mathbf{V}_k) \neq \Phi_0(\tilde{\mathbf{V}}_k)$, from Lemma 16 and the proof of Theorem 4 in [15], we have

$$
\Delta \mathbf{Z}_k = (1 - \beta) \Phi_0(\mathbf{V}_k) + \Phi_0(\tilde{\mathbf{V}}_k)[\text{mod} (1 - \beta)]
$$

(28)

and

$$
\mathbf{K}^T \Delta \mathbf{Z}_k = 0 \Rightarrow \mathbf{K}^T (1 - \beta) = 0\,\!^T
$$

$$
\Rightarrow \mathbf{K}_1^T (\Phi_0(\mathbf{V}_k) + \Phi_0(\tilde{\mathbf{V}}_k)) = 0\,\!^T [\text{mod} (1 - \beta)]
$$

(29)

where $\mathbf{K}_1 = [\Phi_0(\mathbf{V}_k) + \Phi_0(\tilde{\mathbf{V}}_k)]$ from the code linearity follows that the accumulative matrices $\Phi_0(\mathbf{V}_k) + \Phi_0(\tilde{\mathbf{V}}_k)$ also have full rank. Hence, (27) cannot be satisfied, which leads to contradiction.

The previous theorem showed that if the ST code $\mathcal{C}$ satisfies the CPM rank criterion, the matrices $\Delta \mathbf{Z}_k$ have full rank for any two different accumulative matrices $\mathbf{V}_k$ and $\tilde{\mathbf{V}}_k$. Lemma 2 proved that the elements of the differential ST-CPM vector $\Delta \mathbf{s}(t)$ are linearly independent if matrices $\Delta \mathbf{Z}_k$ have full rank. In [9], it is shown that the matrix $\mathbf{U}_s$ has full rank iff the elements of the differential ST-CPM vector $\Delta \mathbf{s}(t)$ are linearly independent. Consequently, the overall ST-CPM system achieves full diversity.

**Remark 1:** The CPM rank criterion requires that different codewords $\mathbf{U} \in \mathcal{C}$ have different modulo-$2^F$ projections $\Phi_0(\mathbf{U})$ because the linear transformation used in (25) to obtain the accumulative matrices $\mathbf{V}_k$ only preserves the rank of the lowest bit code matrix $\Phi_0(\mathbf{U})$. Hence, if $\Phi_0(\mathbf{U}) = \Phi_0(\tilde{\mathbf{U}})$ there is no guarantee that matrices $\Delta \mathbf{Z}_k$ will have full rank even though codewords $\mathbf{U}$ and $\tilde{\mathbf{U}}$ have full rank. The higher bit matrices $\Phi_{1,\ldots,x}(\mathbf{U})$ can be used to optimize coding gain, as discussed in the sequel. Also note that if all modulo-$2^F$ projections are zero, the rank design criterion applies to modulo-$4$ projections, i.e., $\Phi_0(\mathbf{U}/2)$.

**Remark 2:** The proposed rank criterion for $2^F$-ary CPM with modulation index $h = 1/2$ is similar to the BPSK rank criterion [7] and, hence, it provides both necessary and
sufficient conditions for full spatial diversity, whereas the proposed rank criterion for 2\(F\)-ary CPM with modulation indices \(h = \{1/4, \ldots, 1/2^F\}\) provides only sufficient conditions for full spatial diversity. Nonetheless, the CPM rank criterion is a useful design tool for ST-CPM.

**Remark 3:** To derive the CPM rank criterion we used the linear decomposition of CPM signals with tilted phase. The CPM rank criterion applies to the original tilted-phase or excess-phase CPM waveform as well.

### B. Optimization of Coding Gain

Once full diversity is guaranteed, the next objective is to maximize the coding gain, \(\xi_{\text{PEP}}(\Delta s(t))\), over all pairs of distinct codewords \(U\) and \(\tilde{U}\) defined by geometric-mean of the nonzero eigenvalues of the matrix \(U_s\), i.e.,

\[
\xi_{\text{PEP}}(\Delta s(t)) = \left( \frac{1}{L_t} \prod_{i=1}^{L_t} \lambda_i \right)^{1/L_t} = |U_s|^{1/L_t}
\]

where \(\cdot |\cdot\) denotes the determinant operation. From (23), the determinant \(U_s\) can be written as

\[
|U_s| = \left| \sum_{k=0}^{R-1} \sum_{m=0}^{R-1} \Delta Z_k G_{k,m} \Delta Z_k^H \right|
\]

Equation (31) shows that maximization of the coding gain for ST-CPM codes is more difficult than that for linearly modulated ST codes, because the coding gain is not only a function of codewords \(U\) and \(\tilde{U}\), but also a function of the phase shaping pulses used in the vectors \(g_k\). The following theorem introduces the coding gain design criterion for ST-CPM codes.

**Theorem 4: (Coding Gain Design Criterion)** The coding gain \(\xi_{\text{PEP}}(\Delta s(t))\) is maximized if the positive-definite Hermitian matrix \(\sum_{k=0}^{R-1} \Delta Z_k G_{k,m} \Delta Z_k^H\) is a semi-identity matrix with maximized diagonal elements \(\sum_{k=0}^{R-1} \text{tr}(\Delta Z_k G_{k,m} \Delta Z_k^H)/L_t\) (i.e., constrained on the trace \(\text{tr}(U_s)\)).

**Proof:** This claim can be proven by observing that the matrices \(G_{k,m}\) are different from zero only for coefficients \(k = m\) (this will be proven in Section IV-C) and using the results in [23, Example 2.B].

To find the codes, phase shaping pulses, and modulation indices (i.e., the terms in matrix \(\sum_{k=0}^{R-1} \Delta Z_k G_{k,m} \Delta Z_k^H\)) that jointly maximize the coding gain, computer search is required. However, in this paper, we are interested in defining a theoretical framework that leads to coding gain improvement or optimization, as discussed below.

**Proposition 5: (Coding Gain Improvement)** The coding gain \(\xi_{\text{PEP}}(\Delta s(t))\) can be improved (or optimized) if the matrices \(G_{k,k}\) and \(\Delta Z_k \Delta Z_k^H\) are designed to be semi-identity matrices (i.e., \(G_{k,k} = \text{tr}(G_{k,k}) I_{N_c}/N_c\) and \(\Delta Z_k \Delta Z_k^H = \text{tr}(\Delta Z_k \Delta Z_k^H) I_{L_t}/L_t\), where \(I_{N_c}\) and \(I_{L_t}\) are the \(N_c \times N_c\) and \(L_t \times L_t\) identity matrices) with maximized product of traces \(\text{tr}(G_{k,k})\) and \(\text{tr}(\Delta Z_k \Delta Z_k^H)\).

**Reasoning Behind the Proposition:** From Theorem 4 follows that the first step toward coding gain optimization is to design the matrix \(\sum_{k=0}^{R-1} \Delta Z_k G_{k,k} \Delta Z_k^H\) as a semi-identity matrix. Note that the PAM pulse shaping functions \(g_{n,k}(\ell)\) in \(G_{k,k}\) can be separated from the transmitted symbols \(r(l)\) and considered as part of the channel impulse response [24]. This allows us to separate the influence of the pulse shaping functions and codewords on the coding gain and optimize the matrices \(\sum_{k=0}^{R-1} \Delta Z_k G_{k,k} \Delta Z_k^H\) and \(G_{k,k}\) separately. The matrices \(G_{k,k}\) will be optimized if they are designed to be semi-identity matrices, \(G_{k,k} = Q_k I_{N_c}\), where \(I_{N_c}\) is the \(N_c \times N_c\) identity matrix and \(Q_k = \text{tr}(G_{k,k})/N_c\). Then, the matrix \(U_s\) simplifies to \(\sum_{k=0}^{R-1} Q_k \Delta Z_k \Delta Z_k^H\). The matrix \(U_s\) will be optimized if it is designed to be a semi-identity matrix constrained on its trace. There is no unique solution for this problem. To simplify the problem, we choose a solution \(\Delta Z_k \Delta Z_k^H = P_k I_{L_t}\), where \(I_{L_t}\) is the \(L_t \times L_t\) identity matrix and \(P_k = \text{tr}(\Delta Z_k \Delta Z_k^H)/L_t\). Then, it follows that the coding gain \(\xi_{\text{PEP}}(\Delta s(t))\) can be improved if the matrices \(G_{k,k}\) and \(\Delta Z_k \Delta Z_k^H\) are designed to be semi-identity matrices with maximized product of traces \(\text{tr}(G_{k,k})\) and \(\text{tr}(\Delta Z_k \Delta Z_k^H)\), what was our claim.

### C. Improvement of Coding Gain Through Phase Shaping Pulses

In this subsection, we first show that the matrices \(G_{k,m}\) are equal to zero. Then, we investigate conditions under which the matrices \(G_{k,k}\) can be constructed as semi-identity matrices. A general analysis of the matrices \(G_{k,k}\) is difficult because functions \(g_{n,k}(\ell)\) depend on the memory length \(L\). Hence, we analyze matrices \(G_{k,k}\) for cases of practical importance (\(L = 1, 2\)). The results show that the matrices \(G_{k,k}\) are semi-identity matrices for the raised cosine frequency shaping function with memory length \(L = 1, 2\) and the modulation indices \(h = 1/2^F\) for \(1 \leq \gamma \leq (F - 1/2)\). Finally, the modulation indices \(h\) should be chosen to maximize the traces \(\text{tr}(G_{k,k})\).

From (16), observe that functions \(g_{n,k}(\ell)\) are nonzero in the time interval \(nT_c \leq \ell \leq (n + L+1)T_c\). Then, the matrix \(G_{k,m}\) defined in (23) has the form given in (32) at the bottom of the page, where \((\cdot)^*\) denotes complex conjugate operation and the functions \(g_{n,k}(\ell)\) are defined in (16). Appendix D shows that for an arbitrary phase shaping function \(\beta(t)\), matrices \(G_{k,l} = k\neq m, 1<t<8\)
are zero matrices. Then, the matrix $G_{k,k}$ in (32) becomes expression (33) shown at the top of the following page. In general, the matrix $G_{k,k}$ has equal diagonal elements, i.e.,

$$
\int_0^{(L+1)T_c} |g_k(t)|^2 dt = \int_0^{(L+2)T_c} |g_k(t-T_c)|^2 dt = \cdots = \int_0^{(N_c+L)T_c} |g_k(t-(N_c-1)T_c)|^2 dt.
$$

Since all diagonal elements of the matrix $G_{k,k}$ are equal, we just need to define conditions when $G_{k,k}$ is a diagonal matrix. However, a general analysis of the matrices $G_{k,k}$ is difficult because the Laurent functions $g_n(t)$ depend on the memory length $L$. Hence, we will analyze matrices $G_{k,k}$ for cases of practical importance ($L = 1, 2$).

1) Full-Response CPM: For full-response CPM ($L = 1$), the matrix $G_{k,k}$ in (33) becomes (34), shown at the bottom of the page, where the Laurent functions $g_n(t)$ can be written as

$$
g_n(t) = e^{\pi j h W (t-n T_c)} \frac{F-1}{\prod s(t) \left( t - (n - \epsilon_{j,j}^{(k-w_j)}) T_c \right)}.
$$

(35)

Since all diagonal elements are equal, the matrix $G_{k,k}$ will be semi-identity if

$$
I_{n,n+1} = \int_{(n+1)T_c}^{(n+2)T_c} g_{n+1,k}(t) g_n^*(t) dt = 0
$$

and

$$
I_{n+1,n} = \int_{(n+1)T_c}^{(n+2)T_c} g_{n+1,k}(t) g_n^*(t) dt = 0.
$$

We start evaluation of these integrals, by evaluating the products

$$
g_n(t) g_n^{*}(t) = e^{\pi j h W (t-n T_c)} e^{-\pi j h W (t-(n+1) T_c)}
\times \prod_{l=0}^{F-1} s(t) \left( t - (n - \epsilon_{j,j}^{(k-w_j)}) T_c \right)
\times s(t) \left( t - (n + 1 - \epsilon_{j,j}^{(k-w_j)}) T_c \right)^*
$$

for $k \in \{1, \ldots, R-1\}$. Following similar reasoning as in Appendix D, we can conclude that the product $g_n(t) g_n^{*}(t)$ is always zero. A similar argument can be used to show that $g_n^*(t) g_n(t) = 0$. Hence, for an arbitrary phase shaping function $\beta(t)$, matrices $G_{k,k}$, $k \in \{1, \ldots, R-1\}$, are semi-identity matrices.

Previous reasoning cannot be applied to matrix $G_{0,0}$, because all integers $\epsilon_{j,j}^{(k-w_j)} = 0$ for $k = 0$. Hence, we need to evaluate the integrals

$$
I_{n,n+1} = \int_{(n+1)T_c}^{(n+2)T_c} g_{n,0}(t) g_{n+1,0}(t) dt
$$

for particular phase shaping functions. Using (35) and $M = 2^F$, the integral $I_{n,n+1}$ can be written as

$$
I_{n,n+1} = e^{j 2 \pi n h} \int_{(n+1)T_c}^{(n+2)T_c} \frac{2^F \prod s(t) \sin(\pi h T_c)}{\sin(\pi h T_c)} dt
$$

as derived in Appendix E. In general, $I_{n,n+1} \neq 0$. We have observed that these integrals have solutions in the form $\lambda T_c$, where $\lambda$ is a constant that depends on the modulation index $h$ and phase shaping function $\beta(t)$. Hence, the integrals $I_{n,n+1}$ will be minimized if the constant $\lambda$ is made as small as possible. Furthermore, the following proposition shows that for commonly used phase shaping functions $I_{n,n+1} = 0$ and $I_{n+1,n} = 0$.

**Proposition 6:** For the phase shaping functions [25]

$$
\beta(t) = \begin{cases} 
\frac{b}{2 a} \left( \frac{x}{T_c} - 1 \right) + \frac{b}{2}, & 0 \leq t < a T_c \\
\frac{x}{T_c} \left( \frac{x}{T_c} - 1 \right) + \frac{b}{2}, & a T_c \leq t \leq (1-a) T_c \\
\frac{b}{2}, & (1-a) T_c \leq t \leq T_c 
\end{cases}
$$

with adequate selection of parameters $a$ and $b$, $I_{n,n+1} = I_{n+1,n} = 0$. For the raised cosine (1RC) frequency shaping function $\phi(t)$, with the modulation indices $h = 1/2^x$, where $1 \leq x \leq F - 1$, the integrals $I_{n,n+1}$ and $I_{n+1,n}$ are zero.

The proof is shown in Appendix F.

**Remark 4:** If the rectangular pulse (1REC) is selected as the frequency shaping function $\phi(t)$ (i.e., $\beta(t)$ has $a = 1$ and $b = 1$), the integrals $I_{i+1,i}$ and $I_{i+1,i}$ are not always zero. However, for minimum-shift keying (MSK) modulation (1REC with $h =$
2) Partial-Response CPM: For \( L = 2 \) partial-response CPM, the Laurent functions \( g_{n,k}(t) \) become

\[
g_{n,k}(t) = e^{i\pi h_W (t-nT_c)} \prod_{l=0}^{F-1} s(l) (t - (n - e_{jd,l,j} - e_{jd,l,j}^{(k)})) T_c
\]

where \( a_{jd,j} \in \{0,1\} \).

For indices \( k \) having at least one coefficient \( a_{jd,j} = 1 \), \( g_{n,k}(t) \neq 0 \) only on the time interval \( nT_c \leq t \leq (n + 1)T_c \). Since there is no overlap among functions \( g_{n,k}(t) \) and \( g_{n,y}(t) \) for \( y > 0 \), matrices \( G_{k,k} \) are semi-identity for arbitrary phase shaping functions. For indices \( k \) for which \( G_{k,k} \) are not semi-identity for an arbitrary phase shaping function, the integrals

\[
I_{n,n+1} = \int_{(n+1)T_c}^{(n+3)T_c} g_{n,k}(t) g_{n+1,k}(t)^* dt
\]

and

\[
I_{n+1,n} = \int_{(n+3)T_c}^{(n+5)T_c} g_{n+1,k}(t) g_{n,k}(t)^* dt
\]

for integers \( e_{jd,l}^{(k)} \in \{0,1\} \), and

\[
I_{n,n+2} = \int_{(n+2)T_c}^{(n+4)T_c} g_{n,k}(t) g_{n+2,k}(t)^* dt
\]

and

\[
I_{n+2,n} = \int_{(n+4)T_c}^{(n+6)T_c} g_{n+2,k}(t) g_{n,k}(t)^* dt
\]

for \( e_{jd,l}^{(k)} = 0 \) should be minimized. Furthermore, the following proposition shows when the integrals \( I_{n,n+1}, I_{n+1,n}, I_{n,n+2}, \) and \( I_{n+2,n} \) are zero.

**Proposition 7:** For the raised cosine (2RC) frequency shaping function, \( q(t) \), with the modulation indices \( h = 1/2^p \), where \( 1 \leq x \leq F-1 \), the integrals \( I_{n,n+1}, I_{n+1,n}, I_{n,n+2}, \) and \( I_{n+2,n} \) are zero.

The proof is shown in Appendix G.

**D. Improvement of Coding Gain Through ST-CPM Codewords**

As shown in Proposition 5, the coding gain can be improved if matrices \( \Delta Z_k \Delta Z_k^H \) are semi-identity with maximized trace

\[
\text{tr}(U_s) = \sum_{k=0}^{R-1} Q_k \text{tr}(\Delta Z_k \Delta Z_k^H)
\]

In this section, we derive constraints on ST-CPM codewords that lead to trace maximization and construct orthogonal ST-CPM codewords (i.e., an example of codewords with semi-identity matrices \( \Delta Z_k \Delta Z_k^H \)).

1) **Trace Maximization:** The trace \( \text{tr}(U_s) \) is equal to

\[
\text{tr}(U_s) = \sum_{k=0}^{R-1} Q_k \text{tr}(\Delta Z_k \Delta Z_k^H) = \sum_{k=0}^{R-1} Q_k d_{Eh_k}(Z_k, Z_k)
\]

where\( d_{Eh_k}(Z_k, Z_k) \) is defined in (18) and \( d_{Eh_k}(Z_k, Z_k) \) is the squared Euclidean distance between the code matrices \( Z_k = [e^{2n\pi h_W(i)}] \) and \( Z_k = [e^{2n\pi h_W(i+\alpha)}] \). The maximization of \( \text{tr}(U_s) \) is not straightforward because elements in matrices \( V_k \) and \( V_k \) are obtained as linear combinations of elements in the codewords \( U \) and \( \tilde{U} \). Using (25), the trace of the matrix \( U_s \) becomes

\[
\text{tr}(U_s) = \sum_{k=0}^{R-1} Q_k \sum_{i=1}^{L_k} \sum_{n=0}^{N_k-1} 4 \sin^2 \left( \pi h_f (t_{i,n,k} - \phi(i)) \right)
\]

where elements \( t_{i,n,k} \) are defined in (18) and \( d_{Eh_k}(Z_k, Z_k) \) is the squared Euclidean distance between the code matrices \( Z_k = [e^{2n\pi h_W(i)}] \) and \( Z_k = [e^{2n\pi h_W(i+\alpha)}] \). The maximization of \( \text{tr}(U_s) \) is straightforward because elements in matrices \( V_k \) and \( V_k \) are obtained as linear combinations of elements in the codewords \( U \) and \( \tilde{U} \). Using (25), the trace of the matrix \( U_s \) becomes

\[
\text{tr}(U_s) = \sum_{k=0}^{R-1} Q_k \sum_{i=1}^{L_k} \sum_{n=0}^{N_k-1} 4 \sin^2 \left( \pi h_f (t_{i,n,k} - \phi(i)) \right)
\]

where functions \( f_{k,F}(\cdot) \) are defined as \( f_{k,F}(U) = [f_{k,F}(t_{i,n,k})] = V_k, F = 2^p, 1 \leq H \leq F - 1, \) and \( k \in \{0,1,\cdots,p-1\} \). From (41) follows that the trace of the matrix \( U_s \) over all pairs of codewords \( U \neq \tilde{U} \in C \) will be optimized if the squared minimum Euclidean distances

\[
d_{E_{\text{min}}}(Z_k, Z_k) = \min \{d_{E_k}(e^{2n\pi h_W(i,n,k)}), e^{2n\pi h_W(i,n,k)}) \} : U \neq \tilde{U} \in C \}
\]

for \( k \in \{0,1,\cdots,p-1\} \) are maximized. By observing that each matrix of accumulative values \( V_k \) has the form

\[
V_k = f_{k,F}(U) = \sum_{i=1}^{H-1} 2 \Phi_f(f_{k,F}(U))
\]

where \( \Phi_f(\cdot) \) denotes operation \((f_{k,F}(U))^{2}\)mod2, and by using the results in [26], the minimum trace of the matrix \( U_s \) over all pairs of codewords \( U \neq \tilde{U} \in C \) can be written as

\[
\min_{U \neq \tilde{U} \in C} \text{tr}(U_s) = \sum_{k=0}^{R-1} Q_k \sum_{i=1}^{L_k} \sum_{n=0}^{N_k-1} 4 \sin^2 \left( \pi h_f (t_{i,n,k} - \phi(i)) \right)
\]

where elements \( t_{i,n,k} \) are defined in (18) and \( d_{Eh_k}(Z_k, Z_k) \) is the minimum Hamming distance over all code matrices

\[
\Phi_f(f_{k,F}(U)) = \Phi_f(f_{k,F}(U))
\]

From (41) follows that the trace of the matrix \( U_s \) over all pairs of codewords \( U \neq \tilde{U} \in C \) will be optimized if all minimum Hamming distances \( d_{E_{\text{min}}}(Z_k, Z_k) \) are maximized. When the codewords \( U \) cannot be designed to maximize all minimum Hamming distances \( d_{E_{\text{min}}}(Z_k, Z_k) \), then they should be designed to maximize the minimum Hamming distances associated with the matrices \( G_{k,k} \) that contain most of the signal energy. Finally, we note that a design approach relying on the trace maximization with the rank constraint on the codeword matrices can lead to coding gain improvement.

2) **Orthogonal ST-CPM Codewords:** Since orthogonal space-time code rates satisfy the semi-identity requirement in Proposition 5, matrices \( Z_k \) can be designed as orthogonal space-time (OST) codewords. We will start with the orthogonal code for two transmit antennas proposed by Alamouti [18]. The elements in the matrix \( Z_s \) should be equal to elements in the Alamouti’s codeword, i.e.,

\[
Z_k = \begin{bmatrix}
e^{2\pi h_W(i,n,k)} & -x_2 \ne^{2\pi h_W(i,n,k)} \\
e^{2\pi h_W(i,n,k)} & x_2 \\
e^{2\pi h_W(i,n,k)} & -x_1 \ne^{2\pi h_W(i,n,k)} \\
e^{2\pi h_W(i,n,k)} & x_1
\end{bmatrix}
\]
where $x_1$ and $x_2$ are complex numbers from the modulation alphabet. Code elements $v_{1,k}^{(2)}$ and $v_{2,k}^{(2)}$ should be chosen to satisfy

$$v_{1,k}^{(2)} = \left(1/(2h) - v_{1,k}^{(1)}\right) \mod P$$

and

$$v_{2,k}^{(2)} = \left(1/h - v_{1,k}^{(1)}\right) \mod P$$

where the modulation index is $h = 1/P$ and $P = 2^e$ for $1 \leq x \leq F - 1$. The other two code elements $v_{1,k}^{(1)}$ and $v_{2,k}^{(1)}$ should be selected to maximize coding gain, i.e., $|v_{1,k}^{(1)}|^2 + |v_{2,k}^{(1)}|^2$.

For more than two transmit antennas, orthogonal ST codes are unsuitable for CPM modulation, because orthogonal ST codes have some zero elements in the code matrices whereas symbols $e^{j2\pi h x_{m,k}}$ are nonzero. Quasi-orthogonal ST codes may be used instead. The code matrices $V_k$ can be designed using a similar algorithm as for orthogonal ST codes.

V. EXAMPLES AND NUMERICAL RESULTS

Several ST-CPM codewords are constructed using our rank and coding gain design criteria. Simulation results are presented to verify designed criteria. Each spatial channel is modeled as being independently Rayleigh faded. All simulations use a frame length of 100. The receiver is designed as in [21] with $L_r = 1$. As pointed out earlier, we used the linear decomposition of CPM signals with tilted phase to derive the rank and coding gain criteria, but these criteria also apply to the original tilted-phase or excess-phase CPM waveforms. Depending on the CPM waveform that is transmitted, reduced-complexity receivers can be used, but they are beyond the scope of this paper.

The first example uses a full-response raised cosine (HRC) frequency shaping function. Two ST codewords are constructed from a $(4 \times 4)$-ary CPM ST code $C$ that satisfies the rank design criterion. We construct codewords $U \in C$ as $U = f_{0b}^{(2)} \left(\sum_{k=0}^{k-1} 2U_k\right)$, where $f_{0b}^{(2)}(\cdot)$ denotes the inverse of the function $f_{0b}^{(1)}(\cdot)$ defined in (41). $U_k$ are binary codewords from linear $(4 \times 4)$ ST codes $G_t$, all codewords $U_0 \in G_0$ have full rank over $\mathbb{F}$, and all codewords $U_0$ satisfy $U_0 \neq U_0$. Note that any method for constructing CPM ST codes that satisfies the rank design criterion can be used. Some of the methods for constructing binary ST codewords with full rank are described in [7] and [14]. According to the rank design criterion, the binary codewords $U_1, U_1, U_2, U_2$ and $U_2$ can be arbitrary selected. We first construct binary codewords $U_1, U_1, U_2$, and $U_2$ the same way as codewords $U_0$ and $U_0$, and use them as a benchmark to measure further coding gain improvement. Following the method described in [14], we use $\alpha$ as a zero of the primitive polynomial $f(x) = x^4 + x + 1$ over $\mathbb{F}$ and we use the generator matrix $G = \left[ \begin{array}{c} 1 \\ \alpha \\ \alpha^2 \\ \alpha^3 \\ \alpha^4 \end{array} \right]$ to construct benchmark codewords

$$U_{bn} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} = f_{0b}^{(2)} \left[ \begin{array}{c} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right] + 2 \left[ \begin{array}{c} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right]$$

and

$$U_{bm} = \begin{bmatrix} 5 & 1 & 6 & 7 \\ 2 & 1 & 2 & 7 \\ 5 & 0 & 5 & 4 \\ 6 & 4 & 3 & 6 \end{bmatrix} = f_{0b}^{(2)} \left[ \begin{array}{c} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right] + 2 \left[ \begin{array}{c} 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{array} \right] + 4 \left[ \begin{array}{c} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{array} \right]$$

According to the CPM rank criterion, these two codewords will achieve full spatial diversity, if modulation index $h$ takes values from the set $\{1/2, 1/4, 1/8\}$. We now verify this statement for each $h$ in the set.

First, let $h = 1/4$. To verify that codewords (47) and (48) achieve full diversity, we need to check if all functions in the differential matrix $\Delta \phi(t)$, defined in (22), are linearly independent. This will be satisfied if components of the vector $g_h(t)$ are linearly independent and the matrices $\Delta Z_k$ have full rank. From (35), we calculate functions $g_h(t)$. Results are presented in the first column of Table I from which we conclude that they are linearly independent. The first column in Table II shows the matrices $\Delta Z_k = (Z_k - \bar{Z}_k)$, where $\bar{Z}_k = e^{j2\pi h f_{0b}(U_{bn})}$. Observe that all matrices have full rank, implying that this pair of codewords achieves full spatial diversity.

Second, let $h = 1/2$. Functions $g_h(t)$ remain the same as in Table I. The second column in Table II shows the recalculated matrices $\Delta Z_k$. Since all matrices have full rank, this pair of codewords achieves full diversity and also satisfies rank criterion proposed by Zhang and Fitz [9].

Finally, for $h = 1/8$ it can be similarly verified that full diversity is achieved.

Since this pair of codewords achieves full spatial diversity, the next objective is to improve their coding gain. According to Proposition 5, the coding gain will be improved if all the $G_{k,k}$ are designed as semi-identity matrices and the determinants $\left|\sum_{k=0}^{k=1} Q_{k} \Delta Z_{k} \Delta Z_{k}^H\right|$ is maximized, where $Q_k = f_{0b}^{(2t)} ||g_h(t)||^2 dt$. From (34), we calculate matrices $G_{k,k}$ and determinants $|G_{k,k}|$ for modulation index $h = 1/4$. The results are presented in the second and third columns of Table I. Obtained results confirm theoretical results derived in Subsection IV-C. The $G_{k,k}$, $k \in \{1, \ldots, 6\}$, are semi-identity matrices as expected. Finally, the trace obtained using codewords $U_{bn}$ and $\bar{U}_{bn}$, is equal to $\text{tr}(U_{bn})/4 = \sum_{k=0}^{k=1} Q_k/4 |\bar{Z}_k|^2 (Z_k - \bar{Z}_k)^2$. The minimum trace of this ST-CPM code is $\sum_{k=0}^{k=1} Q_k/4 |\bar{Z}_k|^2 (Z_k - \bar{Z}_k)^2 \approx 3.6$. This result is obtained from (43), using the fact that the code proposed in [14] has the minimum Hamming distance $d_{H_{min}} = 4$.

Since the rank design criterion only requires distinct matrices $U_0$ and $U_0$ to have full rank, the higher bit matrices $U_1, U_1, U_2, U_2, U_2$ and $U_2$ can be used for coding gain improvement. In this example, optimization of binary matrices $U_2$ and $U_2$ is ineffective because the modulation index is $h = 1/4$ and, hence, these matrices do not have impact on obtained values in matrices $\Delta Z_k$. Also, we keep matrices $U_0$ and $U_0$ unchanged, since they satisfy the rank criterion. Using the binary ST code proposed in [16] with elementary polynomial $f(x) = 1 + 32r + 288r^2 + \ldots$
TABLE I

| k | $g_k(t)$ | $G_k$ | $|G_k|$ |
|---|---|---|---|
| 0 | $e^{j\phi W(t)}(0)(0)(0)(0)(2)(t)$ | 0.6165 0 0 0 0 0 | 1.444 \times 10^{-1} |
| 1 | $e^{j\phi W(t)}(0)(t+T_c)(0)(0)(2)(t)$ | 0.1489 0 0 0 0 0 | 4.92 \times 10^{-4} |
| 2 | $e^{j\phi W(t)}(0)(0)(t+T_c)(0)(2)(t)$ | 0.1489 0 0 0 0 0 | 4.92 \times 10^{-4} |
| 3 | $e^{j\phi W(t)}(0)(t+T_c)(0)(0)(2)(t+T_c)$ | 0.3082 0 0 0 0 0 | 9.02 \times 10^{-3} |
| 4 | $e^{j\phi W(t)}(0)(0)(0)(0)(2)(t+T_c)$ | 0.3082 0 0 0 0 0 | 9.02 \times 10^{-3} |
| 5 | $e^{j\phi W(t)}(0)(t+T_c)(0)(0)(2)(t+T_c)$ | 0.1489 0 0 0 0 0 | 4.92 \times 10^{-4} |
| 6 | $e^{j\phi W(t)}(0)(0)(t+T_c)(0)(2)(t+T_c)$ | 0.1489 0 0 0 0 0 | 4.92 \times 10^{-4} |

Further enhancement of the coding gain can be obtained if matrices $U_0$ and $U_1$ are constructed not only to have full rank, but also to optimize coding gain. Using the binary ST code proposed in [16] for codewords $U_0$, $U_1$, $\tilde{U}_0$, and $\tilde{U}_1$, we obtain the optimized codewords

$$U_{\text{opt}} = \left[ \begin{array}{cccc} 4 & 4 & 3 & 5 \\ 4 & 7 & 3 & 0 \\ 3 & 3 & 0 & 2 \\ 5 & 1 & 3 & 0 \end{array} \right] = f_{0.8}^{-1} \left( \begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{array} \right),$$

and

$$\tilde{U}_{\text{opt}} = \left[ \begin{array}{cccc} 5 & 1 & 3 & 0 \\ 2 & 1 & 4 & 5 \\ 7 & 6 & 4 & 3 \\ 6 & 4 & 3 & 6 \end{array} \right] = f_{0.8}^{-1} \left( \begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right).$$

The fourth column of Table II shows the matrices $\Delta Z_{k,\text{imp}}$ obtained using codewords $U_{\text{imp}}^\alpha$ and $U_{\text{imp}}^\beta$. Observe that all matrices still have a full rank. The trace obtained using codewords $U_{\text{imp}}^\alpha$ and $\tilde{U}_{\text{imp}}^\beta$ is equal to

$$\text{tr}(U_s)/4 = \sum_{k=0}^{6} (Q_k/4)^2 E_{\text{imp}}(Z_k, \tilde{Z}_k) \approx 75.24.$$
TABLE II
CODING GAIN OPTIMIZATION FOR FULL-RESPONSE ST-CPM CODES WITH $M = 8$

<table>
<thead>
<tr>
<th>$k$</th>
<th>$\Delta Z_k$ - benchmark $h = 1/4$</th>
<th>$\Delta Z_k$ - benchmark $h = 1/2$</th>
<th>$\Delta Z_k$ - improved $h = 1/4$</th>
<th>$\Delta Z_k$ - optimized $h = 1/4$</th>
</tr>
</thead>
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<tr>
<td>0</td>
<td>$[1-j, 0, 2j, 1-j]$</td>
<td>$[1-j, 2, -2j, 1-j]$</td>
<td>$[1-j, 2, -2j, 1-j]$</td>
<td>$[1-j, 2, -2j, 1-j]$</td>
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<tr>
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<td>$0, 2j, 1-j, 0$</td>
<td>$2, 0, 1-j, 2j$</td>
<td>$2, 2j, 1-j, -1j$</td>
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<tr>
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<td>$-2j, 1-j, 0$</td>
<td>$0, 1-j, -2j$</td>
<td>$2j, 1-j, -2j, 0$</td>
</tr>
<tr>
<td></td>
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<td>$[1+j, 0, -2j, 2j]$</td>
<td>$[1+j, 0, 0, 2j]$</td>
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<tr>
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<td>$0, 2j, 1-j, 0$</td>
<td>$2, 0, -1-j, 2j$</td>
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<td>$[1-j, 2, -2j, 1-j]$</td>
<td>$[1-j, 2, -2j, 1-j]$</td>
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<tr>
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<td>$0, 2j, 1-j, 0$</td>
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<td>$2, 2j, 1-j, -1j$</td>
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<td>$[1-j, 2, -2j, 1-j]$</td>
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<td>$0, 2j, 1-j, 0$</td>
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</tr>
</tbody>
</table>

Fig. 2. Frame-error rate of 8-ary ST-CPM with 1RC and $h = 1/4$ in quasi-static fading.

The minimum trace of this ST-CPM code is increased to $\sum_{k=0}^{6} (Q_{k}/4) \| Z_k \|_F \approx 7.31$.

Fig. 2 compares the performance curves obtained for full-response 8-ary, $h = 1/4$, 1RC, CPM with $L_d = \{1,2,3,4\}$ transmit antennas. The codewords used to obtain the second, third, and fourth curve are constructed to satisfy only the rank design criterion. The fifth curve is obtained using the improved codewords. Fig. 2 plots the frame-error rate versus the signal-to-noise ratio ($E_b/N_0$, where $E_b$ denotes the received energy per bit) and shows that full diversity and coding gain improvement are achieved when the ST codes meet both the rank and trace maximization criteria.

The second example uses a partial-response raised cosine (2RC) frequency shaping function. The ST codewords are chosen from a $(4 \times 4)$ 16-ary CPM ST code $\mathcal{C}$, where codewords $U \in \mathcal{C}$ are constructed as $U = \sum_{l=0}^{3} 2^l U_l$, where
$f^{-1}_{0,16}(\cdot)$ denotes the inverse of the function $f_{0,16}(\cdot)$ defined in (41), $U_i$ are binary codewords from linear $(4 \times 4)$ ST codes $C_t$, all codewords $U_0 \in C_0$ have full rank over $\mathbb{F}$, and all codewords $U_0$ satisfy $U_0 \neq \bar{U}_0$.

Fig. 3 compares the performance curve obtained for partial-response 16-ary CPM signals with 2RC frequency shaping function, $h = 1/4$, and $L_t = \{1, 2, 3, 4\}$ transmit antennas. As in the first example, the codewords used to obtain the second, third, and fourth curve are obtained using the binary ST code described in [14]. The codewords used to obtain the fifth curve are obtained using the binary ST code proposed in [16]. Fig. 3 shows that full diversity and coding gain improvement are achieved when the ST codes are constructed to satisfy the rank and trace maximization criteria.

The last example uses a full-response 1RC frequency shaping function. The ST codewords are chosen from a $(2 \times 2)$ 8-ary CPM ST code $C_t$, where codewords $U \in C$ are constructed as $U = f^{-1}_{0,8}(\sum_{i=0}^{q} 2^i U_i)$, where $f^{-1}_{0,8}(\cdot)$ denotes the inverse of the function $f_{0,8}(\cdot)$ defined in (41), $U_i$ are binary codewords from linear $(2 \times 2)$ ST codes $C_t$ described in [14] with the generator matrix $G = [1 \ \alpha]$, where $\alpha$ is a zero of the primitive polynomial $f(x) = x^2 + x + 1$ over $\mathbb{F}$. All codewords $U_0 \in C_0$ have full rank over $\mathbb{F}$ and all codewords $U_0$ satisfy $U_0 \neq \bar{U}_0$. The performance curves for this 8-ary, 1RC, $h = 1/4$, CPM ST code with $L_t = \{1, 2\}$ transmit antennas are plotted in Fig. 4. Results show that full diversity is obtained using this ST code. The minimum trace of this ST-CPM code is $\sum_{k=0}^{6} (Q_k/2) d_{E_{\min}}^2(Z_k, Z_k) \approx 3.66$. The coding gain can be improved if the codewords are constructed as in (44), using the orthogonal code for two transmit antennas [18]. The minimum trace of this ST-CPM code is $\sum_{k=0}^{6} (Q_k/2) d_{E_{\min}}^2(Z_k, Z_k) \approx 7.31$. We refer to this design as orthogonal ST full-response CPM (OST-FCPM) design. Further enhancement of the coding gain can be obtained if the codewords are constructed using the super-orthogonal two-state trellis ST code for two transmit antennas proposed in [17]. The minimum trace of this ST-CPM code is $\sum_{k=0}^{6} (Q_k/2) d_{E_{\min}}^2(Z_k, Z_k) \approx 9.78$. We refer to this design as super-orthogonal ST full-response CPM (SOST-FCPM) design. Fig. 4 compares our OST-FCPM and SOST-FCPM
designs with OST-FCPM design [12] and ST-FCPM design with mapping scheme [13], respectively. Results show that our SOST-FCPM design has similar performance as ST-FCPM with mapping scheme [13] and performs better than the OST-FCPM design [12] and our OST-FCPM design. Fig. 4 also shows that full diversity and coding gain improvement are achieved when the ST codes are constructed to satisfy the rank and coding gain improvement criteria.

In Section IV-C is shown that the raised cosine frequency shaping function satisfies the coding gain improvement criterion. Figs. 5 and 6 illustrate how much the coding gain is sacrificed if other frequency shaping functions are used instead of the raised cosine frequency shaping function. The results are obtained using the (2 × 2) 8-ary orthogonal ST code and modulation index $\tilde{h} = 1/4$. In Fig. 5, the second curve is obtained using the Gaussian frequency shaping function (GSP, frequency shaping function used in GMSK) with normalized filter bandwidth $BT = 0.25$, the third curve is obtained using the full-response rectangular frequency shaping function (1REC), and the fourth curve is obtained using the full-response raised cosine shaping function (1RC). Results show that the full-response raised cosine shaping function improves the coding gain for approximately 0.89 dB compared to the Gaussian frequency shaping function and approximately 0.57 dB compared to the full-response rectangular frequency shaping function. In Fig. 6, the second and the third curve are obtained using the rectangular
frequency shaping function with \( L = 3 \) (3REC) and \( L = 2 \) (2REC), respectively. The fourth curve is obtained using the Gaussian frequency shaping function (GSP) with normalized filter bandwidth \( BT = 0.25 \). Finally, the fifth and the sixth curves are obtained using the raised cosine shaping function with \( L = 3 \) (3RC) and \( L = 2 \) (2RC), respectively. Results show that the Gaussian frequency shaping function with \( BT = 0.25 \) performs similarly to the raised cosine shaping function with \( L = 3 \). The raised cosine shaping function with \( L = 2 \) improves the coding gain for approximately 0.47 dB compared to the Gaussian frequency shaping function and approximately 0.86 dB compared to the rectangular frequency shaping function with \( L = 2 \). Finally, note that the full-response raised cosine shaping function outperforms the partial-response raised cosine shaping function with \( L = 2 \) for approximately 0.34 dB. This is expected result because all matrices \( G_{k,j} \), \( k \in \{ 1, \ldots, R-1 \} \), defined in (23), are exactly diagonal matrices for full-response frequency shaping pulses, whereas as for partial-response frequency shaping pulses, these matrices are semi-diagonal (nondiagonal elements are much smaller than diagonal ones), which decreases the coding gain.

VI. CONCLUSION

This paper derived sufficient conditions under which \( M \)-ary partial- and full-response CPM ST codes will attain both full spatial diversity and improved coding gain. Using a linear decomposition of CPM signals in a tilted-phase representation, we have identified the rank criterion for the \( M \)-ary partial- and full-response CPM that specifies the set of allowable modulation indices. We have also proposed a coding gain design criterion. It is shown that the optimization of the coding gain for ST-CPM depends on selection of phase shaping pulses, modulation indices, and codewords. We have specified the set of allowable modulation indices and phase shaping functions which can be used toward ST-CPM coding gain improvement (or maximization) and we have established rules for ST-CPM and OST-CPM codeword optimization. Several examples and simulation results show that full spatial diversity and optimal coding gain are achieved for ST-CPM systems that meet the proposed CPM rank criterion and the coding gain design criterion.

APPENDIX A

THE OUTLINE FOR DERIVATIONS OF (15)–(17)

Similar to Laurent’s decomposition [22], \( M \)-ary partial-response CPM signals with tilted phase can be presented as

\[
s(t, \mathbf{u}) = \sum_{i=0}^{R-1} \sum_{n=0}^{N_i-1} e^{2 \pi h A_{i,n}^{(t)}} A_{i,n}^{(t)} (t - nT_c)
\]

(53)

where functions \( A_{i,n}^{(t)} \) and symbols \( A_{i,n}^{(t)} \) are defined below. Functions \( A_{i,n}^{(t)} \) are

\[
A_{i,n}^{(t)} = e^{j \pi h W(t)} \sum_{n=0}^{L-1} s(t + (n + L a_{i,n}) T_c)
\]

(54)

where \( W(t) \) is defined in (8), functions \( s(t) \) are defined as in (12), \( 0 \leq i \leq 2^{L-1} - 1 \), \( 0 \leq t \leq T_c \cdot \min \{ L(2 - a_{i,n}) - n \} \), and \( L = 0, \ldots, F - 1 \). Symbols \( A_{i,n}^{(t)} \) are

\[
A_{i,n}^{(t)} = \sum_{m=0}^{n} \gamma_{m,n}^{L-1} - \sum_{m=0}^{L-1} \gamma_{m-m, n} a_{i,m}^{(t)}
\]

(55)

where \( a_{i,m} \in \{ 0, 1 \} \) are coefficients in the radix-2 representation of \( i = \sum_{m=0}^{L-1} 2^{m-1} a_{i,m}^{(t)} \).

The output signal \( s(t, \mathbf{u}) \) in (53) is presented as a product of two sums; that makes our further analysis difficult. To simplify equations, we modify the output signal \( s(t, \mathbf{u}) \) following the algorithm proposed by Mengali and Morelli [20]. We start from radix-2\( F \)-1 representation of an integer \( j \) from the interval

\[
0 \leq j \leq (2^{F-1})^{F-1} - 1
\]

(56)

depend \( j = \sum_{l=0}^{F-1} 2^{(F-1)d_{j,l}} \), \( \in \{ 0, \ldots, 2^{F-1} - 1 \} \). Then, for each pair \( (j, \mathbf{d}) \), we choose the corresponding \( d_{j,l} \) and use it as a subscript for functions \( \mathbf{d}_{j,l}^{(m)}(t) \) defined in (54). As the next step, we form the vector \( \mathbf{T}_j = \{ T_{j,F-1}, T_{j,F-2}, \ldots, T_{j,0} \} \), where elements of the vector \( T_{j,i} \) are durations of the functions \( \mathbf{d}_{j,l}^{(m)}(t) \). Then, for a given \( \mathbf{T}_j \), we choose the \( F \)-tuples

\[
\mathbf{e}_{j,l}^{(m)} = \{ \mathbf{e}_{j,F-1}^{(m)}, \mathbf{e}_{j,F-2}^{(m)}, \ldots, \mathbf{e}_{j,0}^{(m)} \}
\]

(57)

that have integer components which satisfy \( 0 \leq e_{j,l}^{(m)} \leq T_{j,l} - 1 \), \( \sum_{l=0}^{F-1} e_{j,l}^{(m)} = 0 \), and \( m = 0, 1, 2 \ldots \). It can be shown that there are \( M_j \) such \( F \)-tuples [20], where \( M_j \) is equal to \( \prod_{l=0}^{F-1} T_{j,l} - \prod_{l=0}^{F-1} (T_{j,l} - 1) \). This result implies that index \( n \) can take only values in the interval \( 0 \leq m \leq M_j - 1 \).

Now, the signal \( s(t, \mathbf{u}) \) in (53) can be modified to

\[
s(t, \mathbf{u}) = \sum_{k=0}^{R-1} \sum_{n=0}^{N_i-1} B_{k,n} g_k(t - nT_c)
\]

(58)

where \( R = 2^{F-1} \), \( 2^{F-1} \) and the Laurent functions \( g_k(t) \) and symbols \( B_{k,n} \) are defined as

\[
g_k(t) = \prod_{l=0}^{F-1} d_{j,l}^{(k)} \left[ t + e_{j,l}^{(k)} T_c \right]
\]

(59)

and

\[
B_{k,n} = \exp \left( j 2 \pi h \sum_{l=0}^{F-1} 2^{l} A_{j,l,n}^{(k)} - e_{j,l}^{(k)} \right)
\]

(60)

respectively, where \( e_{j,l}^{(k)} \) are defined in (54), \( A_{j,l,n}^{(k)} = e_{j,l}^{(k)} \) are defined in (55), and \( k = m + \sum_{j=0}^{j-1} M_{n} \). Finally, by substituting (54) in (58) and using the equality \( k = m + \sum_{j=0}^{j-1} M_{n} \), we obtain the expression in (16). Similarly, by substituting (55) into (59), we obtain the expression in (17).

APPENDIX B

PROOF OF LEMMA 1

Proof: We will prove this lemma by contradiction. Assume that not all functions in the vector \( \mathbf{g}(t) \) are linearly
independent. Then, there exists some nonzero complex vector \( [K_0 g_0(t), \ldots, K_{N_c-1} g_{N_c-1}(t)]^T \) that satisfies
\[
\sum_{k=0}^{N_c-1} \{ K_{i-k} g_{i-k}(t) + \cdots + K_{i-1-k} g_{N_c-1-k}(t) \} = 0.
\] (60)

Note that every function \( g_{n,k}(t) \) has nonzero value only over the interval \( nT_c \leq t \leq (n + L + 1)T_c \), where \( 0 \leq n \leq N_c - 1 \). Hence, for an arbitrary time interval \( IT_c \leq t \leq (l + 1)T_c \), (60) can be modified to
\[
\sum_{k=0}^{R-1} \{ K_{l-k} g_{l-k}(t) + \cdots + K_{l+L-k} g_{l+L-k}(t) \} = 0.
\] (61)

From (16), observe that functions \( g_k(t) \) and \( g_{k-x}(t) \) have different combinations of functions \( s^{(0)}(t) \), defined in (12). Then, (61) can be rewritten as
\[
K_{l-1,k} g_{l-1,k}(t) + \cdots + K_{l-k} g_{l-k}(t) + K_{l+L-k} g_{l+L-k}(t) = 0
\] for \( 0 \leq k \leq R - 1 \). For \( 0 \leq l = 1 \), (62) simplifies to
\[
K_{l-1,k} g_{l-1,k}(t) = K_{l-1,k} g_{l-1,k}(t)
\] for nonzero coefficients \( K_{l-1,k} \). When \( (L-1)T_c \leq t \leq lT_c \), the right term in (63) is equal to zero, because \( g_{l-k}(t) = 0 \). The left term in (63) is equal to \( K_{l-1,k} g_{l-1,k}(t) \). This equality is satisfied only if \( K_{l-1,k} = 0 \). Similarly, when \( (L+2)T_c \leq t \leq (l+3)T_c \), the right term in (63) is equal to zero, because \( g_{l-k}(t) = 0 \). Following a similar argument as above, it follows that \( K_{l+L-k} = 0 \). The fact that \( K_{l-1,k} = 0 \) and \( K_{l+L-k} = 0 \) causes a contradiction. Therefore, for \( L = 1 \), all functions \( g_{n,k}(t) \) are linearly independent.

Using similar reasoning as above, we now show that all functions in \( g_k(t) \) are linearly independent for an arbitrary \( L \). For \( (l-1)T_c \leq t \leq (l - L + 1)T_c \), (62) simplifies to
\[
K_{l-1,k} g_{l-1,k}(t) = 0.
\] This equality is satisfied only if \( K_{l-1,k} = 0 \). Similarly, for \( (l+3L-1)T_c \leq t \leq (l + 3L)T_c \), (62) simplifies to \( K_{l+L,k} g_{l+L,k}(t) = 0 \). Following a similar argument as above, it follows that \( K_{l+L,k} = 0 \). For \( (l - L + 1)T_c \leq t \leq (l - L + 2)T_c \), (62) simplifies to \( K_{l-1,k} g_{l-1,k}(t) + K_{l+L,k} g_{l+L,k}(t) = 0 \). Similar reasoning (or induction) for other time intervals, it follows that coefficients \( K_{l-1,k}, \ldots, K_{l-1,k}, K_{l+L-k}, \ldots, K_{l+L-k} \) are zero which causes a contradiction. Therefore, all functions \( g_{n,k}(t) \) are linearly independent for an arbitrary \( L \).

APPENDIX C
PROOF OF LEMMA 2

Proof: We will prove this lemma by contradiction. Assume that the elements of the differential ST-CPM vector \( \Delta s(t) \) are not linearly independent. Then there exists some nonzero complex vector \( [k_1, k_2, \ldots, k_L] \) that satisfies
\[
k_1 \Delta s_1(t) + \cdots + k_L \Delta s_L(t) = \sum_{n=0}^{N_c-1} \left[ k_1 \Delta z_{n,0}^{(1)} + \cdots + k_L \Delta z_{n,0}^{(L)} \right] g_{n,0}(t) + \cdots
\]
\[
\sum_{n=0}^{N_c-1} \left[ k_1 \Delta z_{n,0}^{(1)} + \cdots + k_L \Delta z_{n,0}^{(L)} \right] g_{n,0}(t) + \cdots
\] where \( \Delta z_{n,k}^{(i)} \) are elements of the matrices \( \Delta Z_k \) for \( 0 \leq k \leq R - 1 \). Since all functions \( g_{n,k}(t) \) are linearly independent (as shown in Lemma 1), it follows that
\[
k_1 \Delta z_{n,0}^{(1)} + \cdots + k_L \Delta z_{n,0}^{(L)} = 0
\] (65)
for each \( k \in \{0, \ldots, R-1\} \) and \( n \in \{0, \ldots, N_c-1\} \). Equation (65) implies that the matrices \( \Delta Z_k \) do not have full rank, which contradicts our assumption.

APPENDIX D
PROOF OF EQUATION \( G_{k,m} = 0 \)

To show that the matrices \( G_{k,m} \) are equal to zero, we first evaluate the products
\[
g_{n,k}(t) g_{m,n}(t) = e^{-j \pi W (t-nT_c)} e^{-j \pi W (t-nT_c)}
\] and
\[
\sum_{t=0}^{L-1} s^{(0)} \left[ t - \left( n - L a_{d_{j,i}} - e_{j,(m-w)} \right) T_c \right] \times s^{(0)} \left[ t - \left( n - L a_{d_{j,i}} - e_{j,(m-w)} \right) T_c \right] \times \cdots
\] (66)
where the functions \( s^{(0)}(t) \) are defined in (12), \( k, m \in \{0, \ldots, R-1\}, \) and \( k \neq m \). Note that the integers \( e_{j,(m-w)} \) and \( e_{j,(m-w)} \) are chosen to satisfy \( 0 \leq e_{j,(m-w)} \leq T_1, d_{j,1} \) and
\[
\sum_{t=0}^{L-1} s^{(0)} \left[ t + (n + L d_{j,i} - e_{j,(m-w)}) T_c \right] \times \cdots
\] (67)
Then, at least one integer \( e_{j,(m-w)} \) is zero and at least one integer \( e_{j,(m-w)} \) is zero. Since \( k \) and \( m \) cannot be zero at the same time, we assume that \( n \neq 0 \). Hence, at least one integer \( e_{j,(m-w)} \) is zero and at least one pair of integers \( e_{j,(m-w)} \) and \( e_{j,(m-w)} \) satisfies \( e_{j,(m-w)} \neq e_{j,(m-w)} \). Without loss of generality, we can assume that \( e_{j,(m-w)} = a_{d_{j,i}d_{j,0}} = 0 \) for \( l = i = 0 \). Then, (66) can be modified to
\[
g_{n,k}(t) g_{m,n}(t) = s^{(0)}(t-nT_c) s^{(0)} \left[ t - \left( n - L a_{d_{j,i}d_{j,0}} - e_{j,(m-w)} \right) T_c \right] \times \sum_{t=0}^{L-1} s^{(0)} \left[ t - \left( n - L a_{d_{j,i}} - e_{j,(m-w)} \right) T_c \right] \times \sum_{t=0}^{L-1} s^{(0)} \left[ t - \left( n - L a_{d_{j,i}d_{j,0}} - e_{j,(m-w)} \right) T_c \right] \times \cdots
\] (67)
Note that the function \( s^{(0)}(t-nT_c) \) and at least one of the functions \( s^{(0)}(t-nT_c) - e_{j,(m-w)} T_c \) are nonzero in the time intervals \( nT_c \leq t \leq (n + L + 1)T_c \) and
\[
(n - i - L a_{d_{j,i}} - e_{j,(m-w)}) T_c \leq t \leq (n + L + 1 - i - L a_{d_{j,i}} - e_{j,(m-w)}) T_c
\]
, respectively. Since at least one \( g_{j,k}^{(n-w_2)} \), \( G_{k,j}^{(n,w_2)} > 0 \), these two functions cannot be nonzero at the same time. Hence, the product is always \( g_{j,k}^{(n,w_2)} g_{k,n}^{(n,w_2)} = 0 \). A similar argument can be used to show that \( g_{j,k}^{(n,x)} g_{k,n}^{(n,x)} = 0 \) for \( x > 0 \). Hence, for an arbitrary phase shaping function \( \beta(t) \), matrices \( G_{k,j} \neq k \) are zero matrices.

**APPENDIX E**

*DERIVATION OF EQUATION (37)*

Using (35) and \( M = 2^F \), the integral \( I_{n,n+1} \) can be written as

\[
I_{n,n+1} = \int_{(n+1)T_c}^{(n+2)T_c} \prod_{\ell=0}^{F-1} e^{j\pi h W(t-(n+1)T_c)} e^{-j\pi h W(t-nT_c)} dt \\
\times s(t-(n+1)T_c) s(t-nT_c) \ast dt \\
= \int_{(n+1)T_c}^{(n+2)T_c} \prod_{\ell=0}^{F-1} \exp \left(j \pi h (2^F - 1) \frac{t-nT_c}{T_c} \right) \\
\times \frac{\sin(\pi h 2^F - 2 \pi h 2^l \beta(t-(n+1)T_c))}{\sin(\pi h 2^l)} \times \frac{\exp\left(-j \pi h (2^F - 1) \frac{t-(n+1)T_c}{T_c} \right)}{\sin(\pi h 2^l)} \times \left\{ \frac{\exp\left(j \pi h 2^l \beta(t-(n+1)T_c))}{\sin(\pi h 2^l)} \times \frac{\sin(\pi h 2^F - 2 \pi h 2^l \beta(t-(n+1)T_c))}{\sin(\pi h 2^l)} \right\}^* dt. 
\]

(68)

**APPENDIX F**

*PROOF OF PROPOSITION 6*

**Proof:** The proof of this proposition starts by evaluating the integral \( I_{n,n+1} \) with the phase shaping function defined in (38). Choosing parameters \( a = 0.5 \) and \( b = 0 \), the integral \( I_{n,n+1} \) is equal to

\[
I_{n,n+1} = e^{2j\pi h (2^F - 1)} \int_{(n+1)T_c}^{(n+2)T_c} \prod_{\ell=0}^{F-1} \frac{\sin(\pi h 2^l - \pi h 2^l \beta(t-(n+1)T_c))}{\sin(\pi h 2^l)} \frac{\sin(\pi h 2^l)}{\sin(\pi h 2^l)} dt = 0. 
\]

(69)

Also, if we choose parameters \( a = 1, b = 2 \), and the modulation index \( h = 1/2 \), the integral \( I_{n,\ast+1} \) is equal to

\[
I_{n,n+1} = e^{2j\pi h (2^F - 1)} \int_{0}^{T_c} \prod_{\ell=0}^{F-1} \sin\left(\frac{\pi 2^l - \pi 2^l \beta(t-(n+1)T_c)}{T_c}\right) \sin\left(\frac{\pi 2^l T_c}{T_c}\right) dt = 0. 
\]

(70)

Note that the same result can be obtained for other values of parameters \( a \) and \( b \).

If the raised cosine function (1RC) is selected as the phase shaping function \( \beta(t) \) and modulation index is \( h = 1/2^x \), where \( 1 \leq x \leq F - 1 \), the integrals \( I_{n,n+1} \) are equal to

\[
I_{n,n+1} = e^{2j\pi h (2^F - 1)} \int_{0}^{T_c} \prod_{\ell=0}^{F-1} \frac{\sin\left(\pi 2^l - \pi 2^l \beta(t-(n+1)T_c)\right)}{\sin(\pi 2^l)} \times \frac{\sin\left(\pi 2^l - \pi 2^l \beta(t-(n+1)T_c)\right)}{\sin(\pi 2^l)} dt. 
\]

(71)

Using the asymptotic expansion of integrals [29], (71) can be simplified to yield

\[
I_{n,n+1} \approx e^{2j\pi h (2^F - 1)} \prod_{\ell=0}^{F-1} \frac{1}{\sin(\pi 2^l - \pi 2^l \beta(t-(n+1)T_c))} \\
\times \int_{0}^{T_c} \frac{\sin\left(\pi 2^l - \pi 2^l \beta(t-(n+1)T_c)\right)}{\sin(\pi 2^l)} \left\{ \frac{a}{3} \sin\left(\pi 2^l - \pi 2^l \beta(t-(n+1)T_c)\right) \right\}^3 \\
\times \frac{4}{15} \sin\left(\pi 2^l - \pi 2^l \beta(t-(n+1)T_c)\right)^5 \times 2\pi \times (\pi T_c)^3 \\
= 0. 
\]

(72)

By noting that \( I_{n,\ast+1} = \text{a real function} \) and that all functions inside the integrals \( I_{n,\ast+1} \) are real functions, we can conclude that integrals \( I_{n,\ast+1} \) are also equal to zero. \( \Box \)

**APPENDIX G**

*PROOF OF PROPOSITION 7*

**Proof:** Proof of this proposition begins by evaluating the integral \( I_{n,n+1} \) for all integers \( h \neq (k-w_2) \) equal to zero. The integral \( I_{n,n+1} \) can be written as

\[
I_{n,n+1} = e^{2j\pi h (2^F - 1)} \int_{(n+1)T_c}^{(n+2)T_c} \prod_{\ell=0}^{F-1} \frac{\sin(\pi 2^l \beta(t-(n+1)T_c))}{\sin(\pi 2^l)} \\
\times \sin\left(\pi 2^l - \pi 2^l \beta(t-(n+1)T_c)\right) \\
\times \sin\left(\pi 2^l - \pi 2^l \beta(t-(n+1)T_c)\right) dt. 
\]

(73)
For the raised cosine function (2RC) and modulation index $h = 1/2^x$, where $1 \leq x < F - 1$, the integral $I_{n,n+1}$ is equal to

$$I_{n,n+1} = e^{j2\pi nh(2^x-1)} \int_0^{T_c} \frac{1}{\prod_{l=0}^{F-1} \sin (\pi 2^x \tau)} \left( \frac{\tau + T_c}{2T_c} - \frac{1}{4T_c} \sin \left( \frac{2\pi (\tau + T_c)}{T_c} \right) \right)^2 \sin \left( \frac{\pi 2^x \tau}{T_c} \right) \sin \left( \frac{\pi 2^x (1 - \tau / T_c)}{T_c} \right) \sin \left( \frac{\pi 2^x (1 + \tau / T_c)}{T_c} \right) \prod_{l=0}^{F-1} \sin \left( \frac{\pi 2^x \tau}{T_c} \right) \sin \left( \frac{\pi 2^x (1 - \tau / T_c)}{T_c} \right) \sin \left( \frac{\pi 2^x (1 + \tau / T_c)}{T_c} \right) d\tau.$$  

(74)

Using the asymptotic expansion of integrals, (74) can be rewritten as

$$I_{n,n+1} \approx e^{j2\pi nh(2^x-1)} \prod_{l=0}^{F-1} \frac{1}{\sin (\pi 2^x \tau)} \frac{1}{3} \frac{1}{\sin (\pi 2^x \tau)} \frac{1}{\sin (\pi 2^x \tau)} \left( \frac{\tau}{T_c} \right)^3 \left( 1 - \cos \left( \frac{\pi 2^x \tau}{T_c} - 1 \right) \right)^2 \left( \frac{\pi \tau}{T_c} \right)^3 \prod_{l=0}^{F-1} \left( \frac{\pi 2^x \tau}{T_c} \right) \sin \left( \frac{\pi 2^x \tau}{T_c} - 1 \right) \sin \left( \frac{\pi 2^x \tau}{T_c} + 1 \right) \frac{\pi \tau}{T_c} \right) d\tau \left( \frac{\pi \tau}{T_c} \right) \sin \left( \frac{\pi 2^x \tau}{T_c} - 1 \right) \sin \left( \frac{\pi 2^x \tau}{T_c} + 1 \right) \frac{\pi \tau}{T_c} \right) d\tau = 0.$$  

(75)

By noting that the value of the integral $I_{n,n+1}$ is equal to a complex conjugate value of the integral $I_{n,n+1}$ and that all functions inside the integral are real functions, we can conclude that integrals $I_{n,n+1}$ are also equal to zero. Similarly can be shown that integrals $I_{n,n+1}$ and $I_{n,n+1}$ for other combinations of integers $e^{(k,d-w)} \in \{0,1\}$ and integrals $I_{n,n+2}$ and $I_{n,n+2}$ for all $e^{(k,d-w)} = 0$ are equal to zero.

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