A VARIANT OF THE CLASSICAL VON KARMAN FLOW OF AN ELECTRICALLY CONDUCTING SECOND GRADE FLUID

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Received 29 November 2011; accepted 4 April 2012

ABSTRACT

An analysis is made to obtain exact solutions for the flow of a second grade fluid due to an infinite disk rotating with a constant angular speed. The fluid is electrically conducting in the presence of an external uniform magnetic field applied perpendicular to the surface of the disk. Instead of the classical Von Kármán axisymmetric flow assumption, a rotational non-axisymmetric flow is taken into consideration here. The obtained results point out that for the second grade conducting fluid, a steady asymptotic solution exists for the velocity field whose far-field behavior is distinct from the no slip velocities. It is observed that the flow field is significantly influenced by the material parameter of the second grade fluid and the magnetic interaction number. A comparison with the existing results for the conducting Newtonian fluid is also given.

Keywords: Non-axisymmetric flow, MHD second grade fluid, rotating disk, exact solution

1 INTRODUCTION

The flow induced by an infinite disk rotating in an incompressible viscous fluid was first considered by Von Kármán and later by Cochran. Von Kármán assumed that the flow possessed axial symmetry and introduced a similarity transformation which reduced the Navier-stokes equations into a system of coupled nonlinear ordinary differential equations. He also obtained an approximate solution for that problem. Later Cochran (Cochran 1934, Stuart 1954) obtained a more accurate solution by using the matched asymptotic expansion method for the case of zero normal velocity at the disk. This problem has received considerable attentions over the years and different extensions of Von Karman’s flow problem have been made to address various applications, see for instance Millsaps et al (Millsaps et al 1952), Stuart (Stuart 1954), Watanabe (Watanabe 1991), Ariel (Ariel 2003), Bikash (Bikash 2009). In all above studies, the Von Kármán's axisymmetric similarity transformation is used to reduce the governing partial differential equations to a nonlinear system of ordinary differential equations. These ODEs are then solved by some approximate methods either numerical or analytical numerical. However, exact solutions are very important for many reasons. They provide a standard for checking the accuracies of many approximate methods such as numerical or asymptotic. Moreover, the flow induced by an infinite disk rotating with a constant angular speed may not possess axial symmetry (Adabala *et al.* 1987). Recently, Turkyilmazoglu (Turkyilmazoglu 2009) has obtained exact solutions of the Navier-Stokes equations for this classical problem by using a generalized non-similarity transformation which differs qualitatively from the Von Kármán's assumption in such a way that the physical quantities are allowed to develop non-axisymmetrically over a rotating disk. In this study, we have extended the analysis in (Turkyilmazoglu 2011 and A. A. Farooq, 2012) from viscous to a non-Newtonian second grade fluid.

It is also a well-known fact that the Navier-Stokes equations seem to be an inappropriate model for a class of real fluids, called non-Newtonian fluids. During the last few decades, considerable efforts have been usefully devoted to the study of flow of non-Newtonian fluids because of their technological applications. A vast amount of literature is now available for the flow problems associated with non-Newtonian fluids in variety of situations. One important and simple model of non-Newtonian fluids for which one can reasonably hope to obtain analytic solution is the second grade fluid. Keeping in mind this fact, we have chosen this model in this study. Undoubtedly, the equations of motion for the second grade fluid are more complicated with highly non-linear terms which make the question of well-posedness extremely difficult to address. However, in the present case the corresponding boundary value problem is well posed and is integrated exactly. This analysis is important, not only from mathematical point of view, but mainly as an essential test for the underlying physical model. The following structure is pursued in the rest of the paper. In section two basic equations governing the problem are given. Section three concerns with the mathematical formulation of the problem and section four presents the flow analysis. Finally, section five includes some concluding remarks.

2 BASIC EQUATIONS

The basic equations of unsteady, incompressible MHD flow with the generalized Ohm's law and the Maxwell's equations are

$$\nabla.\mathrm{V}=0, \tag{1}$$

$$\rho\left[\frac{\partial \mathrm{V}}{\partial t}+\left(\mathrm{V}.\nabla\right)\mathrm{V}\right]=\nabla.\mathrm{T}+\mathrm{J}\times\mathrm{B}, \tag{2}$$

$$\nabla.\mathrm{B}=0,\ \nabla\times\mathrm{B}=\mu_m\mathrm{J},\ \nabla\times\mathrm{E}=-\frac{\partial \mathrm{B}}{\partial t},\ \mathrm{J}=\sigma\left[\mathrm{E}+\nabla\times\mathrm{B}\right] \tag{3}$$

where $\mathrm{V}=(u,v,w)$ is the velocity field, T is the Cauchy stress tensor, ρ is the fluid density, J is the current density and B is the total magnetic field so that $\mathrm{B}=\mathrm{B_o}+\mathrm{b}$, where $\mathrm{B_o}$ is the applied magnetic field and b is the induced magnetic field. The other parameters appearing in (3) are, respectively, E is the electric field conductivity, μ_m is the magnetic permeability and σ is the electric field conductivity. Under the usual MHD approximations, the magnetohydrodynamic force $\mathrm{J}\times\mathrm{B}$ becomes $\sigma(\mathrm{V}\times\mathrm{B})\times\mathrm{B}$ when imposed and induced electric fields are negligible and only the magnetic field B contributes to the current $\mathrm{J}=\sigma(\mathrm{V}\times\mathrm{B})$.

The fluid considered in this article is a second grade fluid and its constitutive equations are (Ariel 1997, Mishra *et al.* 2011)

$$\mathrm{T}=-p\mathrm{I}+\mathrm{S},\quad \mathrm{S}=\mu \mathrm{A}_1+\alpha_1\mathrm{A}_2+\alpha_2\mathrm{A}^2_1 \tag{4}$$

with p being the pressure, I the unit tensor, μ is the dynamic viscosity, α_1, α_2 are material constants satisfying thermodynamic compatibility model $\alpha_1 \geq 0$, and $\alpha_1+\alpha_2=0$, and A_1 and A_2 are the kinematic tensors defined through

$$\mathrm{A}_1=(\nabla \mathrm{V})+(\nabla \mathrm{V})^T,\ \mathrm{A}_2=\frac{d\mathrm{A}_1}{dt}+\mathrm{A}_1(\nabla \mathrm{V})+(\nabla \mathrm{V})^T\,\mathrm{A}_1\,, \tag{5}$$

where superscript T stands for transpose of a matrix.

3 FORMULATION OF THE PROBLEM

Consider three dimensional steady flow of an incompressible second grade fluid due to an infinite disk which rotates in the plane $z=0$ about its axis of rotation z with a constant angular velocity Ω. Flow is assumed to take place in the semi-infinite space $z \geq 0$. A uniform magnetic field B=B₀ is applied to the system along the direction perpendicular to the disk which is taken as electrically non-conducting. It is natural to describe the flow in the cylindrical coordinates (r,θ,z) and denote the corresponding velocity components by u, v and w. Boundary conditions for the problem are such that the fluid adheres to the disk at $z=0$ with a given axial velocity and that the velocity field is bounded at far distances from the surface of the disk.

Von Kármán flow and the flow in this analysis are different qualitatively in that in the traditional Von Kármán flow, the rotational symmetry assumption is used which removes the θ-dependence of the flow variables. However, in the present study we allow the θ-dependence, enabling the motion of a non-axisymmetric flow to develop, but we further assume $w=0$. Under these stated assumptions, the velocity field for the problem can be taken in the form

$$\mathrm{V}=\left[u(r,\theta,z),v(r,\theta,z),0\right] \tag{6}$$

We introduce the following dimensionless variables:

$$r^*=\frac{r}{L},z^*=\frac{z}{L},u^*=\frac{u}{U},v^*=\frac{v}{U},P^*=\frac{P}{\rho U^2},S^*_{ij}=\frac{S_{ij}}{\left(\frac{\mu U}{L}\right)},i,j=r,\theta,z \tag{7}$$

where L is the length scale and $U=L\Omega$. Hence, the dimensionless form of the continuity and the equations of motion (1)-(3), after dropping the * are given by

$$\frac{\partial u}{\partial r}+\frac{1}{r}\frac{\partial v}{\partial \theta}+\frac{u}{r}=0\,, \tag{8}$$

$$u\frac{\partial u}{\partial r}+\frac{v}{r}\frac{\partial u}{\partial \theta}-\frac{v^2}{r}+mu=-\frac{\partial P}{\partial r}+\frac{1}{\mathrm{Re}}\left[\frac{\partial S_{rr}}{\partial r}+\frac{1}{r}\frac{\partial S_{r\theta}}{\partial \theta}+\frac{\partial S_{rz}}{\partial z}-\frac{S_{rr}-S_{\theta\theta}}{r}\right] \tag{9}$$

$$u\frac{\partial v}{\partial r}+\frac{v}{r}\frac{\partial v}{\partial \theta}+\frac{uv}{r}+m(v-r)=-\frac{1}{r}\frac{\partial P}{\partial \theta}+\frac{1}{\mathrm{Re}}\left[\frac{\partial S_{r\theta}}{\partial r}+\frac{1}{r}\frac{\partial S_{\theta\theta}}{\partial \theta}+\frac{\partial S_{\theta z}}{\partial z}-\frac{2S_{\theta\theta}}{r}\right] \tag{10}$$

$$0=-\frac{\partial P}{\partial z}+\frac{1}{\mathrm{Re}}\left[\frac{\partial S_{rz}}{\partial r}+\frac{1}{r}\frac{\partial S_{\theta z}}{\partial \theta}+\frac{\partial S_{zz}}{\partial z}+\frac{S_{rz}}{r}\right] \tag{11}$$

With the help of (4)-(6) the stress tensor components in (9)-(11) are given by

$$S_{rr}=2\frac{\partial u}{\partial r}+2\lambda_1\left\{\left(u\frac{\partial}{\partial r}+\frac{v}{r}\frac{\partial}{\partial\theta}\right)\frac{\partial u}{\partial r}+\left(\frac{\partial v}{\partial r}-\frac{v}{r}\right)\left(\frac{\partial v}{\partial r}+\frac{1}{r}\frac{\partial u}{\partial\theta}-\frac{v}{r}\right)+2\left(\frac{\partial u}{\partial r}\right)^2\right\}$$
$$+\lambda_2\left\{4\left(\frac{\partial u}{\partial r}\right)^2+\left(\frac{\partial v}{\partial r}+\frac{1}{r}\frac{\partial u}{\partial\theta}-\frac{v}{r}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2\right\} \tag{12}$$

$$S_{r\theta}=\frac{\partial v}{\partial r}+\frac{1}{r}\frac{\partial u}{\partial\theta}-\frac{v}{r}+\lambda_1\left\{\left(u\frac{\partial}{\partial r}+\frac{v}{r}\frac{\partial}{\partial\theta}\right)\left(\frac{\partial v}{\partial r}+\frac{1}{r}\frac{\partial u}{\partial\theta}-\frac{v}{r}\right)+\frac{\partial u}{\partial r}\left(\frac{\partial v}{\partial r}+\frac{3}{r}\frac{\partial u}{\partial\theta}-\frac{v}{r}\right)\right\}$$
$$+\lambda_2\left\{2\frac{\partial u}{\partial r}\left(\frac{\partial v}{\partial r}+\frac{1}{r}\frac{\partial u}{\partial\theta}-\frac{v}{r}\right)+\left(\frac{\partial u}{\partial z}\right)\left(\frac{\partial v}{\partial z}\right)\right\} \tag{13}$$

$$S_{rz}=\frac{\partial u}{\partial z}+\lambda_1\left\{\left(u\frac{\partial}{\partial r}+\frac{v}{r}\frac{\partial}{\partial\theta}\right)\frac{\partial u}{\partial z}+3\frac{\partial u}{\partial z}\frac{\partial u}{\partial r}+\frac{\partial v}{\partial z}\left(2\frac{\partial v}{\partial r}+\frac{1}{r}\frac{\partial u}{\partial\theta}-\frac{2v}{r}\right)\right\}$$
$$+\lambda_2\left\{2\frac{\partial u}{\partial z}\frac{\partial u}{\partial r}+\frac{\partial v}{\partial z}\left(\frac{\partial v}{\partial r}+\frac{1}{r}\frac{\partial u}{\partial\theta}-\frac{v}{r}\right)\right\} \tag{14}$$

$$S_{\theta z}=\frac{\partial v}{\partial z}+\lambda_1\left\{\left(u\frac{\partial}{\partial r}+\frac{v}{r}\frac{\partial}{\partial\theta}\right)\frac{\partial v}{\partial z}-3\frac{\partial u}{\partial z}\frac{\partial u}{\partial r}+\frac{\partial u}{\partial z}\left(\frac{\partial v}{\partial r}+\frac{2}{r}\frac{\partial u}{\partial\theta}-\frac{v}{r}\right)\right\}$$
$$+\lambda_2\left\{\frac{\partial u}{\partial z}\left(\frac{\partial v}{\partial r}+\frac{1}{r}\frac{\partial u}{\partial\theta}-\frac{v}{r}\right)-2\frac{\partial u}{\partial r}\frac{\partial v}{\partial z}\right\} \tag{15}$$

$$S_{\theta\theta}=-2\frac{\partial u}{\partial r}-2\lambda_1\left\{\left(u\frac{\partial}{\partial r}+\frac{v}{r}\frac{\partial}{\partial\theta}\right)\frac{\partial u}{\partial r}-2\left(\frac{\partial u}{\partial r}\right)^2-\left(\frac{v}{r}+\frac{1}{r}\frac{\partial u}{\partial\theta}\right)\left(\frac{\partial v}{\partial r}+\frac{1}{r}\frac{\partial u}{\partial\theta}-\frac{v}{r}\right)\right\}$$
$$+\lambda_2\left\{4\left(\frac{\partial u}{\partial r}\right)^2+\left(\frac{\partial v}{\partial r}+\frac{1}{r}\frac{\partial u}{\partial\theta}-\frac{v}{r}\right)^2+\left(\frac{\partial v}{\partial z}\right)^2\right\} \tag{16}$$

$$S_{zz}=\left(2\lambda_1+\lambda_2\right)\left\{\left(\frac{\partial u}{\partial z}\right)^2+\left(\frac{\partial v}{\partial z}\right)^2\right\} \tag{17}$$

where $\lambda_1=\dfrac{\alpha_1 U}{\mu L}$ and $\lambda_2=\dfrac{\alpha_2 U}{\mu L}$ are the dimensionless material parameters of the second grade fluid, $\mathrm{Re}=\dfrac{\rho U^2 L}{\mu}$ is the Reynolds number and $m=\dfrac{\sigma B^2{}_0}{\rho\Omega}$ is the magnetic interaction parameter.

4 FLOW ANALYSIS

In this section we restrict ourselves to the steady magnetohydrodynamic mean flow relative to the non-conducting rotating disk. Let us therefore introduce a coordinate transformation $\varsigma=\sqrt{\dfrac{\mathrm{Re}}{2}}z$ together with a solution of the following form (Turkyilmazoglu 2009, 2011)

$$u=r_oF(\theta,\varsigma),\ v=r+r_oW(\theta,\varsigma),\ w=0,\ P=\frac{r^2}{2}-rr_o\cos(\theta-\varphi)+r_o^2p(\varsigma) \tag{18}$$

such that, non-axisymmetric and periodic solutions with respect to θ of F and W are determined here, subjected to the pressure field given by (18). This assumption simplifies our subsequent analysis. In the above transformations, the parameters r_o and φ correspond to the polar representation of a fixed point on the disk surface and $p(\varsigma)$ is some function of ς.

On employing the above transformations in (8)-(11) along with (12)-(17), the continuity equation is automatically satisfied, while the momentum equations, with the help of periodicity assumption of F and W with respect to θ give rise to the set of following ordinary differential equations

$$F_{\varsigma\varsigma} + \lambda_1 W_{\varsigma\varsigma} - 2mF + 2W = -2\cos(\theta - \varphi) \tag{19}$$

$$W_{\varsigma\varsigma} - \lambda_1 F_{\varsigma\varsigma} - 2mW - 2F = 2\sin(\theta - \varphi) \tag{20}$$

$$p(\varsigma) = \frac{(2\lambda_1 + \lambda_2)}{2}\left(F_\varsigma^2 + W_\varsigma^2\right) + K \tag{21}$$

where the constant K can be determined from the pressure prescribed at the surface of the disk. The boundary conditions for the problem to be satisfied are

$$F = 0,\ W = 0 \text{ at } \varsigma = 0,\ F,\ W \text{ bounded as } \varsigma \to \infty \tag{22}$$

Combining equations (19)-(20) into a single complex differential equation with real variables, we obtain

$$(1 - i\lambda_1)H_{\varsigma\varsigma} - 2(m + i)H = -2\left(\cos(\theta - \varphi) - i\sin(\theta - \varphi)\right) \tag{23}$$

where $H = F + iW$ and $i = \sqrt{-1}$. The corresponding boundary conditions for H are given as

$$H = 0 \text{ at } \varsigma = 0 \text{ and } H \text{ is bounded as } \varsigma \to \infty \tag{24}$$

We note that for λ_1=0, (23) corresponds to the differential equation for the Newtonian conducting fluid (Turkyilmazoglu 2011).

The solution of the above ordinary second order complex differential equation which is bounded with respect to ς is

$$H = \frac{C(i - m)}{m^2 + 1}\left(-1 + e^{-\Lambda\varsigma}\right) \tag{25}$$

where $C = \cos(\theta - \varphi) - i\sin(\theta - \varphi)$ and $\Lambda = \sqrt{\frac{2}{1 - i\lambda_1}(i + m)}$. Equating real and imaginary parts of the solution given in (25), F and W are found to be

$$F(\theta,\varsigma) = \frac{1}{m^2 + 1}\left[\left(f(\varsigma) - mg(\varsigma)\right)\cos(\theta - \varphi) + \left(g(\varsigma) + mf(\varsigma)\right)\sin(\theta - \varphi)\right] \tag{26}$$

$$G(\theta,\varsigma) = \frac{1}{m^2 + 1}\left[\left(-f(\varsigma) + mg(\varsigma)\right)\sin(\theta - \varphi) + \left(g(\varsigma) + mf(\varsigma)\right)\cos(\theta - \varphi)\right] \tag{27}$$

in which

$$f(\varsigma) = \sin(d_2\varsigma)e^{-d_1\varsigma}, \quad g(\varsigma) = -1 + \cos(d_2\varsigma)e^{-d_1\varsigma} \tag{28}$$

where $d_1 = \sqrt{\frac{m - \lambda_1 + \sqrt{(1 + \lambda^2{}_1)(m^2 + 1)}}{(1 + \lambda^2{}_1)}}$ and $d_2 = \sqrt{\frac{\lambda_1 - m + \sqrt{(1 + \lambda^2{}_1)(m^2 + 1)}}{(1 + \lambda^2{}_1)}}$.

The asymptotic behavior of f and g as $\varsigma \to \infty$ in (27) implies that the velocities far away from the disk turn to be $u = -r_o \sin(\theta - \varphi)$, $v = r - r_o \cos(\theta - \varphi)$ which differ from the no-slip velocities. Thus, the fluid tends to move as a rigid body outside the layer whose thickness varies. The path along which the velocities vanish exactly through the space can be easily determined by setting zero the velocities in (18) with the consideration of (25)-(27). Moreover, when $\theta = \varphi$, the implication is that $F = f$ and $W = g$. It may be easily shown that when λ_1=0, our results are in agreement with those of (Turkyilmazoglu 2011) for Newtonian conducting fluid. Moreover, (27) show that the velocity distribution is in the form of an Ekman spiral representing the flow over a disk in a rotating system.

In order to have better understanding of the flow field, the graphs of $f(\varsigma)$ and $-g(\varsigma)$ are plotted for different values of λ_1 and m in Figures 1 and 2. These graphs clearly indicate that the flow exhibits a boundary layer like structure near the surface of the disk. Figure1 displays the effects of λ_1 on the velocity field. It is observed that when λ_1 increases the oscillatory behavior of the flow becomes more prominent and can be seen upto a considerable distance from the disk. The effect of varying the magnetic interaction number m on the velocity field displayed in Figure2 is almost opposite to that of λ_1. It can be seen that as m getting large, there is a considerable reduction in the three-dimensional nature of the flow field.

The effects of viscosity in the fluid adjacent to the disk tend to develop tangential shear stress which opposes the rotation of the disk. There is also a surface shear stress in the radial direction. The dimensionless expressions for tangential and radial stresses are given as

$$S_{rz} = r_o \sqrt{\frac{\text{Re}}{2}} \left[W_\varsigma - \lambda_1 F_\varsigma \right]_{\varsigma=0} = \frac{-r_o}{m^2+1} \sqrt{\frac{\text{Re}}{2}} \left[\Lambda_a \sin(\theta-\varphi) + \Lambda_b \cos(\theta-\varphi) \right] \tag{29}$$

$$S_{\theta z} = r_o \sqrt{\frac{\text{Re}}{2}} \left[F_\varsigma + \lambda_1 W_\varsigma \right]_{\varsigma=0} = \frac{r_o}{m^2+1} \sqrt{\frac{\text{Re}}{2}} \left[\Lambda_a \cos(\theta-\varphi) - \Lambda_b \sin(\theta-\varphi) \right] \tag{30}$$

where $\Lambda_a = d_2 + md_1 + \lambda_1(md_2 - d_1)$ and $\Lambda_b = d_1 - md_2 + \lambda_1(d_2 + md_1)$. In particular case when $\theta = \varphi$, the results obtained point out the fact that the minimum and maximum resistance offered against the flow within the presence of a transversely applied magnetic field take place, respectively, at the locations $\theta = \tan^{-1}\left(\frac{\Lambda_b}{\Lambda_a}\right)$ and $\theta = \tan^{-1}\left(\frac{\Lambda_b}{\Lambda_a}\right) + \pi$ for the tangential stresses and at the locations $\theta = \tan^{-1}\left(\frac{-\Lambda_b}{\Lambda_a}\right)$ and $\theta = \tan^{-1}\left(\frac{-\Lambda_b}{\Lambda_a}\right) + \pi$ for the radial stresses. From the above equations one can easily find out the locations at which minimum and maximum skin friction takes place against the flow.

The boundary layer thickness in radial and tangential directions are evaluated as

$$\delta_r = \int_0^\infty f(\varsigma) d\varsigma = \frac{d_2}{d_1^2 + d_2^2}, \qquad \delta_\theta = \int_0^\infty (1 + g(\varsigma)) d\varsigma = \frac{d_1}{d_1^2 + d_2^2} \tag{31}$$

Hence, the boundary layer thickness decay in a significant manner (see also Figure2) as the magnetic effect gets considerably large. Moreover, an increase in λ_1 results in the increase of boundary layer thickness.

The vorticity components $(\omega_r, \omega_\theta, \omega_z) = \nabla \times \text{V}$ existing within the fluid can be found out exactly using Eqs. (18)-(19) and are respectively

$$\omega_r = -\frac{\partial v}{\partial z} = -r_o \sqrt{\frac{\text{Re}}{2}} W_\varsigma, \ \omega_\theta = \frac{\partial u}{\partial z} = r_o \sqrt{\frac{\text{Re}}{2}} F_\varsigma, \ \omega_z = 2 \tag{32}$$

In order to get the nature of vorticity near the disk the expressions for ω_r and ω_θ are plotted for different values of λ_1 and m when $\theta = \varphi$. It is observed from Figure3 that ω_r decreases while ω_θ increases near the disk by increasing the value of λ_1 and show oscillatory behavior before approaching the asymptotic limits. Figure4 is to demonstrate the effects of *m* on ω_r and ω_θ. However, the radial component ω_r is observed to be increasingly damped as compared to the tangential component ω_θ. Actually, these vorticity components are responsible for driving the motion of fluid flow considered in the current study.

5 CONCLUDING REMARKS

An incompressible magnetohydrodynamic second grade fluid flow over a single rotating disk has been discussed in such a way that the physical quantities are allowed to develop non-axisymmetrically within a no normal flow assumption. An exact solution of the governing equations in three dimensions has been obtained in closed form. We have worked through cylindrical coordinates which rotates with the disk, whose polar representation is (r_o, φ). The solution is influenced by these parameters. When r_o =0, the associated solution corresponds to that for a rigid body rotation. The non-zero choice of r_o has enabled us to achieve the solutions bounded away from the disk. The obtained results point out the well known damping effect of the transverse magnetic field on the motion of second grade fluid due to a rotating disk. Therefore, it is observed that increase in *m* helps in decreasing the boundary layer thickness. However, an increase of λ_1 causes an increase in the boundary layer thickness.

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6 FIGURES

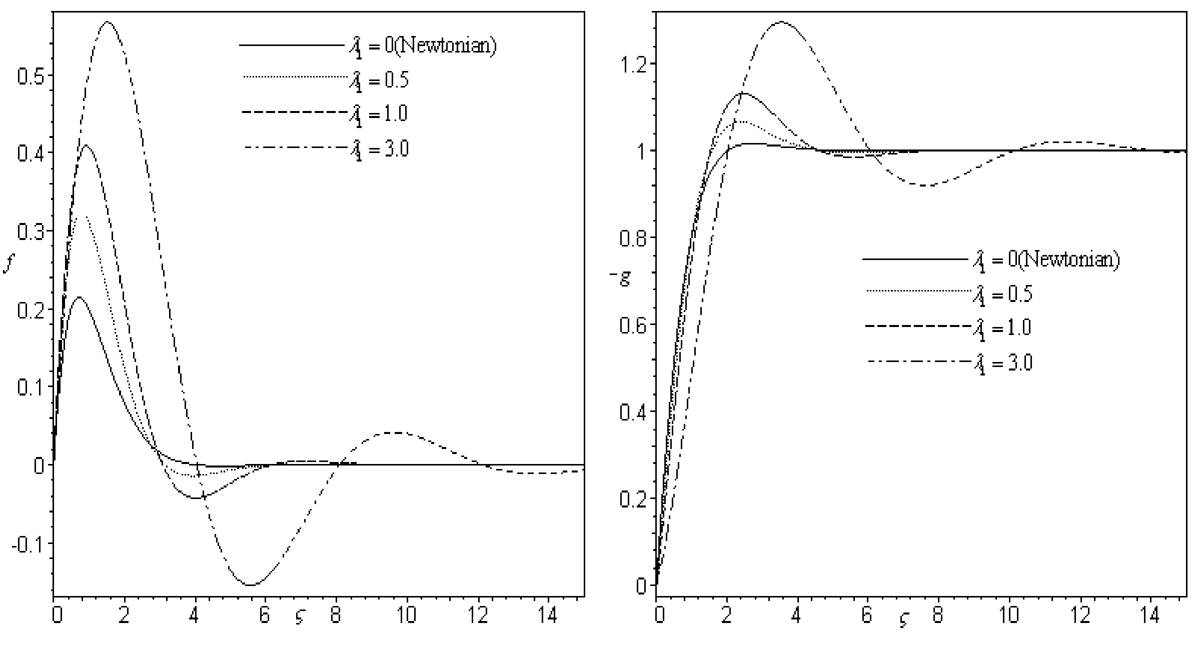

Figure1: Variations of f and $-g$ with ς for several values of λ_1 when m =0.5

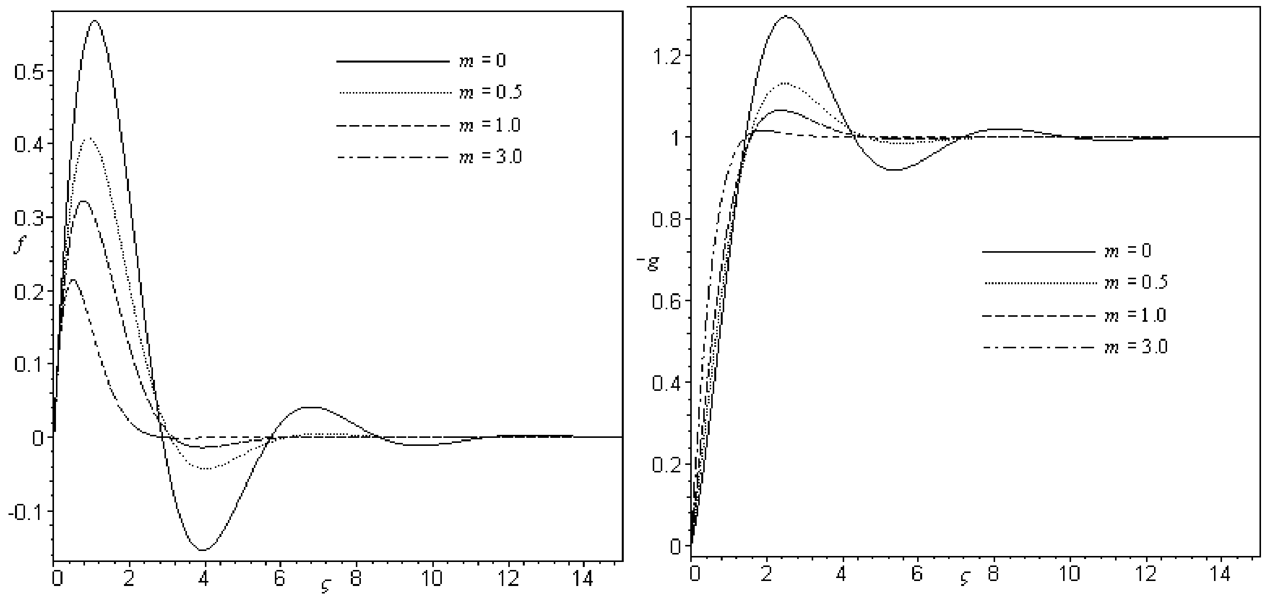

Figure2: Variations of f and $-g$ with ς for several values of m when λ_1 =1

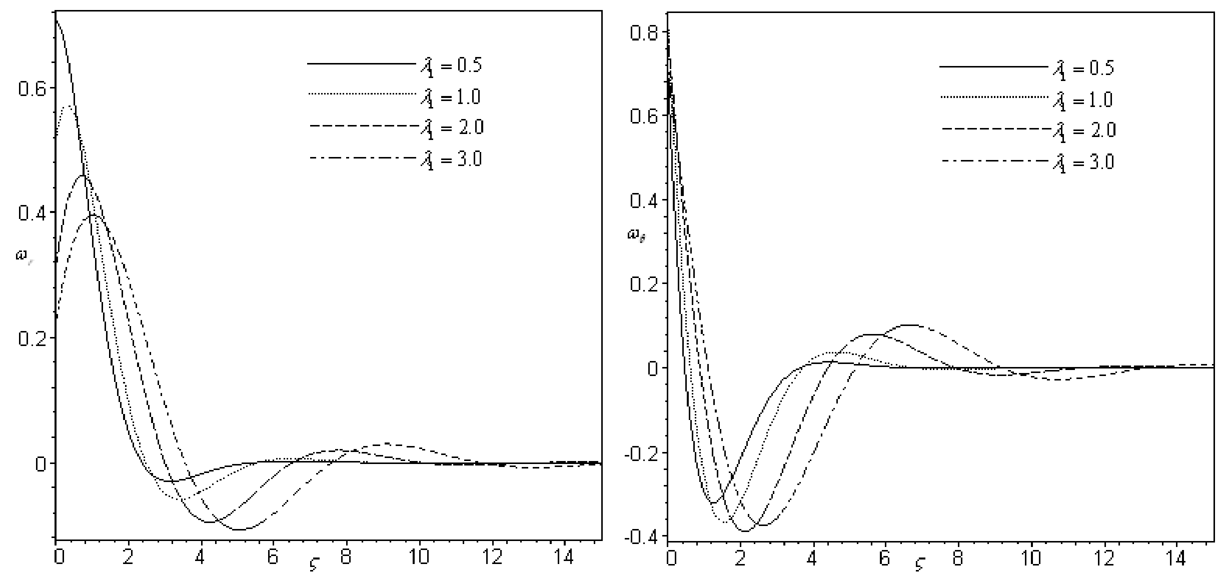

Figure3: Variations of ω_r and ω_θ with ς for several values of λ_1 when m =0.5

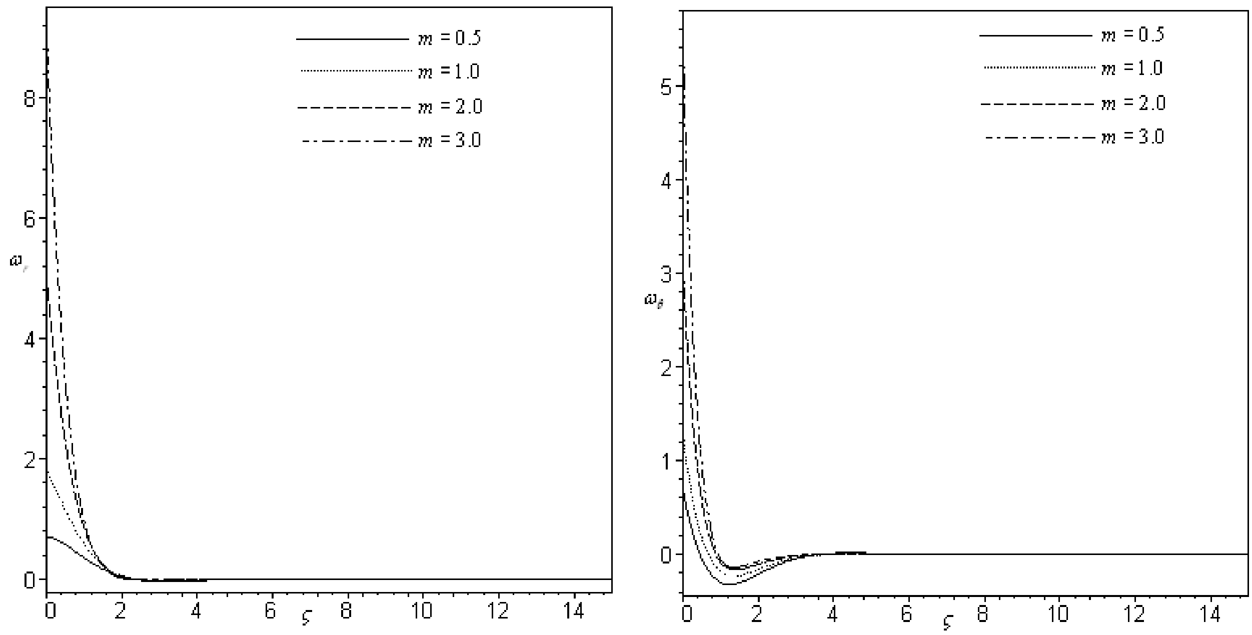

Figure4: Variations of ω_r and ω_θ with ς for several values of m when λ_1 =1