On the Average Redundancy Rate of Adaptive Block Codes under Mixed Sources

Yuriy A. Reznik
RealNetworks, Inc.
2601 Elliott Ave, Seattle, WA 98121
E-mail: yreznik@ieee.org

Anatoly V. Anisimov
Faculty of Cybernetics, Kiev University
2 Acad. Glushkov Ave, 03680 Kiev, Ukraine
E-mail: ava@mi.unicyb.kiev.ua

We study the average redundancy rate of Krichevsky’s sample-based universal block codes [1, 2, 3] in a situation when samples and blocks to be compressed are produced by two different memoryless sources. We prove the following.

**Theorem 1.** The average redundancy rate of adaptive block codes \( \phi_{\ell,T} \) constructed using samples of length \( \ell \) from a source \( T \) and used to encode blocks of length \( n \) from a source \( S \) is asymptotically (with \( \ell, n \to \infty, \ell/(\ell + n) \to 0 \)):

\[
R(\phi_{\ell,T}, n, S) = \frac{1}{n} \left[ \frac{m - 1}{2} \log \frac{n}{\ell} + \ell D(T \| S) + O(1) \right],
\]

where \( m \) is a cardinality of the alphabet, \( D(T \| S) \) is a relative entropy (or Kullback-Leibler distance [4]) between sources \( T \) and \( S \), and \( \log := \log_b \), where \( b \) corresponds to a unit of information (e.g. bits or nats) being used.

**Theorem 2.** If \( D(T \| S) \neq 0 \), then there exists a sample length \( \ell^* \) such that

\[
R(\phi_{\ell^*,T}, n, S) = \min_{\ell} R(\phi_{\ell,T}, n, S).
\]

**Corollary 1.** The optimal length of samples \( \ell^* \) is asymptotically (with \( n \to \infty \)):

\[
\ell^* = \frac{m - 1}{2 D(T \| S)} + O\left(\frac{1}{n}\right).
\]

**Corollary 2.** The minimum average redundancy rate \( R(\phi_{\ell^*,T}, n, S) \) is asymptotically (with \( n \to \infty \)):

\[
R(\phi_{\ell^*,T}, n, S) = \frac{1}{n} \left[ \frac{m - 1}{2} \log n + \frac{m - 1}{2} \log \frac{2 e D(T \| S)}{m - 1} + O(1) \right].
\]

**Theorem 3.** Adaptive block codes constructed using samples from a source \( T \) and applied to a source \( S \) can achieve a lower average redundancy than (pure) universal codes [3] if:

\[
D(T \| S) < \delta_1 = \frac{1}{2} + O\left(\frac{1}{m}\right).
\]

**References**


