Performance of Distributed Space-Time Block Codes

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Abstract— **An extended form of multi-hop communication systems has been introduced recently which allows the application of multiple-input-multiple-output (MIMO) capacity enhancement techniques over spatially separated relaying mobile terminals. It was further shown that such deployment yields significant capacity gains over direct communication or traditional single-input-single-output (SISO) relaying networks. The contribution of this paper is the derivation of the end-to-end bit-error rates (BER) for space-time block encoded M-PSK and M-QAM, assuming full and partial cooperation at each relaying stage. The theoretical analysis is corroborated by simulation results.**

I. Introduction

Relaying systems are known to yield performance gains due to the reduced aggregate pathloss when compared to a direct link transmission [1], [2]. Multiple-Input-Multiple-Output (MIMO) systems offer significant performance gains when compared to a Single-Input-Single-Output (SISO) communication system [3], [4]. The combination of a relaying system with MIMO processing in each relaying stage is a straightforward extension of both concepts and enjoys a widespread popularity within the research community, see e.g. the landmark contributions by [5]−[10].

To overcome the disadvantage of having only one antenna element in a mobile terminal (MT) in such relaying system, spatially adjacent mobile terminals are allowed to communicate with each other and thus form a virtual array of more than one antenna element. The hence introduced concept of Virtual Antenna Arrays (VAAs) [11], [12] is deployed here in a more general context.

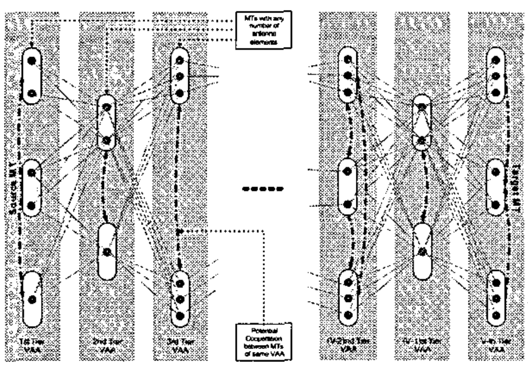

Fig. 1. Distributed-MIMO Multi-Stage Communication Network.

Brief Concept. A generic realisation of a distributed-MIMO multi-stage communication network with the utilisation of VAAs is depicted in Figure 1. Here, a source mobile terminal (s-MT) communicates with a target mobile terminal (t-MT) via a number of relaying mobile terminals (r-MTs). Spatially adjacent r-MTs form a VAA, each of which receives data from the previous VAA and relays data to the consecutive VAA until the t-MT is reached. Each of the terminals may have an arbitrary number of antenna elements; furthermore, the MTs of the same VAA may cooperate among each other. With a proper setup, this clearly allows the deployment of MIMO capacity enhancement techniques.

Brief Background. To the authors' best knowledge, the general concept of distributed-MIMO multi-stage communication systems has been introduced in [11], [12]. It based on previous contributions by [1], [2], [13] on relaying and by [3], [4] on MIMO communication. Independent research has led to similar concepts [5]−[10]. Today, the subject of distributed space-time coding has commenced to flourish, see e.g. [14].

Aim of the Paper. It is assumed here that each relaying stage deploys distributed space-time block codes (STBCs) with a varying degree of cooperation at each relaying stage. The contribution of this paper is the derivation of the end-to-end bit error rate (BER) through such distributed multi-stage network. To this end, expressions of the symbol error rate (SER) of STBCs operating over flat Rayleigh and Nakagami fading channels with different sub-channel gains are given in closed form.

Paper Structure. In Section II, the system model is briefly described. In Section III, the exact symbol error rates for unequally distributed Rayleigh and Nakagami flat fading channels are exposed. In Section IV, the end-to-end bit error rates are derived assuming full cooperation of all mobile terminals at each relaying stage. The latter assumption is relaxed in Section V, where the generic case of partial cooperation is analysed and assessed by means of simulation results. Final remarks and conclusions are given in Section VI.

II. System Model

General Assumptions. Assumed is regenerative relaying, where the terminal selection and routing path is determined prior to transmission. Synchronisation among the terminals involved in the relaying process is also assumed to be perfect. The decision on the correctness of a received packet is performed at the target terminal only.

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The information source passes the information to a cooperative transceiver, which relays the data to spatially adjacent r-MTs belonging to the same VAA. This is assumed to happen over an air interface distinct from the interface used for inter-stage communication, and is not considered further. It is also assumed that these cooperative links are error-free due to the short communication distances. Each of the terminals in the VAA performs distributed encoding of the information according to some prior specified rules. That information is then transmitted from the spatially distributed terminals after having been synchronised.

Any of the relaying VAA tiers functions as follows. First, each r-MT within that VAA receives the data which is optionally decoded before being passed onto the cooperative transceiver. Ideally, every terminal cooperates with every other terminal; however, any amount of cooperation is feasible. If no decoding is performed, then an unprocessed or a sampled version of the received signal is exchanged with the other r-MTs. Note that unprocessed relaying is equivalent to transparent relaying, which is not considered here. After cooperation, appropriate decoding is performed. The obtained information is then re-encoded in a distributed manner, synchronised and re-transmitted to the following relaying VAA tier.

As for the target VAA, the functional blocks are exactly the opposite to the source VAA. All terminals receive the information, possibly decode it, then pass it onto the cooperative transceivers which relay the data to the target terminal. The data is processed and finally delivered to the information sink.

Distributed Transcoder. The functionality of distributed space-time codes (STCs) differs from a traditional deployment because only a fraction of the entire space-time codeword is transmitted from any of the spatially distributed terminals. The transmission across all terminals then yields the complete space-time codeword. Therefore, a control signal to each distributed space-time encoder is essential, as it tells each of them which fraction of the entire space-time codeword to pass onto the transmitting antenna(s). This control information is assumed to be available to the space-time encoder.

The cooperative decoder can be realised as the inversion of all processes at the cooperative transmitter. Here, the space-time decoder is fed with the signals directly received from the available antenna(s), as well as the information received via the cooperative links from adjacent terminals. Again, a control signal is needed which specifies the type of information fed into the space-time decoder, to allow for optimum decoding. For example, the control signal could inform the decoder that the relayed signals are a one bit representation of the sampled soft information available at the respective cooperative relaying terminals.

After the space-time decoding process, the information is passed on to the channel decoder which performs the inverse process to the channel encoder. In a cooperative transcoder, the produced binary information output may then be fed into the cooperative encoder, to get relayed to the next VAA tier.

The error rate behaviour of such system assuming distributed STBCs is analysed in the subsequent sections.

III. Error Rates of Distributed STBCs

Closed-form expressions of the M-PSK and M-QAM symbol error rates (SERs) of STBC systems communicating over flat Rayleigh fading channels with equal sub-channel gains have been elegantly derived in [15]. The analysis has been extended to the generic case of Nakagami fading channel with possibly unequal sub-channel gains in [16]. These cases are briefly summarised below.

Rayleigh Fading - Equal Sub-Channel Gains. With reference to [15], the SER of M-PSK is given through eq. (1) on the top of the next page. Here, t is the number of transmit antennas, r is the number of receive antennas, $u \triangleq t \cdot r$, S is the transmitted signal power, γ is the average channel power gain, R is the rate of the STBC, N is the noise power at detection, $g_{\text{PSK}} \triangleq \sin^2(\pi/M)$, and M is the modulation order.

Furthermore, $\Gamma(x)$ is the complete Gamma function, ${}_2F_1(a,b;c;x)$ is the generalised hypergeometric function with 2 parameters of type 1 and 1 parameter of type 2 [17] (§9.14.1); it is sometimes referred to as the Gauss hypergeometric function [17] (§9.14.2). The function $F_1(a,b,b';c;x,y)$ is a hypergeometric function of two variables [17] (§9.180.1); it is sometimes referred to as the Appell hypergeometric function. Finally, the moment generating function (MGF) is given as

$$\phi_{\frac{1}{R}\frac{\lambda}{t}\frac{S}{N}}(s) = \frac{1}{\left(1 - \frac{1}{R}\frac{\gamma}{t}\frac{S}{N} \cdot s\right)^u} \quad (3)$$

To simplify notation, eq. (1) is denoted as

$$P_{\text{PSK}}(e) = P_{\text{PSK}}(u, t, R, \gamma, S/N, M) \quad (4)$$

Similarly, the SER of M-QAM is given through (2), where $g_{\text{QAM}} \triangleq 3/2/(M-1)$ and $q \triangleq 1 - 1/\sqrt{M}$ [15]. To simplify notation, eq. (2) is denoted as

$$P_{\text{QAM}}(e) = P_{\text{QAM}}(u, t, R, \gamma, S/N, M). \quad (5)$$

Rayleigh Fading - Unequal Sub-Channel Gains. The M-PSK and M-QAM SERs have been derived in [16] as

$$P_{\text{PSK/QAM}}(e) = \sum_{i=1}^{u} K_i \cdot P_{\text{PSK/QAM}}(1, t, R, \gamma_i, S/N, M) \quad (6)$$

where

$$K_i = \prod_{i'=1, i' \neq i}^{u} \frac{\gamma_i}{\gamma_i - \gamma_{i'}} \quad (7)$$

and $\gamma_{i \in (1,u)}$ is the average channel power gain of the i-th sub-channel spanned by the t transmit and r receive antennas.

Nakagami Fading - Equal Sub-Channel Gains. The SERs for the case of Nakagami fading is derived in a similar way as for Rayleigh fading. For a Nakagami fading channel with equal sub-channel gains $\gamma_1 = \ldots = \gamma_u \triangleq \gamma$ and equal fading parameters $f_1 = \ldots f_u \triangleq f$, the respective SERs can be derived as [16]

$$P_{\text{PSK/QAM}}(e) = P_{\text{PSK/QAM}}(fu, ft, R, \gamma, S/N, M). \quad (8)$$

$$P_{\mathrm{PSK}}(e) = \phi_{\frac{1}{R}\frac{\lambda}{t}\frac{S}{N}}(-g_{\mathrm{PSK}})\left[\frac{1}{2\sqrt{\pi}}\frac{\Gamma(u+1/2)}{\Gamma(u+1)}\,{}_2F_1\left(u,1/2;u+1;\left(1+\frac{g_{\mathrm{PSK}}}{R}\frac{\gamma}{t}\frac{S}{N}\right)^{-1}\right)\right.$$
$$\left.+\frac{\sqrt{1-g_{\mathrm{PSK}}}}{\pi}F_1\left(1/2,u,1/2-u;3/2;\frac{1-g_{\mathrm{PSK}}}{1+\frac{g_{\mathrm{PSK}}}{R}\frac{\gamma}{t}\frac{S}{N}},1-g_{\mathrm{PSK}}\right)\right] \tag{1}$$

$$P_{\mathrm{QAM}}(e) = \phi_{\frac{1}{R}\frac{\lambda}{t}\frac{S}{N}}(-g_{\mathrm{QAM}})\frac{2q}{\sqrt{\pi}}\frac{\Gamma(u+1/2)}{\Gamma(u+1)}\,{}_2F_1\left(u,1/2;u+1;\left(1+\frac{g_{\mathrm{QAM}}}{R}\frac{\gamma}{t}\frac{S}{N}\right)^{-1}\right)$$
$$-\phi_{\frac{1}{R}\frac{\lambda}{t}\frac{S}{N}}(-2g_{\mathrm{QAM}})\frac{2q^2}{\pi(2u+1)}F_1\left(1,u,1;u+3/2;\frac{1+\frac{g_{\mathrm{QAM}}}{R}\frac{\gamma}{t}\frac{S}{N}}{1+2\frac{g_{\mathrm{QAM}}}{R}\frac{\gamma}{t}\frac{S}{N}},1/2\right) \tag{2}$$

Nakagami Fading - Unequal Sub-Channel Gains. Finally, the error rates for a Nakagami fading channel with different sub-channel gains $\gamma_{i\in(1,u)}$ and different fading factors $f_{i\in(1,u)}$ can be expressed as [16]

$$P_{\mathrm{PSK/QAM}}(e) = \sum_{i=1}^{u}\sum_{j=1}^{f_i} K_{i,j} P_{\mathrm{PSK/QAM}}(j, jt, R, \gamma_i, S/N, M) \tag{9}$$

where

$$K_{i,j} = \frac{1}{(f_i-j)!\left(-\frac{1}{R}\frac{\gamma_i}{f_i t}\frac{S}{N}\right)^{f_i-j}} \cdot \frac{\partial^{f_i-j}}{\partial s^{f_i-j}}\left[\prod_{\substack{i'=1,\\ i'\neq i}}^{u}\frac{1}{\left(1-\frac{1}{R}\frac{\gamma_{i'}}{f_{i'}t}\frac{S}{N}s\right)^{f_{i'}}}\right]_{s=\left(\frac{1}{R}\frac{\gamma_i}{f_i t}\frac{S}{N}\right)^{-1}} \tag{10}$$

IV. Full Cooperation at Each Stage

Under the assumption of full cooperation, each of the K relaying stages experiences independent BERs $P_{b,v\in(1,K)}(e)$ caused by independent SERs $P_{s,v\in(1,K)}(e)$. A bit from the s-MT is received correctly at the t-MT only when at all stages the bit has been transmitted correctly. (The cases where two or more wrong bits may result again in a correct bit is neglected here.) The end-to-end BER can therefore be expressed as

$$P_{b,\mathrm{e2e}}(e) = 1 - \prod_{v=1}^{K}(1 - P_{b,v}(e)) \tag{11}$$

which, at low BERs at every stage, can be approximated as

$$P_{b,\mathrm{e2e}}(e) \approx \sum_{v=1}^{K} P_{b,v}(e) \tag{12}$$

$$\approx \sum_{v=1}^{K} \frac{P_{s,v}(e)}{\log_2(M_v)} \tag{13}$$

where M_v is the modulation order and $P_{s,v}(e)$ corresponds to any of the above-given SERs at the v-th stage. The end-to-end BERs are now easily calculated utilising (1) and (2). Due to its simplicity, this case is not further illustrated.

V. Partial Cooperation at Each Stage

Partial cooperation at each relaying stage results in parallel MIMO channels, all of possibly different strength. An example of such clustering is depicted by means of Figure 2 with none of the involved r-MTs cooperating among each other.

Here, the first stage spans two independent SISO channels with average attenuation $\gamma_{1,1}$ and $\gamma_{1,2}$, respectively. Each of these channels causes independent BERs, denoted as $P_{1,1}$ and $P_{1,2}$, respectively. Similarly, the second stage spans two independent MISO channels, where the first MISO channel consists of channels with average attenuations $\gamma_{2,1}$ and $\gamma_{2,3}$, and the second MISO channel consists of channels with average attenuations $\gamma_{2,2}$ and $\gamma_{2,4}$. Furthermore, assuming an error free input into the second VAA relaying tier, the BERs at the output of the MISO channels are $P_{2,1}$ and $P_{2,2}$. Finally, the third stage spans a single MISO channel with a BER $P_{3,1}$.

Note that the r-MTs belonging to the same stage need to communicate at the same rate; furthermore, they obviously need to know which part of the space-time block code to transmit.

To obtain the exact end-to-end BER is not trivial, as an error in the first stage may propagate to the t-MT; however, it may also be corrected at the next stage. Referring to Figure 2, for example, it is assumed that the same information bit is erroneously received over the link denoted as $(1,1)$ and correctly for $(1,2)$.

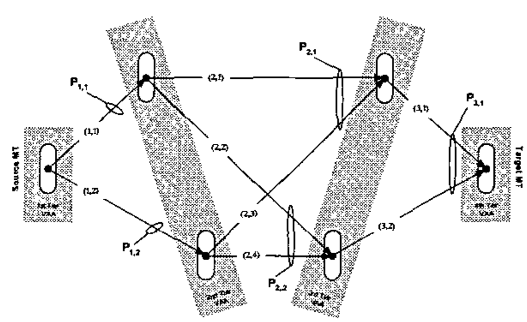

Fig. 2. 3-stage distributed communication system without cooperation.

$$P_{b,\mathrm{e2e}}(e) \approx \left[P_{1,1}(e)\left(\frac{\gamma_{2,1}}{\gamma_{2,1}+\gamma_{2,3}}\frac{\gamma_{3,1}}{\gamma_{3,1}+\gamma_{3,2}}+\frac{\gamma_{2,2}}{\gamma_{2,2}+\gamma_{2,4}}\frac{\gamma_{3,2}}{\gamma_{3,1}+\gamma_{3,2}}\right)+ P_{1,2}(e)\left(\frac{\gamma_{2,4}}{\gamma_{2,2}+\gamma_{2,4}}\frac{\gamma_{3,2}}{\gamma_{3,1}+\gamma_{3,2}}+\frac{\gamma_{2,3}}{\gamma_{2,1}+\gamma_{2,3}}\frac{\gamma_{3,1}}{\gamma_{3,1}+\gamma_{3,2}}\right)\right]+ \left[P_{2,1}(e)\left(\frac{\gamma_{3,1}}{\gamma_{3,1}+\gamma_{3,2}}\right)+P_{2,2}(e)\left(\frac{\gamma_{3,2}}{\gamma_{3,1}+\gamma_{3,2}}\right)\right]+\left[P_{3,1}(e)\right] \quad (14)$$

Then, the STBC formed by (2,1) and (2,3) has as its input one erroneous and one correct information bit. Assuming that $\gamma_{2,3} \gg \gamma_{2,1}$, then the error does not further propagate since it will be outweighed by the correct bit. Alternatively, if $\gamma_{2,3} \ll \gamma_{2,1}$, then there is a large likelihood that the error propagates.

This creates dependencies between the error events at each stage in dependency of the modulation scheme used, the prevailing channel statistics, the average channel attenuations, the SNR, as well as the STBC chosen. The fairly complex interdependencies call for suitable simplifications, which are exposed and justified below.

Generally, it is desirable to develop an approximation which decouples the error events at the respective stages. To this end, it is assumed that the system operates at low error rates which causes only one error event at a time in the entire network. Let us assume that an error occurs in link (1,1); however, (1,2) is error free. Then the probability that the error propagates further is related to the strengths of channels (2,1) and (2,3). It is intuitive and hence conjectured here that the probability that such error propagates is proportional to the strength of the STBC branch it departs from, here (2,1) for one of two MISO channels, and (2,2) for the other one.

Therefore, the probability that an error which occurred in link (1,1) with probability $P_{1,1}$ propagates through the MISO channel spanned by (2,1) and (2,3) is approximated as $P_{1,1} \cdot \gamma_{2,1}/(\gamma_{2,1}+\gamma_{2,3})$, where the strength of the erroneous channel (2,1) is normalised by the total strength of both sub-channels. To capture the probability that such an error propagates until the t-MT, all possible paths in the network have to be found and the original probability of error weighed with the ratios between the respective path gains.

Taking the above-said into account and assuming that at high SNRs only one such error will occur at any link, the end-to-end BER for the network depicted in Figure 2 can be expressed through (14), which is given at the top of this page. Eq. (14) can be simplified to

$$P_{b,\mathrm{e2e}}(e) \approx \left[\xi_{1,1}P_{1,1}(e)+\xi_{1,2}P_{1,2}(e)\right]+ \left[\xi_{2,1}P_{2,1}(e)+\xi_{2,2}P_{2,2}(e)\right]+ \left[\xi_{3,1}P_{3,1}(e)\right] \quad (15)$$

where $\xi_{v,i}$ is the probability that an error occurring in link (v,i) will propagate to the t-MT.

This result is easily generalised to networks of any size and any form of partial cooperation. To this end, assume that there are $Q_{v\in(1,K)}$ cooperative clusters at the v-th stage, each of which will yield an error probability of $P_{v\in(1,K),i\in(1,Q_v)}$. The end-to-end BER is hence approximated as

$$P_{b,\mathrm{e2e}}(e) \approx \sum_{v=1}^{K}\sum_{i=1}^{Q_v}\xi_{v,i}P_{v,i}(e) \quad (16)$$

where the probabilities $\xi_{v,i}$ are easily found from the specific network topology. The BERs $P_{v,i}(e)$ are obtained from the previously derived SERs with an appropriate number of transmit and receive antennas per cluster, as well as prevailing channel conditions. The applicability of the derived end-to-end BER is assessed by means of Figures 3 and 4.

Explicitly, Figure 3 compares the numerically obtained and derived end-to-end BER versus the SNR in the first link for a two-stage network as depicted in Figure 2 without the second stage. For all simulations, QPSK has been used. The graphs are labelled on the respectively utilised channel gains. Clearly, the derived BER differs from the exact one for low SNRs; however, for an increasing SNR, both curves converge.

Figure 4 compares the numerically obtained and derived end-to-end BER versus the SNR in the first link for a three-stage network as depicted in Figure 2. The curves are again labelled on the channel gains. From Figure 4 it is clear that the derived end-to-end BER holds with high precision for a variety of different scenarios.

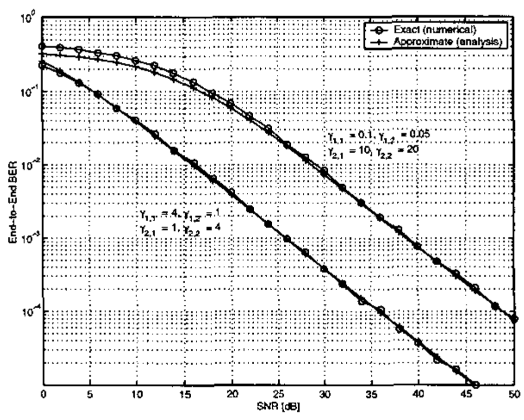

Fig. 3. Numerically obtained and derived end-to-end BER versus the SNR in the first link for a two-stage network without cooperation.

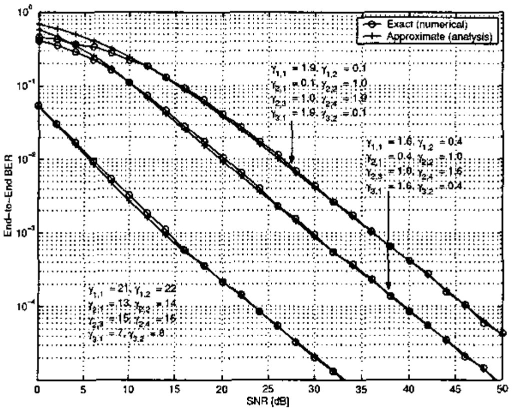

Fig. 4. Numerically obtained and derived end-to-end BER versus the SNR in the first link for a three-stage network without cooperation.

VI. Conclusion

This paper examined the end-to-end bit error rates (BERs) for a distributed-MIMO multi-stage communication network with deployed space-time block codes. To this end, the exact symbol error rates for flat Rayleigh and Nakagami fading channels with potentially different sub-channel statistics have been exposed. This allowed the end-to-end BERs to be analysed assuming either full or partial cooperation of the mobile terminals at each stage. Examples have confirmed the applicability and precision of the derived error rates. The performed analysis proves useful in deriving fractional resource allocation strategies so as to maximise the end-to-end throughput for distributed space-time block encoded wireless communication networks.

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