

## Research Article

# Modelling of an Esaki Tunnel Diode in a Circuit Simulator

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A method for circuit-level modelling a physically realistic Esaki tunnel diode model is presented. A parameterisation technique that transforms the strongly nonlinear characteristic of a tunnel diode into two relatively modest nonlinear characteristics is demonstrated. The introduction of an intermediate state variable results in a physically realistic mathematical model that is not only moderately nonlinear and therefore robust, but also single-valued.

## 1. Introduction

Tunnelling here refers to the phenomenon of movement of charge across a potential barrier with the result that there is a nonunique relationship between current and applied voltage. The effect occurs in semiconductor-based tunnel diodes, molecular diodes, and in current leakage through the thin oxides of advanced metal-oxide-metal (MOS) transistors, see Olivo et al. [1]. Tunnel diodes, see Hall [2], have been used in fast-switching electronic circuits ever since their discovery, Esaki [3]. While tunnelling is a critical effect, it can be difficult to model in a circuit simulator because of the multivalued flux-potential relationship which is also strongly nonlinear. While this paper focuses on the modelling of tunnel diodes, the technique presented can be used to model many types of tunnelling effects.

One unique feature of an Esaki tunnel diode is the Negative Differential Resistance (NDR) region of its current-voltage ( $I$ - $V$ ) characteristic. Under certain circuit conditions, this creates a bistable or multistable circuit, and switching between the two stable states can be rapid. In other situations, tunnelling is undesirable and sets limits on performance. Determining the onset of tunnelling is important in designing integrated circuits using sub-65 nm MOS transistors where the gate oxide is necessarily thin. The typical current-voltage ( $I$ - $V$ ) characteristic of an Esaki tunnel diode can be divided into three regions in the direction of increasing voltage, see Figure 1. The first region

involves an increase in the output current until it reaches a peak value and represents a Positive Differential Resistance (PDR). The region following this involves a decrease in current with increasing voltage and this corresponds to the NDR region. Here the current decreases until it reaches a minimum or valley value. The third region corresponds to conduction of a forward-biased p-n junction diode and is the second PDR region. All three regions are represented by exponential functions and are therefore strongly nonlinear. These exponential functions are derived from a quantum-physical description of the diode, Sze [4], and the total current is a sum of these exponential functions. In order to simulate circuits consisting of Esaki tunnel diodes, it is essential to be able to accurately model the nonlinear behaviour of the  $I$ - $V$  characteristic as well as the negative resistance. Ideally, one would like to model the Esaki tunnel diode in a circuit simulator with minimal changes to the quantum-physical representation of the device.

An Esaki tunnel diode is not directly modelled in commercial circuit simulators. Three strategies have been used to mitigate the bistable and/or strongly nonlinear physical description of the diode. Techniques used include: macromodelling approximation by curve-fitting to a relatively smooth function, and modifications to the underlying algorithms that solve a circuit containing a tunnel diode. The macro-modelling was used in Kuo et al. [5] wherein a tunnel diode was represented as a set of linear current sources which were appropriately switched on or off depending on bias

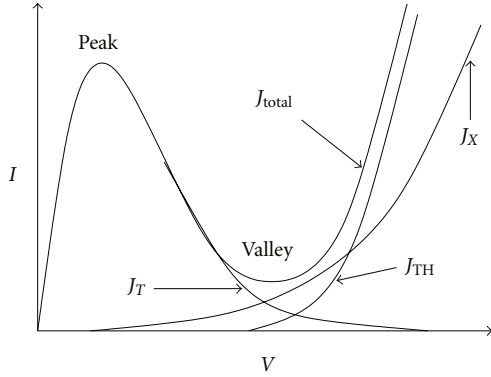


FIGURE 1: Esaki tunnel diode  $I$ - $V$  characteristic.

conditions. This approach uses combinations of switches, resistors, capacitors, voltage sources, and voltage-controlled current sources arranged such that they will approximately mimic the tunnel diode  $I$ - $V$  characteristic. Curve-fitting has also been used extensively wherein the  $I$ - $V$  characteristic was approximated by a piecewise-linear model, see Mohan et al. [6], or by a piecewise nonlinear model utilising a diode for each of the PDR regions, see Neculoiu and Tebeanu [7]. Similarly, polynomial curve-fitting techniques, Chang et al. [8], have also been used. Macro-modelling and curve fitting are essentially nonphysical techniques introducing well-behaved functions to model a device whose operation is quite complex.

Physics-based modelling is essential when a strong correlation between simulation and measurement is desired—particularly with steadily decreasing device geometries. A physics-based description of the diode was used in Bhat-tacharya and Mazumder [9] where the iterative technique used to solve the nonlinear set of differential equations describing a circuit was modified. In particular, algorithmic modifications were made to the Newton-Raphson iterative method and implemented in a circuit simulator. Based on the onset of certain conditions during simulation, the algorithm switches from voltage-based iterations to current-based iterations. Since the  $I$ - $V$  characteristic of the tunnel diode can have multiple values of voltage for a single value of current, the authors proposed a technique to approximately assign a single value of voltage and resolve this uncertainty. In order to calculate this voltage value, the authors introduced additional parameters in the diode model. Other techniques proposed to model circuits with tunnel diodes can be thought of as variations to homotopy-based methods as described in Melville et al. [10], and in general require a steep learning curve on the part of the user to be able to understand and implement the model of a nonlinear device. Also, the generality of the modified iterative techniques and the homotopy methods is limited.

In this paper, a technique for modelling a physically based tunnel diode is demonstrated that makes no changes to the basic equations describing the operation of a device and requires no modification to the underlying algorithms of a circuit simulator. A similar approach can be used for other strongly nonlinear devices.

## 2. Parameterised Device Modelling with State Variables

Simulating circuits with strongly nonlinear devices can often be plagued by nonconvergence. In the well-established circuit simulator tradition, modifications are made at the individual element level to obtain convergence. This philosophical procedure is used in the SPICE simulator to obtain convergence for circuits with junction diodes that have exponential characteristics. In SPICE, heuristics are used including limiting the fraction by which current and voltage can change from one iterative step to the next. This cannot be extended to handling tunnel diodes. In essence, the technique described in this paper introduces an intermediate state variable between the current and voltage variables in order to mitigate the nonlinear diode  $I$ - $V$  characteristics. The iterative solver then acts on this intermediate state variable while trying to minimize the error function associated with the circuit under consideration. As outlined in Christofferssen et al. [11], when an element is modelled using state variables, the current and voltage are expressed as a function of these state variables. This is,

$$\begin{aligned} \mathbf{v}(t) &= \mathbf{v}\left(\mathbf{x}(t), \frac{d\mathbf{x}}{dt}, \dots, \frac{d^m\mathbf{x}}{dt^m}, \mathbf{x}_D(t)\right), \\ \mathbf{i}(t) &= \mathbf{i}\left(\mathbf{x}(t), \frac{d\mathbf{x}}{dt}, \dots, \frac{d^m\mathbf{x}}{dt^m}, \mathbf{x}_D(t)\right), \end{aligned} \quad (1)$$

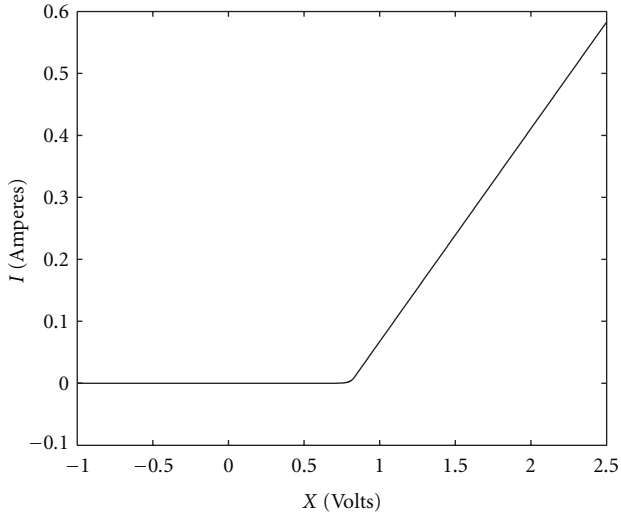
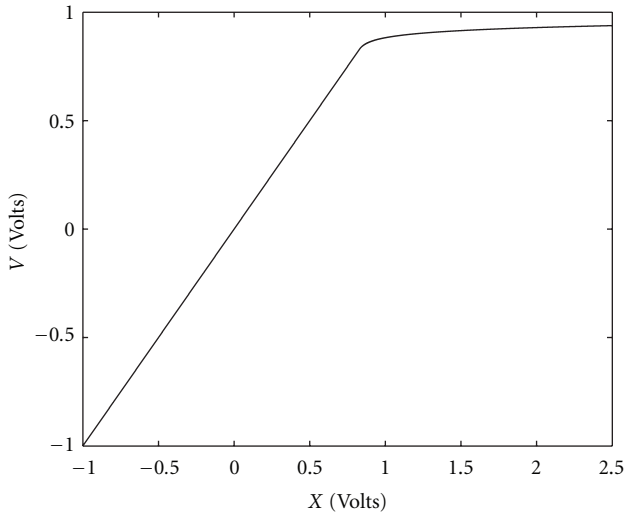
where  $\mathbf{v}(t)$  and  $\mathbf{i}(t)$  are vectors of voltages and currents at the ports of the nonlinear device,  $\mathbf{x}(t)$  is a vector of state variables, and  $\mathbf{x}_D(t)$  is a vector of time-delayed state variables, that is,  $[\mathbf{x}_D(t)]_k = x_k(t - \tau_k)$ . Equation (1) defines the modelling scope. This can be a problem if the describing equations are strongly nonlinear as local convergence control cannot be used. However, avoiding the use of local convergence control, and the ad-hoc modifications of the iterative algorithm required, enables state-of-the-art, off-the-shelf numerical libraries to be employed.

Consider the equation for a microwave diode whose  $I$ - $V$  relationship is described by an exponential function. Once the diode is forward biased, a small change in input voltage can produce a large change in output current. This nonlinear behaviour can be a source of serious simulation errors that can manifest themselves as an incomplete or non-convergent solution, or worse still, an incorrect solution. In order to continue to use the physics-based equations for a nonlinear device and overcome the problem of strong nonlinear behaviour, the technique of parameterisation was introduced in Rizzoli and Neri [12]. This is illustrated by considering the example of a microwave diode.

The conventional current equation for a diode is expressed as

$$i(t) = I_s[\exp(\alpha v(t)) - 1], \quad (2)$$

where  $v(t)$  is the voltage across the diode,  $I_s$  is the reverse saturation current, and  $\alpha$  is a parameter representing the slope of the conductance curve of the diode characteristic. Exponential relationships of this type are responsible for most of the numerical instabilities in circuit simulation. To


 FIGURE 2: Parameterised  $I$ - $X$  relationship for a diode.

 FIGURE 3: Parameterised  $V$ - $X$  relationship for a diode.

circumvent this, the state variable in (2) is changed from voltage to a fictitious nonconventional variable,  $x(t)$ . This state variable is identical to the junction voltage below some nonzero threshold value, say  $V_0$ , and is defined as a linear function of the current in (2) above  $V_0$ . Care is taken to ensure that the current and its derivatives are continuous at  $x = V_0$ . The equations for diode voltage and current are now expressed as

$$v(t) = \begin{cases} V_0 + \frac{1}{\alpha} \ln\{1 + \alpha[x(t) - V_0]\}, & V_0 \leq x(t), \\ x(t), & x(t) \leq V_0, \end{cases}$$

$$i(t) = \begin{cases} I_s \exp(\alpha V_0) \{1 + \alpha[x(t) - V_0]\} - I_s, & V_0 \leq x(t), \\ I_s [\exp(\alpha v(t)) - 1], & x(t) \leq V_0, \end{cases} \quad (3)$$

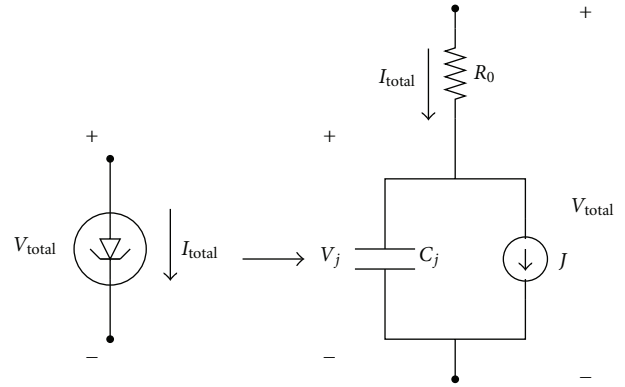


FIGURE 4: Esaki tunnel diode circuit model.

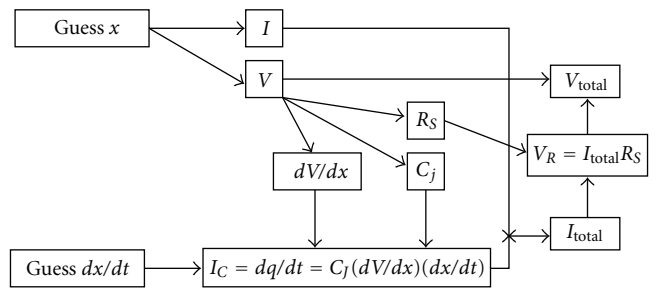


FIGURE 5: Evaluation of total current and voltage as a function of the state variable.

where (3) is the exact parametric representations of the diode  $I$ - $V$  characteristic. The highly nonlinear  $I$ - $V$  characteristic is now transformed into current-state-variable ( $I$ - $X$ ) and voltage-state-variable ( $V$ - $X$ ) characteristics that are not as strongly nonlinear, without any changes to the essential mathematical model of the device. The solution is well behaved and local convergence or heuristics within the diode model are not required. After the transformations of (3), the reduced nonlinear nature of the relationship is illustrated in Figure 2, the  $I$ - $X$  relationship, and in Figure 3, the  $V$ - $X$  relationship, with  $I_s = 100$  aA,  $\alpha = 38.686$ , and  $V_0 = 0.8$  V.

### 3. Modelling the Esaki Tunnel Diode

The intrinsic Esaki tunnel diode model consists of an ideal tunnel diode a nonlinear capacitor and a nonlinear resistor, see Figure 4. There are two controlling variables in this model, which are an as yet unspecified time-dependent state variable, and its time derivative. The ideal tunnel diode is modelled using current and voltage as a function of the primary state variable  $x$ . The capacitance is modelled by using the evaluated voltage and the time derivative of the state variable,  $dx/dt$ , to calculate the derivative of charge as a function of time or  $dq/dt$ . From this, the current through the capacitor can be determined. The resistor is modelled using the evaluated voltage and the total current. An outline of the model calculation of the total current and voltage is shown in Figure 5.

The tunnel diode current density is typically described as the sum of three exponential functions derived from quantum mechanical considerations. This formulation appears in Sze [4], although here the physics is limited only to the forward-bias direction. Referring to Figure 1, this is expressed as

$$J_{\text{total}} = \frac{V(t)}{V_P} J_T + J_X + J_{\text{TH}}, \quad (4)$$

where

$$J_T = J_P \exp\left(1 - \frac{V(t)}{V_P}\right), \quad (5)$$

$$J_X = J_V \exp(A_2(V(t) - V_V)), \quad (6)$$

$$J_{\text{TH}} = J_S \left( \exp\left(\frac{qV(t)}{kT}\right) - 1 \right). \quad (7)$$

The first term is a closed-form expression of the tunnelling current density which describes the behaviour particular to the tunnel diode. This includes the negative resistance region which captures the core functionality of the tunnel diode. The second term describes the excess tunnelling current density while the third term is the normal diode characteristic. In (5),  $J_P$  is the peak current density and  $V_P$  is the corresponding peak voltage. In (6),  $J_V$  is the peak current density and  $V_V$  is the corresponding peak voltage. The parameter  $A_2$  represents an excess current prefactor. Finally, in (7),  $J_S$  is the saturation current density,  $q$  is the charge of an electron,  $k$  is Boltzmann's constant, and  $T$  is the temperature in degrees Kelvin.

As mentioned in the previous section, the exponential functions can lead to a rapid change in current for a relatively small change in voltage which can create convergence issues for the simulator's nonlinear solver. The tunnel diode equation cannot be solved for current as a single and unique function of voltage. Recognising that the three major components of current density can be thought of as three diodes with exponential  $I$ - $V$  characteristics, they are parameterised as previously described. After individual parameterisation of the three regions based on equating current density and first derivatives at threshold points, the tunnel diode equations become

$$J_T(t) = \begin{cases} J_P \exp\left(1 - \frac{V_T}{V_P}\right) \times \left(1 - \frac{1}{V_P}(x_T(t) - V_T)\right), & x_T(t) \leq V_T, \\ J_P \exp\left(1 - \frac{x_T(t)}{V_P}\right), & x_T(t) > V_T, \end{cases}$$

$$V(t) = \begin{cases} V_T - V_P \times \ln\left(1 - \frac{1}{V_P}(x_T(t) - V_T)\right), & x_T(t) \leq V_T, \\ x_T(t), & x(t) > V_T, \end{cases}$$

$$J_X(t) = \begin{cases} J_V \exp(A_2(x_X(t) - V_V)), & x_X(t) \leq V_X, \\ J_V \exp(A_2(V_X(t) - V_V)) \\ \quad \times (1 + A_2(x_X(t) - V_X)), & x_X(t) > V_X, \end{cases}$$

$$V(t) = \begin{cases} x_X(t), & x_X(t) \leq V_X, \\ V_X + \frac{1}{A_2} \times \ln(1 + A_2(x_X(t) - V_X)), & x_X(t) > V_X, \end{cases}$$

$$J_{\text{TH}} = \begin{cases} J_S \exp\left(\frac{q}{kT} x_{\text{TH}}\right) - J_S, & x_{\text{TH}} \leq V_{\text{TH}}, \\ J_S \exp\left(\frac{q}{kT} V_{\text{TH}}\right) \times \left(1 + \frac{q}{kT}(x_{\text{TH}} - V_{\text{TH}})\right) - J_S, & x_{\text{TH}}(t) > V_{\text{TH}}, \end{cases}$$

$$V(t) = \begin{cases} x_{\text{TH}}(t), & x_{\text{TH}} \leq V_{\text{TH}}, \\ V_{\text{TH}} + \frac{kT}{q} \ln\left(1 + \frac{q}{kT}(x_{\text{TH}}(t) - V_{\text{TH}})\right), & x_{\text{TH}} > V_{\text{TH}}. \end{cases} \quad (8)$$

As can be seen from the expressions above, each current is a function of a different  $x(t)$ . Since the voltage across each current source is the same, the voltage equations can be set equal to each other to obtain a relationship between  $x_T(t)$ ,  $x_X(t)$ , and  $x_{\text{TH}}(t)$ . The linear version of  $J_T$  is used for  $x_T(t) \leq V_T$  while the linear versions of  $J_X$  and  $J_{\text{TH}}$  are respectively used for  $x_X(t) > V_X$  and  $x_{\text{TH}}(t) > V_{\text{TH}}$ . Therefore, when combining the terms as a function of a single state variable  $x(t)$ , they should be a function of  $x_T(t)$  for  $x_T(t) \leq V_T$  and a function of  $x_X(t)$  or  $x_{\text{TH}}(t)$  for  $x_X(t) > V_X$  and  $x_{\text{TH}}(t) > V_{\text{TH}}$ , respectively.

Careful study of the various possible relationships between  $V_T$ ,  $V_X$ , and  $V_{\text{TH}}$  reveals that the transition between the various regions will be continuous as long as  $V_T$  is chosen to be less than or equal to  $V_X$  and  $V_{\text{TH}}$ . Using this constraint, continuous equations for the current densities and voltage as a function of a single  $x(t)$  are derived. Note that  $\min\{V_X, V_{\text{TH}}\}$  has been used as a criteria to determine whether the  $x_X(t)$  or  $x_{\text{TH}}(t)$  should be used for large  $x(t)$  where  $V_X$  and  $V_{\text{TH}}$  are calculated based on the first derivative of the current. The expressions for the diode current density and voltage as a function of a single state variable  $x(t)$  are expressed as

$$J_T(t) = \begin{cases} J_P \exp\left(1 - \frac{V_T}{V_P}\right) \times \left(1 - \frac{1}{V_P}(x(t) - V_T)\right), & x(t) \leq V_T, \\ J_P \exp\left(1 - \frac{x(t)}{V_P}\right), & V_T < x(t) \leq V_{\text{XTH}}, \\ A_{T2} \times (1 + B(x(t) - V_{\text{XTH}}))^{C_T}, & x(t) > V_{\text{XTH}}, \end{cases}$$

$$\begin{aligned}
J_X(t) &= \begin{cases} J_V \exp(A_2(V_T - V_V)) \\ \times \left(1 - \frac{1}{V_P}(x(t) - V_T)\right)^{-A_2 V_P}, & x(t) \leq V_T, \\ J_V \exp(A_2(x(t) - V_V)), & V_T < x(t) \leq V_{XTH}, \\ A_{X2}(1 + B(x(t) - V_{XTH}))^{C_X}, & x(t) > V_{XTH}, \end{cases} \\
J_{TH}(t) &= \begin{cases} J_S \exp\left(\frac{qV_T}{kT}\right) \\ \times \left(1 - \frac{1}{V_P}(x(t) - V_T)\right)^{-qV_P/kT} - J_S, & x(t) \leq V_T, \\ J_S \exp\left(\frac{qx(t)}{kT}\right) - J_S, & V_T < x(t) \leq V_{XTH}, \\ A_{TH2} \times (1 + B(x(t) - V_{XTH}))^{C_{TH}} - J_S, & x(t) > V_{XTH}, \end{cases} \\
V(t) &= \begin{cases} V_T - V_P \times \ln\left(1 - \frac{1}{V_P}(x(t) - V_T)\right), & x(t) \leq V_T, \\ x(t), & V_T < x(t) < V_{XTH}, \\ V_{XTH} + \frac{1}{B} \times \ln(1 + B(x(t) - V_{XTH})), & x(t) > V_{XTH}, \end{cases}
\end{aligned} \tag{9}$$

$$\begin{aligned}
B &= A_2, \\
C_T &= -\frac{1}{A_2 V_P}, \\
C_X &= 1, \\
C_{TH} &= \frac{q}{kT A_2},
\end{aligned} \tag{11}$$

and when  $V_X > V_{TH}$ ,

$$\begin{aligned}
V_{XTH} &= V_{TH}, \\
A_{T2} &= J_P \exp\left(1 - \frac{V_{TH}}{V_P}\right), \\
A_{X2} &= J_V \exp(A_2(V_{TH} - V_V)), \\
A_{TH2} &= J_S \exp\left(\frac{qV_{TH}}{kT}\right), \\
B &= \frac{q}{kT}, \\
C_T &= -\frac{kT}{qV_P}, \\
C_X &= \frac{A_2 kT}{q}, \\
C_{TH} &= 1.
\end{aligned} \tag{12}$$

which include a number of constant terms. The expressions for the voltage thresholds,  $V_T$ ,  $V_X$ , and  $V_{TH}$ , define the boundary between the current regions. These are derived by taking the first derivative of the current equation and solving for voltage as a function of slope while maintaining current continuity:

$$\begin{aligned}
V_T &= V_P - V_P \ln\left(\frac{-m_T V_P}{J_P}\right), \quad \text{slope } m_T < 0, \\
V_X &= V_V + \frac{1}{A_2} \ln\left(\frac{m_X}{A_2 J_V}\right), \quad \text{slope } m_X > 0, \\
V_{TH} &= \frac{kT}{q} \ln\left(\frac{m_{TH} kT}{J_S q}\right), \quad \text{slope } m_{TH} > 0.
\end{aligned} \tag{10}$$

When  $V_X \leq V_{TH}$  the other constants are,

$$\begin{aligned}
V_{XTH} &= V_X, \\
A_{T2} &= J_P \exp\left(1 - \frac{V_X}{V_P}\right), \\
A_{X2} &= J_V \exp(A_2(V_X - V_V)), \\
A_{TH2} &= J_S \exp\left(\frac{qV_X}{kT}\right),
\end{aligned}$$

The tunnel diode model is completed by incorporating the diode's junction capacitance  $C_j$ . Referring to the parameter values for the diode shown in Table 1, the model of  $C_j$  depends on whether the zero-bias depletion capacitance parameter CTO is set. If it is set, then

$$C_j = \begin{cases} \frac{CTO}{(1 - V_j/FI)^{GAMA}}, & -V_j > 0, \\ \frac{CTO + V_j(k_1 + CTO V_j)}{1 + k_3} + \frac{k_2 k_3}{1 + k_3}, & -V_j < 0, \end{cases} \tag{13}$$

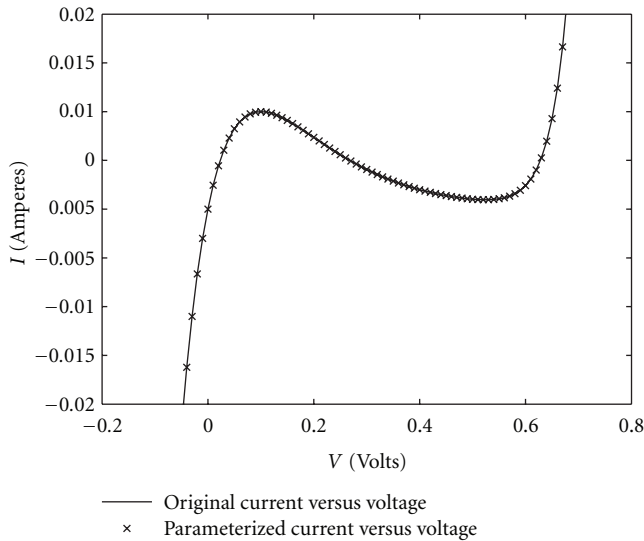
where  $k_1 = CTO \times GAMA/FI$ ,  $k_2 = CTO/GAMA^{0.2}$ ,  $k_3 = \exp(10(V_j - 0.8FI))$ , and  $V_j$  is the diode junction voltage. If the depletion capacitance parameter, CDO, is set, then  $C_j$  is incremented by the amount  $CDO \times \exp(AFACV_j)$ .

The parameterised  $I$ - $V$  relationship using the constants listed in Table 1 are compared with the original  $I$ - $V$  equation and in Figure 6. For the sake of brevity, only the total current equation is plotted instead of the individual current components. As can be seen, the results are identical and the parameterisation process has not affected the accuracy of the underlying equations for the diode.

The benefits of parameterisation are demonstrated by plotting total current and voltage with respect to the suitably introduced state variable  $x$ . As can be seen in Figure 7, which plots tunnel diode voltage with respect to a state variable, and Figure 8, which plots tunnel diode current with respect to  $x$ , the highly nonlinear relationship between current and voltage is replaced by two milder nonlinearities.

TABLE 1: Esaki tunnel diode parameter table.

Parameter	Description	Default	Units
JS	Saturation current	$1 \times 10^{-16}$	A
CTO	Zero bias depletion capacitance	0	F
FI	Built-in barrier potential	0.8	V
GAMA	Capacitance power law parameter	0.5	—
CDO	Zero bias diffusion capacitance	0	F
AFAC	Slope factor of diffusion capacitance	38.696	1/V
RO	Series resistance in forward bias	0	$\Omega$
TAU	Intrinsic depletion layer time constant	0	sec
AREA	Area multiplier	1	—
JV	Valley current	$1 \times 10^{-4}$	A
JP	Peak current	$1 \times 10^{-3}$	A
VV	Valley voltage	0.5	V
VPK	Peak voltage	0.1	V
A2	Excess current prefactor	30	—
MT	Tunnel current slope factor	-1	—
MX	Excess current slope factor	1	—
MTH	Thermal current slope factor	1	—
TEMP	Temperature	300°	K

FIGURE 6: Original Esaki tunnel diode  $I$ - $V$  characteristic compared with the parameterised  $I$ - $V$  characteristic.

#### 4. Simulation and Results

The parameterised Esaki tunnel diode model described above was implemented in an open-source circuit simulator (<http://www.freda.org/>). Simulated  $I$ - $V$  characteristics of the tunnel diode using the parameter values in Table 1 are generated by sweeping the voltage from  $-0.05$  volts to  $0.6$  volts. This voltage range is the typical range of operation for this device. The simulated curve is compared with measured  $I$ - $V$  characteristics and this is displayed in Figure 9. The diode used in measurements is a general-purpose germanium diode with a nominal peak voltage

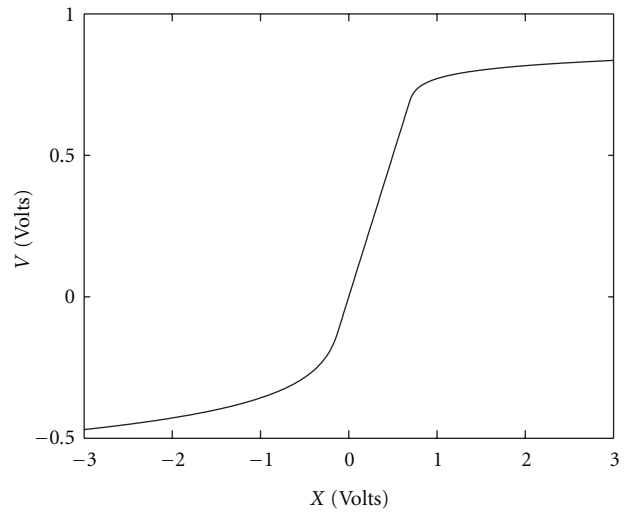


FIGURE 7: Esaki tunnel diode voltage with respect to state variable.

$V_p$  of 105 mV, valley voltage  $V_v$  of 380 mV peak-to-valley current ratio of 8. A match obtained between simulation and measurement thereby validates the effectiveness of this modelling technique. In other words, it is possible to easily and efficiently implement a highly nonlinear device keeping the physics of the device intact while requiring no change to the underlying algorithm that solves the nonlinear equations associated with the circuit.

A transient simulation was also performed on a canonical oscillator circuit containing the tunnel diode to demonstrate the switching characteristics of the device. The circuit used along with the values of the components is shown in Figure 10. A snapshot of the simulated output voltage versus time in Figure 11 shows the circuit oscillating at



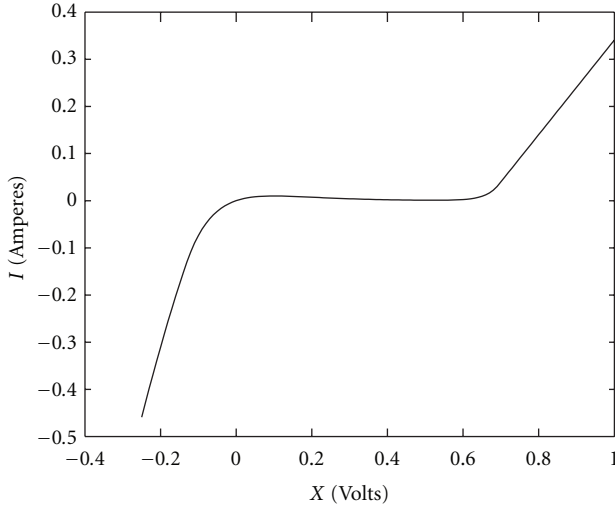
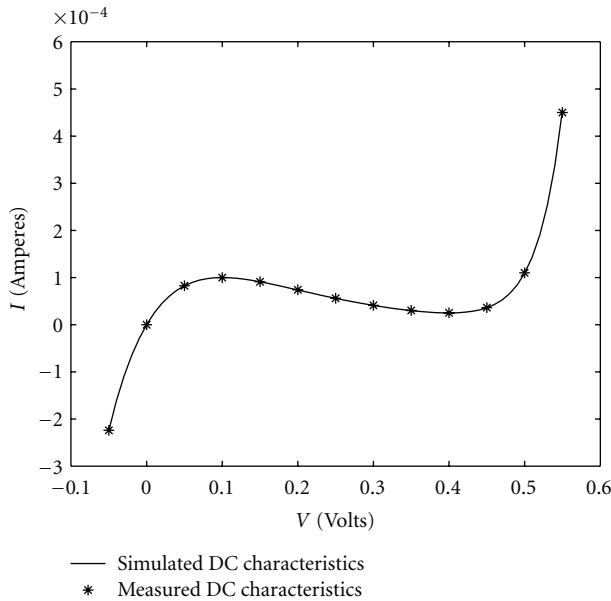


FIGURE 8: Esaki tunnel diode current with respect to state variable.

FIGURE 9: Simulated  $I$ - $V$  characteristics of the Esaki tunnel diode compared with measurement.

approximately 150 MHz. The values of the diode parameters used for this simulation that differ from those in Table 1 are indicated in Table 2.

## 5. Conclusions

Tunnelling is an important aspect of charge transport in semiconductor and molecular devices. While a semiconductor tunnel diode with characteristics described by sums of exponential functions was considered, the parameterisation technique introduced can be used with any analytic expression. The central result is transcribing a circuit simulation problem from one dealing with strong nonlinearities to one working with well-behaved, moderately nonlinear element

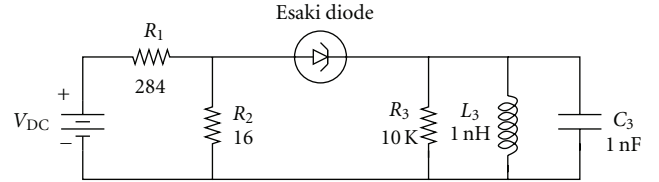


FIGURE 10: Oscillator circuit used for transient simulation.

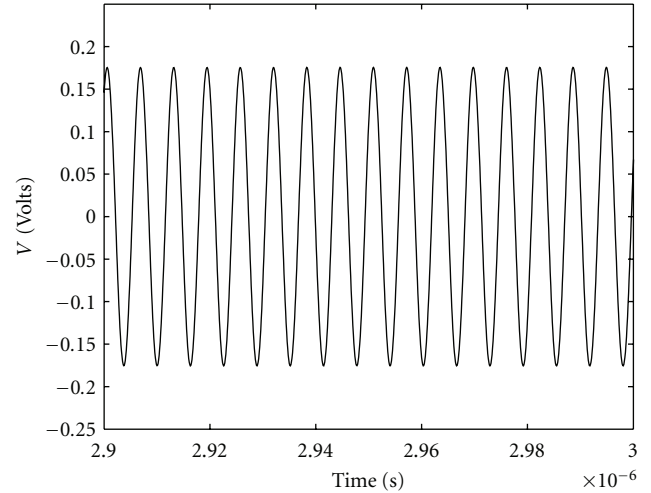


FIGURE 11: Simulated transient response of oscillator circuit.

TABLE 2: Nondefault Esaki tunnel diode parameters used in transient simulation.

Parameter	Value
JS	$2 \times 10^{-12}$
JV	$1.1 \times 10^{-4}$
VV	0.35
VPK	0.065
A2	10
CTO	$0.03 \times 10^{-12}$
GAMA	0.19
FI	0.0368
RO	1.5

characteristics. This is achieved without an increase in problem size. The introduction of an alternative state (or state variable) enables the strongly nonlinear character of tunnelling to be modelled, with full accuracy, by intermediate equations which are relatively well behaved and moderately nonlinear. Thus, off-the-shelf numerics can be used in a circuit simulator and there is no need for local heuristics, homotopy, or functional approximation to obtain convergence. The fundamental requirement is that the circuit simulator support state variables. The state variables replace the (usual) nodal voltages as the unknowns and the error function becomes the energy norm rather than solely based on Kirchoff's Current Law while the total number of unknowns remains unchanged.

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