Automated surface subdivision and tool path generation for $3\frac{1}{2}$-axis CNC machining of sculptured parts

Zezhong C. Chen, Zuomin Dong, Geoffrey W. Vickers

Abstract

As an innovative and cost-effective method for carrying out multiple-axis CNC machining, $3\frac{1}{2}$-axis CNC machining technique adds an automatic indexing/rotary table with two additional discrete rotations to a regular 3-axis CNC machine, to improve its ability and efficiency for machining complex sculptured parts. In this work, a new tool path generation method to automatically subdivide a complex sculptured surface into a number of easy-to-machine surface patches; identify the favorable machining set-up/orientation for each patch; and generate effective 3-axis CNC tool paths for each patch is introduced. The method and its advantages are illustrated using an example of sculptured surface machining. The work contributes to automated multiple-axis CNC tool path generation for sculptured part machining and forms a foundation for further research.

Keywords: Intelligent tool path generation; Multi-axis CNC programming; 5-Axis CNC machining; Fuzzy pattern clustering; Voronoi diagram

1. Introduction

Sculptured parts have found broad applications in the aeronautical, automotive and die/injection moulding industries. Extensive research on automated 3-axis CNC tool path generation, such as those conducted in [3–6,19], allows most of these parts to be machined efficiently. However, a regular 3-axis CNC machine is not capable of producing some sculptured parts with complex shape, and is quite inefficient for machining many others. For example, an occult surface cannot be machined without gouging and/or interference in a 3-axis set-up. Sculptured parts with dramatic curvature change and high surface-finish require long machining time if a 3-axis CNC machine is used [7].

To produce these complex shapes efficiently, the cutter needs to change its approaching direction or orientation to reach the local areas to be machined. Advanced 5-axis CNC machines provide a solution. These machines can re-orient the cutter or re-set the part during machining to increase machining efficiency, eliminate surface gouging, and avoid part-and-cutter interference.

To fully take this advantage, research has been carried out on tool path planning and cutter orientation determination [1,7,8,13–15]. Bedi et al. [1] proposed an algorithm of principal curvature alignment to determine the cutter orientation in 5-axis CNC machining. Lee [13] explored the non-isoparametric
tool path planning for 5-axis CNC machining. However, 5-axis CNC machines have their own disadvantages, including high investment and operation costs, less rigid and chatter-prone structure, and complex tool path planning.

As a low-cost alternative to 5-axis CNC machining, a discrete 5-axis CNC machining technique was introduced [16,17]. This approach uses a regular 3-axis CNC machine plus an automatic indexing table. The axis number indicating cutter/part movement is increased by the discrete rotation of the table along two of the three existing axes (X-, Y-, and Z-axis). Although the table rotations do not occur simultaneously with the cutter movement, the machining capability of this technique is nearly the same as that of a 5-axis CNC machine. In this approach, tool paths are generated in several steps. First, tool paths are planned without considering gouging and interference and divided into many segments. Then an indexing table orientation defines a part set-up. Valid part set-ups in which the cutter can machine the surface without gouging and interference are identified. Each tool path segment is labeled to indicate its cutter accessibility. A least number of valid part set-ups are selected in order to minimize the total set-up time during machining. The part surface is then subdivided into patches by re-grouping the tool path segments of the chosen part set-ups. The method can generate tool paths to facilitate the discrete 5-axis CNC machining scheme. However, since all possible part set-ups must be tested, the approach requires extensive computation and cannot ensure effective tool paths. Optimal surface subdivision and favorable part set-ups have not been addressed.

To improve the stated tool path generation method, a new and radically different approach is introduced in this work. The proposed automated $3\frac{1}{2}$-axis CNC tool path generation method divides a complex sculptured surface into an optimal number of surface patches; determines optimal and favorable part set-ups (or orientations) of the two discrete rotations for convex and concave patches; and automatically generates 3-axis CNC tool paths for each simple-shaped patch. Specifically, geometric features and machinability of a sculptured surface are first calculated to identify the surface shape for a rough surface division. Fuzzy pattern clustering techniques are then used to determine a reasonable patch number, to optimize the surface divisions, and to identify the most favorable patch set-up orientations. Voronoi diagram is later used to define patch boundaries. Combining the automated 3-axis CNC tool path generation scheme for a simple surface patch, an automated and effective $3\frac{1}{2}$-axis CNC tool path programming method is introduced. The proposed approach and its advantages are illustrated using a sculptured surface example.

2. Comparison among 3-, 5-, and $3\frac{1}{2}$-axis CNC machining

3-Axis CNC machines are commonly used in manufacturing due to their low cost and high accuracy. In 3-axis CNC machining, the machine tool simultaneously controls the relative movement between the part and the cutter along its three primary axes, X-, Y-, and Z-axis, as shown in Fig. 1.

A 5-axis CNC machine is more versatile because it controls five motions continuously and simultaneously. Two continuous rotations are synchronized with the X-, Y-, and Z-axis movements and these motions are either applied to the cutter spindle, the machine table, or both. One of its unique advantage is the continuous adjustment of the cutter orientation (or part set-up) while the tool is cutting. When machining a sculptured surface, the machine tool can tilt and/or incline the cutter in order to avoid gouging and interference, as well as to improve cutting efficiency and to reduce machining time.

![Fig. 1. $3\frac{1}{2}$-Axis CNC machining with 3-axis CNC machine and tilt-rotary table.](image-url)
The 3\(\frac{3}{2}\)-axis CNC machining scheme consists of a regular 3-axis CNC machine and a tilt-rotary table to produce a discrete 5-axis CNC machining. Among the five axes, the three linear motions, continuous X-, Y-, and Z-axis motions, are provided by the 3-axis CNC milling machine. A tilt-rotary table (or automated index table) supplies two additional rotations, A- and B-axis. Since these two rotations are not synchronized with the three linear motions, these rotations are thus represented as \(1\) axis motions, leading to the new term, 3\(\frac{3}{2}\)-axis CNC machining, as illustrated in Fig. 1.

3-Axis CNC machining can be used to machine prismatic parts and sculptured parts of simple shape. In 3-axis CNC machining, the cutter orientation and tool path direction affect machining efficiency significantly. For tool path direction, Chen et al. [4,6] discovered that in 3-axis CNC machining, a cutter removes the maximum amount of material when it cuts along the steepest tangent direction on the surface at the cutter contact (CC) point. Furthermore, the steepest-directed tool path, in which the tool path tangent at every CC point is in line with the steepest tangent direction on the surface at that point, is the most efficient tool path. When a cutter strays away from the steepest-directed tool paths, its machining efficiency diminishes, causing a longer machining time. Therefore, it is important to plan tool paths along the steepest tangent directions.

Cutter orientation also influences machining efficiency. If the angle between the surface normal and the tool axis at a CC point is small, the machining capability of a flat or torus end-milling cutter is better utilized. For a convex surface, the cutter is in its best orientation for machining when the axis of the cutter aligns with the surface normal, providing no gouge or interference with the surface. When the angle between the axis of the cutter and the surface normal at the CC point increases to 90°, the cutter is in the worst orientation. In this case, only a small amount of material is removed each time, many parallel tool paths are needed, and the tool paths efficiency reaches its minimum. One of the major limitations of 3-axis machining is its inability to dynamically adjust the angle between the tool axis and the surface normal at the CC point.

The 3\(\frac{3}{2}\)-axis machining overcomes this limitation by constantly changing the part set-up orientation to reduce the angle. After the surface is divided into patches with similar shape, the most representative surface normal orientation of the patch serves as the most favorable part set-up for machining the patch. The proposed approach divides a complex surface into a number of simple-shaped patches and carries out 3-axis CNC machining on each patch in the part set-up of the calculated most favorable patch orientation to ensure maximum tool path and machining efficiency.

In comparison to 5-axis CNC machining, the 3\(\frac{3}{2}\)-axis CNC machining re-sets the part for each divided patch, causing a longer set-up time. However, the benefits of significantly lower initial and operation costs, a more rigid machine structure, and ease of automated CNC tool path programming make this new approach a very promising alternative.

3. Surface features representation

A sculptured surface can be divided automatically into surface patches according to surface features, including surface shape and machinability. Points on each surface patch need to have consistent shape and similar machinability. Surface features are identifiable using several geometric parameters, and the identification criteria are summarized in this section. The machinability parameters represent the possibility for the surface to be machined in correct shape and desired accuracy.

3.1. Representation of sculptured surfaces

To find the geometric parameters, the mathematical model of a sculptured surface is required. In this work, the commonly used B-spline surfaces in CAD systems are used. The surfaces are represented as

\[
S(u,v) = \sum_{i=1}^{n+1} \sum_{j=1}^{m+1} C_{ij} N_i(u) N_j(v)
\]

\[
\begin{align*}
\min u & \leq u \leq \max u, & \min v & \leq v \leq \max v
\end{align*}
\]

(1)

where, \(u\) and \(v\) are two independent parameters, and vector points, \(C_{ij}, i = 1, 2, \ldots, n + 1; j = 1, 2, \ldots, m + 1\), are a set of \((m + 1)\) by \((n + 1)\) control points, which compose a control polyhedron. The terms,
$N_k(u)$ and $N_k(v)$, are $k$th-order B-spline bases in the $u$ and $v$ directions, respectively. A sculptured surface is illustrated in Fig. 2.

In practice, a complex sculptured part consists of many composite surfaces, and each surface can be represented in any other form. The assumption of B-spline representation is made to illustrate the approach and to calculate the geometric properties of the surface. The selection of this surface representation method does not influence the calculated surface properties and the result of the proposed surface division method.

### 3.2. Geometric parameters

Geometric parameters are relevant to surface point, surface curvature, and surface normal, including surface point parameters ($u$ and $v$), Gaussian, mean, minimum, and maximum curvature, and surface normal. Local surface shape around a point can be recognized by the first four geometric parameters at that point; the whole surface shape can be quantified using the parameters at many points crossing the surface. The other three parameters are necessary in calculating the machinability parameters.

#### 3.2.1. Gaussian curvature, $K$, and mean curvature, $H$

The Gaussian curvature, $K$, of the surface, $S(u,v)$, at a point $[x,y,z]^T$, is formulated [10] as

$$K = \frac{LN - M^2}{EG - F^2}$$  \hspace{1cm} (2)

where $E$, $F$, and $G$ are the components of the first matrix of a surface, $A$,

$$A = \begin{bmatrix} \frac{\partial S}{\partial u} & \frac{\partial S}{\partial v} \\ \frac{\partial S}{\partial u} & \frac{\partial S}{\partial v} \end{bmatrix} = \begin{bmatrix} E & F \\ F & G \end{bmatrix}$$  \hspace{1cm} (3)$$

and $L$, $M$, and $N$ are the components of the second matrix of a surface, $B$,

$$B = \begin{bmatrix} \frac{n \partial^2 S}{\partial u^2} & \frac{n \partial^2 S}{\partial u \partial v} \\ \frac{n \partial^2 S}{\partial u \partial v} & \frac{n \partial^2 S}{\partial v^2} \end{bmatrix} = \begin{bmatrix} L & M \\ M & N \end{bmatrix}$$  \hspace{1cm} (4)

Here, vector, $n$, is the unit surface normal at the point. The expression of the mean curvature, $H$, is

$$H = \frac{1}{2} \left( \frac{EN - 2FM + GL}{EG - F^2} \right)$$  \hspace{1cm} (5)

### 3.3. Machinability parameters

Surface machinability at a CC point measures whether or not the point is accessible and the possibility of gouging. Two machinability parameters, gouging and interference, carry this information. The gouging parameter can be calculated based on the relationship between gouging and the geometric parameters. The interference parameter is related to the surface normal.

#### 3.3.1. Gouging parameter

Gouging parameter is a Boolean variable, one for gouging and zero for no gouging. The shapes of the surface and the cutter jointly determine the value of this parameter.

The maximum and minimum curvatures of the sculptured surface at each CC point, $K_{\text{min}}$ and $K_{\text{max}}$, can be calculated with Eqs. (6) and (7).

$$K_{\text{min}} = H - \sqrt{H^2 - K}$$  \hspace{1cm} (6)$$

and

$$K_{\text{max}} = H + \sqrt{H^2 - K}$$  \hspace{1cm} (7)$$

The curvature is positive, negative or null, if the surface shape is concave, convex, or plane, respectively. The maximum and minimum curvatures of the
cutter surface at the same point \( (K_{\text{min}}^c, K_{\text{max}}^c) \) can be calculated with Eqs. (6) and (7). For a torus end-mill, the cutter surface is convex, so \( K_{\text{min}}^c \) and \( K_{\text{max}}^c \) are negative. For a flat end-mill, \( K_{\text{min}}^c \) is infinity, and \( K_{\text{max}}^c \) is the reciprocal of the cutter radius. The \( K_{\text{min}}^c \) and \( K_{\text{max}}^c \) of a ball end-mill are both equal to the reciprocal of the cutter radius.

At present, the introduced method focuses on local gouging. Gouging can be avoided when the local shape of a sculptured surface is either planar or convex. The curvatures of the part and cutter surfaces can predict possible local gouging according to the criteria summarized in Table 1. Rear gouging, which is beyond the scope of this work and is to be addressed in the author’s future research, might be introduced when a surface of concave or saddle shape is machined. A rear gouging problem can be prevented at this moment by carefully choosing the size of the cutter when a concave surface is machined.

### Interference parameter

Interference in machining can be classified into two types: (a) the cutter cannot reach a region when the region faces away from the cutter; and (b) the cutter collides or over-cuts the surface in another region when it cuts a region on the surface. Zero in interference parameter represents non-interference, while its value becomes 1 when any interference is introduced.

The first type of interference can be identified by checking the dot product \((N \cdot V)\) of the surface normal \((N)\) and the cutter axis \((V)\). If this product value is less than zero, the projection of the surface normal along the cutter-axis direction is opposite to its positive direction. The surface point is thus not accessible to the cutter. If the product value is greater than or equal to zero, the cutter is able to access and machine the point.

The second type of interference is way more difficult to detect, requiring extensive computation. In this work, the sculptured surface is assumed not to contain any occult surface that requires more effort to detect.

In surface feature recognition, surface shape is identified using geometric parameters, and surface machinability is determined by machinability parameters. In execution surface, machinability is treated with a higher priority than surface shape, and plays a key role in the surface subdivision, since it determines gouging and interference that will damage the surface.

### 3.4. Calculation of geometric and machinability parameters using surface mesh

In this work, the calculations of the geometric and machinability parameters of local features on a sculptured surface are carried out using a mesh of discrete grid points that represent the sculptured surface. The grids of these discrete points that cover the entire surface are generated using isoparametric curves, \( u- \) and \( v- \) curves, with a proper density. The grid density is determined by many factors, including the size of the surface, the complexity of the shape, the needed accuracy of the representation, and the computation capability and time. When the grid points are too sparse, the number of these points will affect the number of the surface patches in subdivision of the sculptured surface. However, if the grid density is up to a certain level, the number of grid points will have almost no influence on the number of the created surface patches. The use of discrete points for surface representation in the implementation of the algorithm has extended the application of the introduced method from a single B-spline surface to compound sculptured surfaces of various forms. After the grid points are cast on the sculptured surface as shown in Fig. 3, and

<table>
<thead>
<tr>
<th>Part surface principal curvature ((K_{\text{min}}^s, K_{\text{max}}^s))</th>
<th>Cutter surface principal curvature ((K_{\text{min}}^c, K_{\text{max}}^c))</th>
<th>Gouging criteria</th>
<th>Gouging possibility</th>
</tr>
</thead>
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<tr>
<td>( K_{\text{min}}^s \leq K_{\text{max}}^c \leq 0 )</td>
<td>( K_{\text{min}}^c \leq K_{\text{max}}^c \leq 0 )</td>
<td>None</td>
<td>Impossible</td>
</tr>
<tr>
<td>( K_{\text{min}}^s \geq K_{\text{max}}^c &gt; 0 )</td>
<td>(</td>
<td>K_{\text{min}}^c</td>
<td>\geq</td>
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<td>( K_{\text{max}}^s \geq K_{\text{min}}^c &gt; 0 )</td>
<td>(</td>
<td>K_{\text{min}}^c</td>
<td>\geq</td>
</tr>
<tr>
<td>( K_{\text{max}}^c &gt; 0, K_{\text{max}}^c &gt; 0 )</td>
<td>(</td>
<td>K_{\text{min}}^c</td>
<td>\geq</td>
</tr>
</tbody>
</table>
the geometric and machinability parameters at each grid point are calculated.

4. Surface subdivision

The objective of surface subdivision is to divide a sculptured surface into a number of simple patches so that each patch can be properly accessed in a part setup and efficiently machined through 3-axis machining. The subdivision is carried out on the sculptured surface that is represented by grid points through two steps: (a) rough subdivision, in which the surface is divided into regions of one of the several major shape categories, according to the calculated geometric and machinability parameters at each grid point; and (b) fine subdivision, in which all roughly divided surface grid points are further clustered into an adequate number of groups using the fuzzy pattern clustering method, each group forming a surface patch.

4.1. Rough surface subdivision

Regions of the surface are first classified into three major geometric shape categories: convex, concave and saddle shapes (see Fig. 2). The relationship between surface shape and Gaussian/mean curvatures is used to recognize the surface shape, as shown in Table 2.

Using the relationship given in Table 2, all grid points on the sculptured surface are roughly grouped, and each group represents a region of a particular surface shape. To facilitate the rough subdivision, a hierarchical data structure is used to organize all grid points with different surface shapes [18]. The root of the data structure is the original grid; the first level consists of three geometric shape branches: convex, concave, and saddle; and the second level consists of two machinability shape branches: (a) accessible and no gouging, and (b) inaccessible or gouging.

With this data structure established, the fuzzy pattern clustering techniques are applied to each of the leaf-level categories to perform fine surface subdivision.

4.2. Fine surface subdivision

In the fine surface subdivision, two fuzzy pattern clustering methods, the subtractive fuzzy clustering method and the fuzzy C-means method, are applied sequentially. The subtractive fuzzy clustering method is an effective tool for estimating the number and centers of the subdivided groups, or the surface patches [11]. With the calculated patch number and patch centers, the optimization-based fuzzy C-means method [12] carries out the fine subdivision of the surface patches.

4.2.1. Patch number determination

With a group of grid points in rough subdivision, the subtractive fuzzy clustering method works in the following manner. For a set of \( n \) points, \( \{x_1, x_2, \ldots, x_n\} \), in a multiple-dimension space, the data points are first normalized in each dimension. These data points are then contained in a hyper-cube. Second, each data point is regarded as a potential cluster center and each cluster has only one data point, the center itself. The potential of a data point is calculated using a function that incorporates the distances between this point and all other points.
The shorter the distance, the higher the contribution it has to the potential. Therefore, the potential of a data point in a dense area is higher than the potential of a data point in a sparse area. Third, the data point with the highest potential is selected as the first cluster center because this point is closely surrounded by a maximum number of data points. The potential of every point is then updated by subtracting the potential of the cluster center. The subtracted value of each point is nonlinear and reversely proportional to the distance between the point and the first cluster center. The potentials of data points located closer to the first cluster center are reduced much more, and the potential of the first cluster center becomes null. On the other hand, the points located far away from the first center will now have higher potentials.

Among the updated data points, the data point with the highest potential is then selected as the second cluster center. The same procedure of locating the cluster center repeats until the criteria for accepting and rejecting cluster centers terminate the iterations.

If \(k\)th cluster centers, \(\{x_1^k, x_2^k, \ldots, x_n^k\}\), have been found before the procedure finally terminates, the number of clusters and their centers are then determined. Each cluster consists of a group of grid points, occupies a region, and forms a small surface patch. The cluster number is the patch number and the cluster centers are the patch centers.

In this work, the seven geometric parameters, calculated at each grid point, form a seven-dimensional (7D) parameter space for fuzzy pattern clustering. The subtractive fuzzy pattern clustering method is used to find the patch numbers and patch centers for the regions of convex, concave, and saddle shape, respectively.

Several operational parameters, including the preset cluster and penalty radiuses, and reject and accept ratios, exert influence on the cluster number and cluster centers. For instance, an increase of the cluster radius or accept ratio will lead to a reduction of the cluster number. These operational parameters are determined experimentally. Due to the different significance of each geometric parameter on the geometric shape and machinability, proper weighting factors may be used to adjust their influences on the clustering. In this work, equal weights are used.

4.2.2. Optimized surface subdivision
The fuzzy C-means method is a well-established technique for optimizing clustering and cluster centers [2]. The previously discussed subtractive fuzzy clustering method identifies the initial cluster number and cluster centers for the grid points of a particular surface shape region. The fuzzy C-means method is then applied to optimize the locations of cluster centers for the grid points of the surface shape region. Each of these clusters contains the grid points with very similar surface characteristics, forming a surface patch for a part orientation set-up. Each cluster center best represents all grid points in its cluster, and the surface normal direction at this cluster center presents the ideal surface patch orientation of the part set-up. The method now divides a given sculptured surface into patches for discrete part set-ups, as illustrated in Fig. 3. Four clusters and their cluster centers are shown.

5. Surface patch boundaries definition
As a result of the grid data clustering and surface subdivision, the entire surface is divided into patches that are joint seamlessly. However, the boundaries of the patches have not yet been exactly defined. One solution to define the patch boundaries is to increase the grid density at the beginning, and connect the grid points on the border of each region after clustering. However, adding more grid points is not a proper solution since to do so will increase computing time exponentially.

To form smooth and closed boundaries for the surface patches using the grid points of the formed clusters, the Voronoi diagram algorithm that is capable of sorting discrete points based on their similarity to the cluster centers is used in this work to finally define the surface patches. The similarity is defined using the characteristic of grid points on the surface. All grid points on a patch have similar surface features and their surface features are best represented by the patch center. From the perspective of the characteristic of the surface features, the points on this patch should be closer, or more similar, to the center of this patch than to any other patch centers. This logic matches the definition of Voronoi region. In the Voronoi diagram, a Voronoi region consists of all points that are at least as close as to a site, as to any other site (see Fig. 4) [9].
If a Voronoi diagram based on the sites of the patch centers is built, the Voronoi regions coincide with the patches correspondingly. The Voronoi diagram actually becomes the boundaries of the patches.

Given a number of sites, each site lies in a Voronoi region contained by its boundaries, and the boundaries of all Voronoi regions comprise the Voronoi diagram. Any Voronoi region is closed by the Voronoi diagram and/or the circumference of the site space, and the point on the region boundary has more than one nearest site. An example of Voronoi diagram with 37 sites is illustrated in Fig. 4. To compute the Voronoi diagram, a dual structure of the Voronoi diagram, Delaunay triangulation, is considered. Since it is easier to construct, the Delaunay triangulation is normally carried out first. Then the medians of the triangle sides form the needed Voronoi diagram. Over the years, many algorithms have been developed to generate the Voronoi diagram, and computation complexity and implementation complexity are the major concerns. The computational complexity of the most naive algorithm is $O(n^4)$. Although the algorithm is easy to implement, its long computing time hampers its wide application. On the other hand, the advanced Fortune’s algorithm has the fast complexity of $O(n \log n)$, but it is more demanding in programming. In this work, an incremental construction algorithm is employed due to its acceptable computation complexity of $O(n^2)$ and relatively simple implementation.

The Voronoi diagram is applied in the 2D parametric surface space. All 3D cluster centers are mapped into 2D parametric surface space, and the 2D cluster centers are treated as the sites. The incremental construction algorithm produces a Voronoi diagram on the sites, and the surface patch boundaries in the 2D space are determined. These boundaries are then transformed back into 3D Cartesian space to subdivide the sculptured surface. The created boundaries of surface patches are also illustrated in the example shown in Fig. 3.

6. Part set-up, tool path planning, and surface machining

Given a surface patch and its characteristic center, a favorable part set-up for the patch can be determined and executed through the $\frac{1}{2}$-$\frac{1}{2}$-axis motions of the $3\frac{1}{2}$-axis machining; effective 3-axis CNC tool paths for the patch in this part set-up can be generated; and the patch can be machined using the 3-axis CNC of the $3\frac{1}{2}$-axis machining scheme.

First, the surface normal at the patch center determines the favorable part set-up or orientation for the patch, providing it is machinable. If the patch is convex, the ideal situation to machine a point happens when the cutter axis coincides with the surface normal at that point. Similarly, to machine a point on a concave patch it is always desired to set up the part so that the angle between the cutter axis and the surface normal at that point lies within $5–10^\circ$. The favorable part orientation for a surface patch is determined based on the fact that the angle between the surface normal of every point on the surface patch and the cutter will reach minimum collectively, since the patch center, identified by the Fuzzy C-mean clustering method, best represent every point on the patch and the surface normal direction is used as a key measure in the clustering.

The determined favorable part set-ups for all surface patches are executed by adjusting the rotation angles of the tilt-rotary table, $A$ and $B$, patch after patch. The surface shown in Fig. 3 is divided into four different set-ups. Fig. 5 shows a part set-up in a 3-axis CNC machine, and the patch marked out is in the favorable part set-up. In the surface machining, the part is re-set after a patch is machined. Too many subdivided patches will lead to lengthy part set-up and machining time, reducing productivity. The optimization on the number of the subdivided patches is an issue to be addressed in further research.
The 3-axis CNC tool paths of all patches can be generated in their own part set-up using one of the proposed tool path generation algorithms, i.e. SDIC [6], isoparameter [3], or iso-cusped tool path generation algorithms. Due to the simple shape of each subdivided patch, the tool path generation task is greatly simplified and efficient 3-axis CNC tool paths can be generated at ease.

At last, the 3 1/2-axis CNC machining is executed on the part. The tilt-rotary table sets up the part for the first patch according to its part set-up (see Fig. 5), and the 3-axis CNC machine mills the patch along the planned tool paths. After finishing the first patch, the table rotates the part to the part set-up of the second patch, and the machine works on the second patch. Patch by patch, this process continues until all surface patches are machined.

In the 3 1/2-axis CNC machining, the machining time of most surface patches drops considerably comparing with the machining time of the same patches in 3-axis machining, due to the more efficient tool paths and the much improved machining conditions. Although the re-set-up costs some time, the total machining time is still considerably reduced in the 3 1/2-axis CNC machining since re-set-up time is only a fraction of machining time. The more important advantage is that the 3 1/2-axis CNC machining can be used to produce complex sculptured parts that cannot be machined using a regular 3-axis CNC machine.

7. Example of the 3 1/2-axis CNC programming

The 3 1/2-axis CNC tool path programming method is illustrated using an example sculptured surface. Results from each step of the introduced method are shown and explained. The selected example is defined using a B-spline surface with a four-by-four control polyhedron, as given in Table 3, and represented by a grid of points (30 by 30) cast on the surface. The coordinates of the points are calculated with the non-uniform knot vector \([0, 0, 0, 0, 1, 1, 1, 1]^{T}\). Fig. 6 shows the control polyhedron and the grid points of the surface.

To obtain the surface features at a grid point, the geometric parameters and machinability parameters are calculated. According to the relationship between the geometric parameters and the surface shape (see Table 1), the local shape at the grid point can be identified as the whole surface. The example surface is recognized to contain two shapes, convex and saddle shape (see Fig. 7). The grid points at convex region are labeled with asterisk (*), and the grid points of saddle-shaped

<table>
<thead>
<tr>
<th>$u$ direction</th>
<th>(0, 5, 5)</th>
<th>(0, 13, 8)</th>
<th>(0, 22, 7)</th>
<th>(0, 30, 5)</th>
</tr>
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<tbody>
<tr>
<td>$v$ direction</td>
<td>(5, 6, 8)</td>
<td>(5, 12, 11)</td>
<td>(5, 22, 10)</td>
<td>(5, 31, 6)</td>
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<td></td>
<td>(10, 5, 11)</td>
<td>(10, 15, 15)</td>
<td>(10, 23, 11)</td>
<td>(10, 30, 13)</td>
</tr>
<tr>
<td></td>
<td>(15, 6, 3)</td>
<td>(15, 15, 6)</td>
<td>(15, 25, 4)</td>
<td>(15, 31, 5)</td>
</tr>
</tbody>
</table>
region are labeled with circle (○), to illustrate the shape variation of the surface. All grid points in Fig. 7 are in the 2D parametric space (u–v space) of the surface.

Using the subtractive fuzzy clustering method, the grid points in the convex region are clustered into nine groups, and the grid points in the saddle-shaped region into five groups. The estimated cluster centers of these groups are also shown in Fig. 7.

Based on the initialized cluster number of 9 and the estimated cluster centers found above, the fuzzy C-means clustering method is used to optimize the convex region subdivision. Subdivision of the saddle-shaped region is optimized in a similar way. As a result, all grid points on the surface are clustered. The grid points in a cluster define a closed region, or a subdivided surface patch. In Fig. 8, points on different patches are labeled using different marks, and all 14 patches are shown. These include five patches of saddle points in the upper portion of the surface and nine patches of convex points in the lower portion. Within each patch, its center is located and labeled using an star (⋆).

Using the optimized cluster centers as the sites, a Voronoi diagram is generated to define the patch boundaries, as shown in Fig. 9. With this step, the

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**Fig. 6.** B-spline surface and its control polyhedron.

**Fig. 7.** Subtractive fuzzy clustering on grid points of the surface.
surface subdivision is finished and the surface patches are completely defined. Since the Voronoi diagram is in the 2D parametric space of the surface, the subdivided patches, including the cluster centers and the boundaries, are transferred back into the 3D Cartesian space on the 3D sculptured surface, as shown in Fig. 10.

In the last step, the surface normal at a cluster centre are calculated and shown with arrows (see Fig. 10). Fourteen part set-ups are found for the 14 patches in this example. Isoparameter tool paths are then generated for every patch due to the simplicity of the approach. Fig. 10 illustrates the isoparameter tool paths of each patch. To machine the surface, the part is set up for the first patch according to its part set-up, and 3-axis CNC machining is then carried out on the patch. The part set-up is adjusted 13 times for all patches to complete the machining of the entire surface. After each $\frac{1}{2}$ axis set-up adjustment, a patch
is machined using 3-axis CNC machining. Machining of the sculptured surface is completed when all 14 surface patches are machined.

8. Conclusions

The $3\frac{1}{2}$-axis CNC machining technique is a cost-effective method for carrying out the discrete 5-axis CNC machining using a low-cost 3-axis CNC machine. This research is aimed at developing the innovative $3\frac{1}{2}$-axis CNC sculptured part machining technique.

In this paper, the concept of the $3\frac{1}{2}$-axis CNC machining is first presented. A new method to automatically subdivide a complex sculptured surface into a number of easy-to-machine patches; identify the favorable part set-up of each patch; and generate effective 3-axis CNC tool paths is introduced.

Specifically, the geometric feature and machinability of the sculptured surface are first calculated to identify the surface shape and to form a rough subdivision. Fuzzy pattern clustering techniques are then used to optimize the surface subdivisions, partitioning the surface into simple patches, and locating the cluster centers. A Voronoi diagram is later used to define patch boundaries. The method integrates automated 3-axis CNC tool path generation techniques to create tool paths for all patches.

The proposed method provides an alternative to the costly and complex 5-axis CNC machining using a number of 3-axis CNC machining on a modified CNC milling machine without dropping high surface quality and part productivity. In addition, the method also provides an easy approach for generating 5-axis CNC tool paths for complex sculptured surface. The work contributes to the automated multiple-axis CNC tool path generation for sculptured part machining and forms a foundation for further research.

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References


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