A Non-Coherent AF Scheme for Two-Way Wireless Relay Networks based on Packings in Grassmann Manifolds

by

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Two-Way Relaying

- The two-way relaying channel has been recently studied in the literature [Rankov et al.], [Popovski et al.]
- The schemes described in the literature, require channel knowledge at the terminals and/or the relays.
- Here we focus on two-way relaying *without* channel knowledge requirements at the terminals and the relays.
- The capacity of the point-to-point MIMO block Rayleigh fading channel without channel knowledge assumption at the receiver (neither at the transmitter) was analyzed by Zheng and Tse, where the capacity of the unknown channel (also denoted as non-coherent capacity) was interpreted as sphere packing problem in Grassmann manifolds.
- Motivation for the two-way relaying channel without channel knowledge.
A network with $K$ relays $R_i$, $i = 1, \ldots, K$ and 2 terminals $T_m$, $m = 1, 2$

Each node has $M$ antennas and cannot transmit and receive simultaneously

We assume block Rayleigh model where the channel is constant in a certain time block.

Terminal 1 sends a $T \times M$ matrix $X \in \mathcal{X}$.

Terminal 2 sends a $T \times M$ transmit matrix $U \in \mathcal{U}$

Power constraints- $E[\text{tr}(X^H X)] = E[\text{tr}(U^H U)] = T$.

$P_1$ and $P_2$ are the average transmit powers for one transmission of Terminal 1 and Terminal 2 respectively.
System Model

- We assume an AF (Amplify-and-Forward) scenario.
- It can be proven that the AF scheme is optimal in the non-coherent two-way relaying setup. The AF scheme uses the degrees of freedom offered by the network.
- It can be shown that the upper and the lower bound for the achievable rates coincide and are equal to

\[ R_{12} = \frac{M}{2} \left( 1 - \frac{M}{T} \right) \log_2 \frac{P_R}{\sigma^2} + \frac{1}{2} c(M, M) + o(1) \]

- \( \frac{M}{2} \left( 1 - \frac{M}{T} \right) \) degrees of freedom.
- The degrees of freedom can be achieved by an AF scheme.
System Model

In the AF scenario the relay $k$ receives
\[ \mathbf{R}_k = \sqrt{P_1} \mathbf{XH}_k + \sqrt{P_2} \mathbf{UG}_k + \mathbf{N}_k, \]

where $\mathbf{N}_k$ is the noise contribution at the relay with entries which are i.i.d $CN(0, \sigma^2)$.

In the second step (broadcast stage) the relay $k$ sends
\[ \sqrt{\gamma_k} \mathbf{T}_k = \sqrt{\frac{\gamma_k}{P_1 + P_2 + \sigma^2}} \mathbf{R}_k, \]

where a normalization is performed such that $E[\text{tr}(\mathbf{T}_k^H \mathbf{T}_k)] = T$.

Terminal 2 receives
\[ \mathbf{Y} = \sum_{k=1}^{K} \sqrt{\frac{P_1 \gamma_k}{P_1 + P_2 + \sigma^2}} \mathbf{XH}_k \mathbf{G}_k^{(r)} + \sum_{k=1}^{K} \sqrt{\frac{P_2 \gamma_k}{P_1 + P_2 + \sigma^2}} \mathbf{UG}_k \mathbf{G}_k^{(r)} + \sum_{k=1}^{K} \sqrt{\frac{\gamma_k}{(P_1 + P_2 + \sigma^2)}} \mathbf{N}_k \mathbf{G}_k^{(r)} + \mathbf{Z} \]
Motivation for the Subspace-based Code Construction

We denote

\[
H' = \sum_{k=1}^{K} \sqrt{\frac{P_1 \gamma_k}{P_1 + P_2 + \sigma^2}} H_k G_k^H \\
G' = \sum_{k=1}^{K} \sqrt{\frac{P_2 \gamma_k}{P_1 + P_2 + \sigma^2}} G_k G_k^H \\
N = \sum_{k=1}^{K} \sqrt{\frac{P_2 \gamma_k}{P_1 + P_2 + \sigma^2}} N_k G_k^H + Z
\]

The received signal at terminal 2 can be thus written as

\[
Y = XH' + UG' + N
\]
Geometric Approach to Non-Coherent MIMO

Two-Way Relaying System Model

\[ Y = X'H + U'G + N \]

Non-coherent point-to-point MIMO communication - no CSIR, no CSIT

\[ Y = XH + W \]
The Grassmann Manifold as Coding Space

- Grassmann manifolds are coding spaces for the unknown MIMO channel.
- The (complex) Grassmann manifold $G_{M,T}^C$, i.e. the set of all $M$-dimensional linear subspaces of $\mathbb{C}^T$.

$$G_{M,T}^C := \{ \langle \Phi \rangle | \Phi^H \Phi = I_M \},$$
Motivation for the Subspace-based Code Construction

- Let $\mathcal{X}$ and $\mathcal{U}$ be the codebooks of user 1 and user 2 respectively.
- The codebooks represent subsets of the Grassmann manifold $G_{T,M}^C$.
\[
Y = X'H + U'G + N
\]
- Given $U'$, i.e. $U$, we have an idea about the "direction" in which the interference acts.
- Example: $M = 1$ and block length (vector size) $T = 2$.
- $\Omega_x$ and $\Omega_u$ - the subspaces spanned by $x$ and $u$
- $u$ determines the direction, or more accurately the subspace (line) along which the interference is added.
- The uncertainty of the channels $h$ and $g$ is imaged in the received vector $y$, i.e. the subspace it represents $\Omega_y$, even without the noise term $n$.
- Joint design of $\mathcal{X}$ and $\mathcal{U}$ is needed.
Motivation for the Subspace-based Code Construction

Note that the system model can be written as

\[ Y = \begin{bmatrix} X' & U' \end{bmatrix} \begin{bmatrix} H \\ G \end{bmatrix} + N \]

We can think of \( \begin{bmatrix} X' & U' \end{bmatrix} \) as an equivalent transmit matrix and of \( \begin{bmatrix} H \\ G \end{bmatrix} \) as an equivalent channel.

This is similar to the system model for the non-coherent MIMO point-to-point block fading channel with \( 2M \) transmit, \( M \) receive antennas and coherence time \( T \).

\( \Omega_Q \) - the \( 2M \)-dimensional subspace spanned by the columns of \( \begin{bmatrix} X' & U' \end{bmatrix} \).

\( Q \) is the codebook obtained by the above concatenation and \( |Q| = |X||U| \).

A \( 2M \)-dimensional subspace \( \Omega_Q \) collapses into a \( M \)-dimensional subspace after the channel action.
Code Construction

- Decoding by looking for the most likely transmitted subspace, having the received matrix $Y$
  \[ Q = \arg \max_{Q_i \in \mathcal{Q}} \|Y^H Q_i\|_F^2 \]

- Having $\Omega_Q$, we get the pair $(X, U)$.

- When looking for the most likely subspace $\Omega_Q$ we can use the fact that we know $U$ in advance, which limits the number of the subspaces we have to search.

- The performance of the code depends on the properties of the codebook $\mathcal{Q}$, in the context of the well established criteria for construction of codes for the non-coherent channel, such as chordal distance and diversity product.

- The subtle difference (which makes the problem more difficult) is the fact that the codebook $\mathcal{Q}$ is obtained as a result of the concatenation of the transmit matrices $X$ and $U$.

- The codebooks $\mathcal{X}$ and $\mathcal{U}$ are not chosen independently, but rather as two "well separated" subsets of a larger codebook.
**Code Construction - One Antenna Case**

The signal transmitted from Terminal 1 is a $T \times 1$ vector $x$. Similarly, Terminal 2 transmits a $T \times 1$ vector $u$. For the case $T = 4$, codes for both terminals can be constructed as

$$
\begin{bmatrix}
\cos(\alpha_1) \\
0 \\
\sin(\alpha_1)x_1 \\
\sin(\alpha_1)x_2
\end{bmatrix},
\begin{bmatrix}
0 \\
\cos(\alpha_2) \\
\sin(\alpha_2)u_1 \\
\sin(\alpha_2)u_2
\end{bmatrix}
$$

(1)

with symbols from Terminal 1 $x_1, x_2 \in \text{QPSK}$, symbols from Terminal 2 $u_1, u_2 \in \text{QPSK}$. This construction is motivated from the non-coherent codes constructed in [Utkovski et. al].
Code Construction - Two Antennas Case

The signal transmitted from Terminal 1 is a $T \times 2$ matrix $X$. Similarly, Terminal 2 transmits a $T \times 2$ matrix $U$. For case that $T = 8$, codes for both terminals can be constructed as

$$X = \begin{bmatrix} \cos(\alpha_1) A_x \\ \sin(\alpha_1) B_x \end{bmatrix}, \quad U = \begin{bmatrix} \cos(\alpha_2) A_u \\ \sin(\alpha_2) B_u \end{bmatrix}$$

(2)

where

$$A_x = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad A_u = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B_x = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \\ -x_3^* & -x_4^* \\ x_4 & -x_3 \end{bmatrix}, \quad B_u = \begin{bmatrix} u_3 & u_4 \\ -u_4^* & u_3^* \\ u_1^* & u_2^* \\ -u_2 & u_1 \end{bmatrix}$$

with symbols from Terminal 1 $x_1, x_2, x_3, x_4 \in \text{QPSK}$, symbols from Terminal 2 $u_1, u_2, u_3, u_4 \in \text{QPSK}$. 

Code Construction - Two Antennas Case

The acquisition of the construction of $B_x$ and $B_u$ comes from the quasi-orthogonal space-time block code introduced by Jafarkhani:

$$
C = \begin{bmatrix}
  c_1 & c_2 & c_3 & c_4 \\
  -c_2^* & c_1 & -c_4^* & c_3^* \\
  -c_3^* & -c_4^* & c_1^* & c_2^* \\
  c_4 & -c_3 & -c_2 & c_1
\end{bmatrix}
$$

With different combinations of columns of $C$, variants of $B_x$ and $B_u$ can be obtained. Let’s denote the above-mentioned $B_x$ and $B_u$ as $B_x(1,2)$ and $B_u(3,4)$. There are also

$$
B_{x(1,4)} = \begin{bmatrix}
  x_1 & x_4 \\
  -x_2^* & x_3^* \\
  -x_3^* & x_2^* \\
  x_4 & x_1
\end{bmatrix} \hspace{1cm} B_{u(2,3)} = \begin{bmatrix}
  u_2 & u_3 \\
  u_1^* & -u_4^* \\
  -u_4^* & u_1 \\
  -u_3 & -u_2
\end{bmatrix} \hspace{1cm} B_{x(1,3)} = \begin{bmatrix}
  x_1 & x_3 \\
  -x_2^* & -x_4^* \\
  -x_3^* & -x_1^* \\
  x_4 & -x_2
\end{bmatrix} \hspace{1cm} B_{u(2,4)} = \begin{bmatrix}
  u_2 & u_4 \\
  u_1^* & u_3^* \\
  -u_4^* & u_2^* \\
  -u_3 & u_1
\end{bmatrix}
$$
Examples and Simulations Results: M=1, T=4, K=1

- $\alpha_1 = \alpha_2 = \pi/4$
- $M = 1, T = 4, \alpha_1 = \alpha_2 = 0.5535$
- $M = 1, T = 4, \alpha_1 = 0.5535, \alpha_2 = \pi/2 - 0.5535$
Examples and Simulations Results: M=1, T=4, variable K

![Graph showing Bit Error Rate vs. SNR for different values of K.](image-url)
Examples and Simulations Results: \( M=2, T=8, K=1 \)
Examples and Simulations Results: \( M=2, T=8, \text{variable } K \)
Conclusions and Future Work

- We presented a scheme for Non-coherent Two-Way Relaying
- Possibility to extend to multiple users
- Intuition for non-coherent capacity characterization of other networks
- Non-coherent interference alignment?
- Connection to Information Geometry?
References