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Three-dimensional point spread function measurement of cone-beam computed tomography system by iterative edge-blurring algorithm

Zikuan Chen and Ruola Ning

Department of Radiology, University of Rochester, Box 648, 601 Elmwood Avenue, Rochester, NY 14642, USA

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Abstract
With separability assumed, we decompose a three-dimensional point spread function (3D PSF) into two-dimensional (2D) PSFs and further into one-dimensional (1D) PSFs. Based on the observation of the location invariance of a step edge under convolution, we propose a rectification procedure to automatically establish the step-edge function from a blurred edge profile. The 1D PSF is modelled as a single-parameter Gaussian function, which is determined by iteratively blurring a step-edge function into a spread edge profile. A plastic solid ball (diameter $\sim 6$ mm) is used to provide double-edged rectangular functions along scanlines passing through the ball centre, and correspondingly, the reconstructed digital volume provides the blurred rectangular profiles. Experimenting with a cone-beam computed tomography system, we demonstrate the iterative edge-blurring algorithm for PSF measurement. By repositioning the ball phantom in the object support space, we measure the system’s spatial variance in terms of full-width-at-half-maximum (FWHM) of the local PSFs. Specifically, we obtained the FWHMs for three specific locations at $(0, 0, -40$ mm), $(0, 0, 0)$ and $(0, 0, 40$ mm), which are given by $0.92 \pm 0.10$ mm, $0.65 \pm 0.08$ mm and $0.93 \pm 0.10$ mm, respectively.

1. Introduction

The performance of an imaging system can be quantitatively characterized by its point spread function (PSF), which is defined as the spread spot at the output of a system with an input of a point source. For a computed tomography (CT) system, the concept of PSF is not so intuitive as that for an optical imaging system, because the CT reconstruction involves not only the x-ray imaging process (CT scan) but also a computation procedure (tomographic reconstruction). Nevertheless, a CT system is essentially a linear system (both the Radon transform and the inverse Radon transform involved in a CT scan and reconstruction are linear),
therefore its imaging performance can be conceptually characterized by the PSF (Wang et al 1992). Adopting the concept of PSF, we can grasp (or assess) the overall performance of a CT system without knowing the specifics associated with the x-ray scan and reconstruction. Besides characterizing a CT system’s blurring effect, the 3D PSF can also be used for image enhancement or restoration (Lehr et al 1998).

Theoretically, the PSF of a system is defined as its output with the input of a delta point (infinitesimal point) source. This definition suggests a direct 3D PSF measurement using a tiny ball (or a microbead (Lehr et al 1998)). For a traditional CT system that reconstructs a volume via the reconstruction of a stack of slice images, the 2D PSF is naturally used to evaluate the image quality within slice images (Nickoloff and Richard 1985, Nickoloff 1988, Boone and Seibert 1994). In experiment, a thin wire or slit has been used to measure the line spread function (LSF) (Nickoloff and Richard 1985, Nickoloff 1988), where a scanline at a cross section of the wire provides a step-edge distribution and the corresponding scanline at the reconstructed slice image provides the spread edge distribution (Lehr et al 1998, Boone and Seibert 1994). Instead of slice-wise 3D reconstruction, a cone-beam CT (CBCT) system can directly produce a volume through the FDK reconstruction algorithm (Feldkamp et al 1984). Therefore, the 3D PSF is necessary to characterize a CBCT system.

To avoid the difficulty in the pursuit of analytic formulation of CBCT PSF (Wang et al 1992), we can procure the numeric results by experiment. Adopting the edge-based technique, we use a small solid ball to provide step edges along any orientations in 3D space. A scanline passing through the ball centre is used to determine the 1D PSF on that line, and a cross section across the centre is used to determine the 2D PSF at that plane.

Noise is unavoidable in an imaging system. To resist the noise associated with spread edge profiles, the PSF is usually modelled as a mathematically tractable function. Commonly, a Gaussian function is a good model for the PSF of a CT system (Lehr et al 1998, Nickoloff and Richard 1985, Nickoloff 1988, Boone and Seibert 1994). Although the Gaussian PSF model can be refined (Bruni et al 2001), for example, by including an exponential term for accounting for x-ray scattering (Boone and Seibert 1994), the refined model needs multivariate nonlinear optimization. For the purposes of unsophisticatedness and emphasis on reporting the new technique, we will, in this paper, adopt the single-parameter Gaussian model for the PSF.

The PSF describes a system’s blurring effect on image formation, and accordingly it can be obtained by a deblurring or deconvolution technique (Jain 1989, Gonzalez and Woods 2002). Specifically, with a test object (input) and its image (output), the deblurring procedure is to find the transformation kernel that bridges the input and the output. This is a typically ill-posed inverse problem, which is especially sensitive to noise (Gonzalez and Woods 2002). The PSF model and iterative implementation are useful in finding a robust solution in the presence of noise (Bruni et al 2001, Jain 1989, Gonzalez and Woods 2002, Biemond et al 1990, Ning et al 2000). During the iterative process, each iteration is interpreted as an imaging process, i.e., generating an output by blurring the input with the PSF model. The iterative procedure updates the model parameters until minimizing the difference between the iterative output and the experiment output.

For PSF measurement, we use a plastic solid ball (6 mm in diameter) as a prop, which is placed at a position in the object support space. Any scanline passing through the ball centre offers a step-edge distribution in terms of x-ray absorption at the ball surface. Through cone-beam x-ray scanning and FDK-based volumetric reconstruction (Feldkamp et al 1984, Ning et al 2000, Chen and Ning 2003), we reproduce a digital ball from which we extract the digital scanline profile, thereby obtaining the output. Since the analog input and digital output are represented in different measure metrics (the input in millimetres and the output in
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It is necessary to render a geometrical calibration procedure to unify the input and the output in the same metrics. Based on the observation of edge point invariance to convolution, we suggest an edge rectification procedure for automatically establishing the input edge from the blurred edge (appearing as a sigmoidal curve in the output), thereby avoiding the system calibration procedure. With both the input and the output represented in voxel metrics, as well as the Gaussian model, we can find the numerical PSF by an iterative algorithm. The main reason for adopting the iterative algorithm is that it can produce a robust and optimal solution in the presence of noise (Jain 1989, Gonzalez and Woods 2002, Biemond et al 1990).

2. Methodology

2.1. Overall concept

For a linear spatial invariant system, the 3D input–output relationship, in the presence of additive noise, can be mathematically described by a triple convolution

\[ g(x, y, z) = \int \int \int f(x', y', z') h(x - x', y - y', z - z') \, dx' \, dy' \, dz' + n(x, y, z) \]

(1)

where `*` denotes convolution, and \( h(x, y, z) \) represents the integral kernel, or the 3D PSF in the context of an imaging system. For a spatial variant system, if the variance over the object support region changes slowly and smoothly, the convolution formula in equation (1) approximately holds for a local region. Let \( \Omega(x_0, y_0, z_0) \) denote a local region at \((x_0, y_0, z_0)\), then the local spatial invariance assumption is expressed by

\[ g(x, y, z; x_0, y_0, z_0) = \int \int \int_{\Omega(x_0, y_0, z_0)} f(x', y', z') h(x - x', y - y', z - z'; x_0, y_0, z_0) \, dx' \, dy' \, dz' + n(x, y, z) \]

\[ = f(x, y, z) \ast \ast \ast h(x, y, z; x_0, y_0, z_0) + n(x, y, z) \]

\[ \forall (x, y, z) \in \Omega(x_0, y_0, z_0) \]  

(2)

where the position \((x_0, y_0, z_0)\) in the object support space is explicitly specified in the integral kernel. In this section, our goal is to determine the local PSF of a CBCT system via equation (2), through the use of a prop of a small solid ball. In the presence of noise, we can find the optimal solution to the integral transform in equation (2) by an iterative procedure. For implementation, we should provide experimental data for the input \((f)\) and output \((g)\), and specify a model for the kernel \((h)\). Since the scanline of a ball may assume any orientation in 3D space, the 3D PSF problem can be solved via the 1D PSF measurement along an arbitrary scanline.

2.2. PSF model

In the presence of noise, it is preferable to solve the integral equation in equation (1) or (2) by an iterative procedure with an integral kernel model. For a CBCT system, the integral kernel or PSF is assumed separable, i.e., a high-dimension function can be factorized into low-dimension functions. The CT PSF's separability can be perceived from the following facts. For slice-reconstruction-based CT, the intra-slice image quality is an area of concern, therefore we are interested in 2D PSF at the reconstruction plane. This reveals the fact that the 3D PSF is decomposed into 2D PSFs. The 2D PSF at a plane is in turn decomposed into 1D PSF, such as the LSF measurement through the use of a spread edge function (Boone and Seibert 1994).
For a CBCT system (Ning et al. 2000, Chen and Ning 2003), it may recon-struct the digital volume in an isotropic resolution representation, i.e., the grid resolutions are identical in three major axial directions (x-axis, y-axis and z-axis). In terms of the volume slicing operation, the 3D volume can be decomposed into 2D images on the slice planes, where the 2D images can be studied separately, without interfering with other slices. To simplify the problem further, the 2D image at a slice plane can in turn be decomposed into 1D signals in orthogonal directions. Above all, since the CT system aims at faithfully reproducing a digital representation of an analog object, it neither introduces interference among orthogonal directions, nor brings skew distortions such as shearing and twisting, except the local blurring inherently associated with a real system. Therefore, the 1D specifications in three orthogonal directions can be used to roughly represent the overall three-dimensional specifications in 3D space.

With the separability assumption, a 3D PSF can be decomposed into 2D PSFs in three mutually orthogonal planes, and a 2D PSF can in turn be decomposed into 1D PSFs in two orthogonal directions. For a slice-reconstruction-based CT system, this decomposition scheme gives rise to

\[ h(x, y, z) = h_1(x, y)h_2(z) = h_1(x)h_1(y)h_2(z) \]  

(3)

where \( h_1 \) is usually different from \( h_2 \), reflecting a fact that the intra-slice PSF may be different from the inter-slice PSF. However, the 1D PSFs along two orthogonal directions in a plane are assumed identical in equation (3). For the 3D PSF of a CBCT system, we adopt the general separation formula:

\[ h(x, y, z) = h_x(x)h_y(y)h_z(z) \]  

(4)

where \( h_x, h_y \) and \( h_z \) respectively represent 1D PSFs along the x-, y- and z-axes passing through the centre of the 3D PSF, which are used in general to reveal anisotropy of the 3D PSF. If a 3D PSF is of isotropic or spherical symmetry, all three components, \( h_x, h_y \) and \( h_z \) in equation (4) are identical in formulation. The edge-based 2D PSF measurement assumes that the 1D PSFs at arbitrary orientations are identical. With this assumption, the 1D PSFs determined from scanlines at different orientations can be averaged to produce a robust result. The separability assumption can be considered as sparsely sampling a 3D space in three orthogonal directions. As a result of the 3D-to-1D decomposition, the triple convolution in equation (2) is reduced to a 1D convolution

\[ g(t; t_0) = f(t) * h(t; t_0) + n(t) \quad \forall t \in \Omega(t_0) \]  

(5)

or in a succinct form

\[ g(t) = f(t) * h(t) + n(t) \]  

(6)

where the study is implicitly confined to a local region \( \Omega(t_0) \).

To solve equation (6) for \( h(t) \) by an iterative procedure, \( h(t) \) should be instantiated by an appropriate model. As aforementioned, it is adequate to model a system’s PSF by a Gaussian function (Lehr et al. 1998, Boone and Seibert 1994). The 1D Gaussian function is given by

\[ h_\sigma(t) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{t^2}{2\sigma^2}\right) \]  

(7)

with

\[ \int_{-\infty}^{\infty} h_\sigma(t) \, dt = 1 \quad \text{since} \quad \int_{-\infty}^{\infty} \exp\left(-\frac{t^2}{2\sigma^2}\right) \, dt = \sqrt{2\pi\sigma} \]  

(8)

where \( \sigma \) represents the standard deviation of the Gaussian distribution. The shape of the Gaussian distribution is solely determined by a single parameter, \( \sigma \).
Besides the experimental support for the Gaussian PSF model, another rationale is its asymptotical behaviour approaching a delta impulse (the ideal imaging system’s PSF), i.e.,

$$\lim_{\sigma \to 0} h_\sigma(t) = \delta(t).$$

(9)

Coincidently, the Gaussian model satisfies the separability since

$$\exp\left(-\frac{x^2 + y^2 + z^2}{2\sigma^2}\right) = \exp\left(-\frac{x^2}{2\sigma^2}\right) \exp\left(-\frac{y^2}{2\sigma^2}\right) \exp\left(-\frac{z^2}{2\sigma^2}\right).$$

(10)

By substituting $h(t)$ in equation (6) by $h_\sigma(t)$, we can solve equation (6) by an iterative algorithm with the objective function defined by

$$\varepsilon = \|g(t) - f(t) * h_\sigma(t)\|^2.$$

(11)

The iterative procedure updates the parameter $\sigma$ so as to minimize the objective function. In the result, the PSF is uniquely specified by the optimal parameter $\sigma_{opt}$, as given by

$$\sigma_{opt} = \arg\min_{\sigma} \|g(t) - f(t) * h_\sigma(t)\|^2.$$

(12)

In order to find the solution to equation (12), we need to provide the input, $f(t)$, and the output, $g(t)$, from the CT system to be evaluated. For simplicity, we provide $f(t)$ by a step-edge function, and $g(t)$ by the blurred edge profile of $f(t)$ in the reconstructed volume, as reported below.

2.3. Edge rectification

We instantiate $f(t)$ as a step-edge function through the use of a solid ball, and $g(t)$ as the corresponding digital edge profile extracted from the reconstructed volume of the ball by a CT system. The step edges are formed by the density distribution across the surface of the ball, reflecting the density difference between interior and exterior (or surroundings). A scanline passing through the ball centre produces a double-edged profile, in the shape of a rectangle, thus referred to as a rectangular profile. The sharp rectangle consists of two parameters: width and height. The width refers to the geometrical distance between the left edge point and the right edge point, corresponding to the ball diameter. The height refers to the x-ray absorption difference between the top and bottom plateaus, determined by the density difference between the ball interior and its surrounding material. Due to spherical symmetry, a solid ball can provide identical sharp rectangular profiles along all scanlines passing through the centre.

The digital reproduction of the ball object is carried out by a cone-beam x-ray scan and volumetric reconstruction on a CBCT system. The gantry geometry is sketched in figure 1, with the experimental settings listed in table 1. In the reconstructed volume, we extract a blurred profile and consider it as the output, $g(t)$, which corresponds to the input, $f(t)$, predefined by the analog ball. However, this issue is not so simple as it looks.

The $f(t)$ and $g(t)$ thus obtained are represented in different metrics, i.e., $f(t)$ in millimetres and $g(t)$ in voxels. When $f(t)$ and $g(t)$ are put into the same formula for computation, specifically in equation (12), it is necessary to unify their representations in a common measure. For example, the geometry of the analog input, $f(t)$, which is usually measured in units of millimetres, needs to be digitally represented in the grid space where its output image, $g(t)$, is presented. This analog-to-digital conversion is generally referred to as the calibration of an imaging system. The calibration process can be conceptually described by the identity

$$f(t) = f(t) * \delta(t)$$

(13)
where $\delta(t)$ is interpreted as the PSF of an ideal imaging process (no blurring). Mathematically, the identity in equation (13) only shows the perfect preservation between the input and the output, but it hides the important calibration for a real system. The calibration links the analog input and digital output, as explained in the following: $f(t)$ on the right-hand side represents the analog input predefined by a solid ball, and the $f(t)$ on the left-hand side represents the digital output provided by the reconstructed digital volume. In other words, suppose a perfect CT system can provide a faithful object reproduction, the input and the output are however represented in different measures. In our discussion, the analog step-edge profile provided by a solid ball (measured in units of millimetres) corresponds to a spread edge profile in a digital ball (represented in pixels or voxels) by a CBCT system. Since the ideal $\delta(t)$ in equation (13) is not physically reachable, the relationship between the analog input and digital output should be determined by a calibration procedure, rather than a simple digitization process.

Calibration itself is a laborious task because it demands precise measurements on both the input analog object and the output digital image, not only in object/image size but also in density/signal magnitude. Here, we propose a technique to derive a digital form of $f(t)$ represented in the output grid space, without calibration. This technique is purely a digital image processing technique, which is based on the observation of the invariance of edge locations to convolution blurring. In the absence of noise ($n(t) = 0$), from the convolution definition in equation (6) we have

$$g'(t) = \int f'(t-s)h(s)\,ds = f'(t)*h(t)$$  \hspace{1cm} (14)
where \( g'(t) \) denotes the derivative operation. In the derivation of equation (15), we have used equation (8). An edge point is defined at a maximum of \( |g'(t)| \) and \( |f'(t)| \) in a local region. This observation suggests the edge rectification principle: the edge point of \( f(t) \) can be determined from that of its blurred version, \( g(t) \). In the context of edge detection by derivative operation, the edge rectification is expressed by

\[
\arg \max_t |g'(t)| = \arg \max_t |f'(t)|
\]

which indicates the edge-location invariance to convolution. This conclusion is valid for the blurring of a step edge in the absence of noise, where the spread edge appears as a sigmoidal curve, rather than a slope. Figure 2 illustrates the edge-location invariance under two convolution operations. In the presence of noise, the condition in equation (16) may be observed if the noise magnitude is far less than the edge transition. To safely use equation (16) for edge point detection, it is helpful to smooth the signal when necessary. In digital image processing, the smooth preprocessing is often applied to a digital image before edge detection (Jain 1989, Gonzalez and Woods 2002). This practice implicitly assumes the edge-location invariance to a smoothing procedure. For our spread edge cases, the blurring serves as a smoothing effect. Another robust approach to edge identification is to use a water-gauge algorithm (Chen and Molloi 2003) by defining the water gauge as the height of the edge transition and setting the waterline at the middle of the water gauge, that is, the edge point is defined as the location at the half height of the edge transition. Since the technique is free of derivative operation, it is highly immune to noise. What we are going to do is to restore a sharp rectangular function from a blurred rectangular profile by edge point identification, without the need of calibration. Let \( g(t) \) denote the blurred rectangular function, then its two edge points can be identified by

\[
t_l = \arg \max_{t < t_0} |g'(t)|
\]
\[
t_r = \arg \max_{t > t_0} |g'(t)|
\]

with \( t_0 = \arg \max_t |g(t)| \)

where \( t_0 \) represents a point at the top plateau of the rectangular function (assuming the maximum of \( g(t) \)), \( t_l \) and \( t_r \) correspond to the left-side and right-side edge points with respect to \( t_0 \), respectively, as depicted in figure 2. For a sigmoidal profile, \( t_l \) and \( t_r \) are uniquely defined. However, a slope profile, or a sigmoidal profile corrupted by noise, may lead to multiple \( t_l \) and \( t_r \) in equation (17). This non-uniqueness can be captured by a procedure. If this is achieved, the water-gauge algorithm (Chen and Molloi 2003), together with a smoothing procedure, may be used to remove the uncertainty.

To completely describe a rectangular function, we also need to specify its height, which is determined by the top level (\( I_{\text{top}} \)) and the bottom level (\( I_{\text{bot}} \)). In a digital image, both \( I_{\text{top}} \) and \( I_{\text{bot}} \) are represented by pixel values. In the presence of additive noise, we can determine the levels by local average, as given by

\[
I_{\text{top}} = \frac{\text{mean}_{t_0-W < t < t_0+W} \{g(x)\}}{\text{mean}_{t_0-W < t < t_0+W} \{g(x)\} + \text{mean}_{t_0+W < t < t_0+W} \{g(x)\}}
\]
\[
I_{\text{bot}} = \frac{\text{mean}_{t_0-W < t < t_0+W} \{g(x)\}}{2}
\]

(18)
Figure 2. Illustration of edge-location invariance to convolution. A rectangular signal is rounded by two Gaussian blurrings \( (g(t) = f(t) * h(t)) \) with \( \sigma_1 > \sigma_2 \), while its edge points (marked by ‘o’) are fixed. The sharp rectangular function can be recovered from the blurred signal by edge rectification.

where mean[] denotes average; \( w > 0 \) is an appropriate constant to dodge the blurring region at an edge point, as depicted in figure 2. Usually, \( w > 2\sigma_{opt} \) is required so that \( I_0 = I_0 \ast h(t) \) holds for the central part of the top flat plateau, where \( I_0 \) represents the top level. The second formula in equation (18) implies that both the left-hand and right-hand sides of the rectangular function assume the same value. (This assumption may be dispelled when the left and right edges are dealt with separately.) As a result, the sharp rectangular shape is established by

\[
\hat{f}(x) = \begin{cases} I_{top} & t_1 < t < t_r \\ I_{bot} & \text{otherwise} \end{cases}
\]  

(19)

In analogy with the signal rectification in an electronic circuit, we refer to the procedure of establishing a step edge from a sigmoidal distribution as edge rectification. The edge rectification consists of edge identification (by equation (17)) and edge shape establishment (by equations (18) and (19)). It is noted that the edge rectification only involves the digital signal, \( g(t) \), without the need of millimetre-to-pixel and density-to-voxel-value conversions. In fact, the capped \( f(t) \) in equation (19) is the digital format of \( f(t) \), represented in the output grid space. This is what we are seeking. At this point, we have fulfilled the unification of the analog input and the digital output in the output grid space, thus paving the way for iterative implementation of equation (6).

Observing the rectangle shape in equation (19), we note that it has an offset of \( I_{bot} \), which reflects the surroundings of the test object in terms of x-ray absorption. The height of the rectangular function, \( |I_{top} - I_{bot}| \), is determined by both the material the ball is made of and the material surrounding the ball, in terms of x-ray absorption. This consideration can be characterized by a contrast defined by

\[
\text{contrast} = \frac{|I_{top} - I_{bot}|}{I_{top} + I_{bot}}.
\]  

(20)

In application, noise is unavoidable in an imaging system. The noise strength can be characterized by the standard deviation of voxel values at a flat region. For the rectangular signal, we express the noise influence by defining a signal-to-noise ratio (SNR) by

\[
\text{SNR} = \frac{2|I_{top} - I_{bot}|}{\text{std(bot)} + \text{std(top)}}
\]  

(21)
where 'std' denotes 'standard deviation', and 'top' and 'bottom' respectively represent signals at the top and bottom regions. The SNR in equation (21) is interpreted as the ratio of the height of the rectangle over the average noise at its top and bottom, where the noises at both the top and at the bottom are accounted for. If std\(_{\text{bot}} = \text{std}\(_{\text{top}}\), equation (21) reduces to the conventional definition of SNR (Jain 1989, Gonzalez and Woods 2002).

2.4. Implementation

Overall, the 3D PSF measurement of a CBCT system is implemented by the flowchart in figure 3. It consists of three main stages: (1) CB x-ray scan and volumetric reconstruction; (2) edge rectification and (3) iterative edge-blurring process. The CBCT geometry is sketched in figure 1. The test object is a small plastic ball (diameter \(\sim 6\) mm), which is placed at a position in the object support space for the local PSF measurement. With a set of cone-beam projections, we reconstruct the digital volume by the FDK algorithm (Feldkamp et al. 1984). In the digital volume, we recognize the digital ball and extract the blurred rectangular profile in three orthogonal planes passing through the ball centre. The edge rectification is to identify the edge points, followed by the calculation of top and bottom levels, hence establishing a digital rectangular profile. The rectified rectangle is conceptually treated as the true input profile, which is now represented in the output grid space. With the Gaussian model, the 1D PSF is generated by solving equation (12) using an iterative algorithm. Starting with a small initial \(\sigma\) (for example = 0.5), the iterative process blurs the sharp rectangular profile progressively, and eventually terminates when the criterion in equation (11) is met.

We have assumed the 3D PSF separability, which makes possible measuring the 3D PSF via the 1D PSF. The PSF decomposition is shown in figure 4, where a ball is decomposed into three cross-section images, i.e., three 2D disc-like images. Each disc image is in turn
Figure 4. The 3D PSF decomposition scheme. (a) Visualization of the reconstructed digital ball in a subvolume of 140 × 140 × 70 voxels; (b) z-axis cross-section image (140 × 140), (c) z-axis cross-section image (140 × 70) and (d) y-axis cross-section image (140 × 70). The 3D PSF is decomposed into 2D PSFs at three orthogonal cross sections, and the 2D PSF is in turn decomposed into two 1D PSFs at two orthogonal radial scanlines (highlighted). Other radial scanlines are used for robustness. The inverse procedure is used for 1D-to-2D-to-3D assembling scheme.

decomposed into a set of radial scanlines across the disc centre. In principle, the 2D-to-1D decomposition only requires two orthogonal scanlines, usually, along the horizontal and vertical directions as highlighted in figures 4(b)–(d). Other radial scanlines are used, by an ensemble average, for noise reduction or robustness. Working with the profiles extracted from the radial scanlines, we carry out the 1D PSF measurement by an iterative edge-blurring algorithm. Figure 5 demonstrates this algorithm with a typical blurred rectangular profile. Specifically, figure 5(a) shows a blurred noisy rectangular profile, \( g(t) \). The edge rectification establishes a sharp rectangular function, \( f^\wedge(t) \), as shown in figure 5(b), where both \( g(t) \) and \( f^\wedge(t) \) are represented in the same grid space. The iterative blurring procedure (via equation (12) with \( f^\wedge(t) = f(t) \)) produces a sequence of errors, as plotted in figure 5(c). The minimum at the error curve indicates the termination condition, which produces the optimal solution. As seen from figure 5(c), the objective function diverges after passing the minimum point. From either direction (a large or a small initial \( \sigma \)), the iterative procedure can consistently reach the minimum, thus producing the optimal solution as plotted in figure 5(d).

By reversing the decomposition procedure, we can assemble 1D PSFs into 2D PSFs according to the orientation information recorded during the decomposition stage. The 2D PSFs are in turn assembled into a 3D PSF. As a result of the assembling scheme in figure 4, a 3D PSF is constructed, which consists of three 2D PSFs at its three mutually orthogonal cross sections. With the separability assumption, a 3D PSF can be roughly represented by three 2D PSFs at three orthogonal cross sections, with each 2D PSF being further decomposed into two 1D PSFs in two orthogonal directions at the cross-sectional plane. The reason for which we use more than one 2D PSF, as well as a cluster of 1D PSFs for a 2D PSF, is to average out the fluctuations resulting from noise and discreteness, and thereby to produce a robust solution. With the single-parameter Gaussian model, the PSF is numerically specified.
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Figure 5. Demonstration of the iterative edge-blurring algorithm. (a) A blurred profile extracted from a radial scanline of a reconstructed digital ball, (b) a sharp rectangular profile established from (a) by edge rectification, (c) the iterative behaviour of equation (11) and (d) the optimal PSF solution.

by the parameter of standard deviation (σ\text{opt} in equation (12)), which can be converted to full-width-at-half-maximum (FWHM) by

\[ \text{FWHM} = 2.3547 \sigma_{\text{opt}} \]  

(22)
as a result from

\[
\begin{align*}
\exp\left[-t_0^2/(2\sigma_{\text{opt}}^2)\right] &= 1/2 \\
\text{FWHM} &= 2t_0.
\end{align*}
\]  

(23)

Since an iterative edge-blurring technique does not require the calibration, both σ\text{opt} and FWHM obtained so far are represented in units of voxel spacing. Given calibration information, the results can be readily converted into a convenient measure metrics. For example, with the calibration formula, 1 voxel = 0.1843 × 0.1843 × 0.1843 mm³, the result ‘σ\text{opt} = 2.0 voxel-spacing’ is equivalent to ‘FWHM = 4.71 voxel-spacing’, or ‘FWHM = 0.8686 mm’ via ‘1 voxel-spacing = 0.1843 mm’.

3. Results and discussions

Since it is difficult or impractical to derive an analytic expression for the 3D PSF of a CBCT system (Wang et al 1992), the expedient is to seek the numeric solution through experiment. With the implementation scheme as reported in section 2.4, we measured the PSFs at three locations at (0, 0, −z₀), (0, 0, 0) and (0, 0, z₀), in the object support space, as sketched in figure 1, where z₀ = 40 mm. The experiment used a prop of a plastic ball (diameter ∼6 mm) made of polytetrafluoroethylene (PTFE). One reason for us to use a PTFE ball is that the PTFE
is a suitable material for the phantom study on breast imaging, in terms of x-ray absorption. Surrounded by air, the solid ball provided rectangular signals of contrast = 0.68 (calculated by equation (20)) and SNR = 26.6 (calculated by equation (21)). (During the x-ray scan, the ball was supported by a piece of foam whose disturbance to the background, in terms of x-ray absorption, was ignorable). The experimental settings are listed in table 1; other specifications can be found in Ning et al (2000) and Chen and Ning (2003).

From the reconstructed digital volume, we cropped out a subvolume containing the digital ball in a 140 × 140 × 70 array, as shown in figure 4(a). Through the 3D-to-2D-to-1D decomposition scheme in figure 4 and the iterative edge-blurring algorithm, we obtained the 2D PSFs for each cross-section image. Figure 6 shows a z-axis cross-section image of the ball at (0, 0, 0) and the 2D PSF in the 3D mesh plot, which was obtained by assembling the 1D PSFs generated from a cluster of 32 radial scanlines (evenly distributed on a circle). In this figure, a vertical scanline and its corresponding 1D PSF curve are highlighted for a better understanding of the decomposition and assembling scheme. In the same way, we obtained the 2D PSFs at two other orthogonal cross sections. The first column in figure 7 shows the three 2D PSFs of the 3D PSF at (0, 0, 0), which are displayed as image arrays in the second column. Considering a 2D PSF as a grey-level image, its half-maximal cross-section area can be easily obtained by a thresholding operation with a threshold being equal to half of the maximum pixel value. The binary images resulting from thresholding are displayed in the right column. By summing up the 1-pixels (bright pixels) in the binary images, we obtained their half-maximal cross-section areas, denoted by AREA. With the central symmetry assumption on a 2D PSF, we calculated the FWHM by a formula of AREA = 4π(FWHM)^2. The non-circularity and pixelization appearance in the binary images are due to image noise and digital geometry. For the sake of simplification, a 3D PSF is decomposed into three 2D PSFs at three orthogonal cross sections, and each of the 2D PSFs is calculated by averaging over a set of 1D PSFs at the cross-sectional plane. By repeating the same procedure, we obtained the PSFs at two other locations (0, 0, z_0) and (0, 0, -z_0). The result of the PSF measurements at three locations is provided in table 2, in terms of σopt and FWHM. With the system calibration (listed in table 1), we also present the FWHMs in units of millimetres in the last column. It is seen that our cone-beam CT system is a spatial variant imaging system, where the central regions give better imaging performance, in the sense of smallest FWHM.

For the purposes of comparison, we calculated the 2D PSF with a cross-sectional image of the ball at (0, 0, 40 mm) by both a data-fitting technique (Boone and Seibert 1994) and our iterative technique, using 64 radial scanlines this time. Both techniques adopted the
Three-dimensional point spread function measurement

Figure 7. Three 2D PSFs of the 3D PSF at (0, 0, 0). In the left-side column are the topographical plots of the 2D PSFs corresponding to (a1) z-axis, (b1) x-axis and (c1) y-axis cross sections, respectively; in the middle column are the corresponding image arrays; in the right-side column are the binary images resulting from thresholding.

Table 2. Local PSF measurements at three locations.

<table>
<thead>
<tr>
<th>Location</th>
<th>σ_{opt} (voxel)</th>
<th>FWHM (voxel)</th>
<th>FWHM (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 0, z_0)</td>
<td>2.11 ± 0.23</td>
<td>4.98 ± 0.54</td>
<td>0.92 ± 0.10</td>
</tr>
<tr>
<td>(0, 0, 0)</td>
<td>1.49 ± 0.18</td>
<td>3.52 ± 0.42</td>
<td>0.65 ± 0.08</td>
</tr>
<tr>
<td>(0, 0, -z_0)</td>
<td>2.13 ± 0.24</td>
<td>5.03 ± 0.56</td>
<td>0.93 ± 0.10</td>
</tr>
</tbody>
</table>

single-parameter Gaussian PSF model. The resultant parameters, σ_{opt}, corresponding to 64 radial scanlines, are displayed in figure 8, where the data-fitting technique produces σ_{opt} = 1.97 ± 0.28 voxels, and our technique produces σ_{opt} = 2.09 ± 0.21 voxels. The results show that in this particular case our technique was slightly more stable and produced a somewhat larger FWHM in comparison with the data-fitting technique.

So far, we have demonstrated the 3D PSF experimental measurement of our cone-beam CT system using an iterative edge-blurring algorithm. We have reported focusing on the implementation details using typical experimental settings as listed in table 1. Based on the preliminary results, we would like to briefly address the following issues.
Figure 8. A comparison of two PSF measurement techniques. With 64 scanline profiles extracted from the \( z \)-axis image in figure 6(a) and the Gaussian model, the data-fitting technique produces \( \sigma_{opt} = 1.97 \pm 0.28 \), indicated by circles, and the iterative edge-blurring technique produces \( \sigma_{opt} = 2.09 \pm 0.21 \), indicated by stars.

The use of a plastic solid ball (diameter \( \sim 6 \) mm) alleviates the burden of manufacturing a microbead (\( \sim 100 \) nm in diameter). The ball provides step edges in the form of a rectangular function. The width of the rectangle corresponds to the diameter of the ball, and the height of the rectangle is determined by the difference between the interior and exterior in terms of x-ray absorption. For a spatial variant system, local PSF measurement prefers a small ball, whereas the edge rectification prefers a big ball. For our cone-beam CT system whose FWHM is about 1 mm, it is adequate to use a ball of 6 mm in diameter.

We demonstrated the iterative edge-blurring algorithm with a single-parameter Gaussian blurring model. This simple model can be refined by including an exponential term with more parameters. Alternatively, the PSF may also be modelled by a Lorentzian lineshape by \( a/(a^2 + t^2)\pi \), where \( a \) is the model parameter (the Lorentzian half width). In these pursuits of a more accurate PSF model, the single-parameter iterative algorithm evolves as an issue of multivariate nonlinear optimization, which demands more efforts in the search for the optimal solution.

Our experiment was conducted on our cone-beam CT system, in a particular setting of contrast = 0.68 (defined in equation (20)) and SNR = 26.6 (defined in equation (21)). The edge identification may be disturbed as the noise level increases. In this case, we may consider the use of a ‘water-gauge algorithm’ (Chen and Molloi 2003) to resist the noise influence, which locates edge points by local minima and maxima, without the noise-sensitive derivative operation. In our experiment, a PTFE ball provides SNR = 26.6, which is stable enough for edge identification by equation (17).

Since a CBCT system involves a reconstruction algorithm, its imaging performance is algorithm-dependent. In our experiment, we carried out cone-beam reconstruction by the practical Feldkamp algorithm, with the ramp filter. The PSF may vary with the cone-beam reconstruction algorithm, as well as with the selection of backprojection filter.

The cone-beam CT system is a spatial variant system. We only examined three locations along the rotational axis, thereby revealing the inter-slice variance along the rotational axis of
the scanner. In principle, the PSFs over the whole object support space can be measured by repositioning the test ball and repeating the same procedure as sketched in figure 3.

Other factors influencing the CT PSF include the x-ray source (characterized by kVp, mAs), ball material and surroundings (characterized by equation (20)), noise level (characterized by equation (21)) and detector (characterized by the detector array). Through experiment, the numeric representations of these influences are obtainable.

4. Summary

In cone-beam computed tomography (CBCT), a three-dimensional point spread function (3D PSF) is needed to characterize the performance of volume reconstruction. In this paper, we have reported an iterative edge-blurring algorithm for 3D PSF measurement, and demonstrated its implementation in a cone-beam CT system. The 3D PSF measurement consists of three main stages: (1) cone-beam x-ray scan and volumetric reconstruction, (2) edge rectification and (3) iterative edge-blurring procedure. Through the use of a plastic solid ball (diameter ~6 mm) under the cone-beam x-ray scan, we reconstructed a digital volume by the Feldkamp algorithm. With the separability assumption, a 3D PSF was decomposed into 2D PSFs on three orthogonal slice images across the ball centre, and each 2D PSF was in turn decomposed into 1D PSFs. At a cross-section image of the digital ball, we extracted 1D scanline profiles along the diameter directions. For each scanline profile, a rectification procedure was applied to automatically establish its own sharp rectangular function. The iterative edge-blurring method was automatically implemented, without bothering with the arduous calibration procedure. By placing the ball at three different locations, we examined the spatial variances of our CBCT system, which are quantitatively characterized by the standard deviation of the Gaussian PSF, or equivalently by the full-width-at-half-maximum (FWHM). The 3D PSF measurement method reported in this paper is easy, automated and robust.

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