Predictive Control for Dual-rate Systems Based on Lifted State-space Model Identified by N4SID Method

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Abstract—We address a novel predictive control strategy for dual-rate systems in which the input updating period is different from the output sampling period based on lifted state-space model identified by a modified Numerical Subspace State-Space IDentification (N4SID). There are three steps in the predictive control strategy. Firstly, lifted state-space models are identified for dual-rate systems by the modified N4SID. Based on the identified lifted state-space model, we construct predictors which can predict the output of dual-rate systems in multi-step ahead. Combining the predictors with an objective function minimization, predictive control laws for dual-rate systems are derived.

I. INTRODUCTION

The technological and the economical reasons often make the researchers and engineers encounter the so-called multi-rate sampled-data systems which include multiple sampled-data mechanisms with different updating and sampling periods. As a particular case of multi-rate sampled-data systems, dual-rate systems in which the input updating period differs from the output sampling period are very attractive because the contributions of dual-rate systems can be easily extended to the whole scope of multi-rate sampled-data systems. The problems of controller design and system analysis for dual-rate systems have been studied extensively in the literature [1], [2], [3], [4].

Model Predictive Control (MPC) designates a wide range of control algorithms which make an explicit use of a plant model in an objective function minimization to obtain the control law [5]. For instance, Clark et al. developed the Generalized Predictive Control (GPC) based on solving Diophantine equations [6], [7]. In [8], Ling et al. proposed a state-space based GPC for dual-rate systems. However, they only considered two special dual-rate systems in which the input updating period \( T_1 \) and the output sampling \( T_2 \) satisfy \( T_1 = mT_2 \) or \( mT_1 = T_2 \) (\( m \) is a positive integer). Without loss of generality, in this paper, we consider the dual-rate systems in which \( T_1 \) and \( T_2 \) satisfy \( qT_1 = pT_2 \) (\( p \) and \( q \) are coprime positive integers).

It is well known that plant models are crucial for MPC and system identification methods are often used to obtain plant models. However, most traditional identification methods only handle the identification problems of single-rate sampled-data systems. To handle this problem, the lifting technique [9] are often used to obtain lifted systems in which the input updating period equals to the output sampling period for dual-rate systems.

Most recently, Ding et al. handled the identification problems of the state-space models of lifted systems by estimating state vectors and parameters iteratively by the bootstrap algorithm [10]. Although satisfactory results were obtained through numerical simulations, the iterative algorithm still requires strict analysis of convergency properties. Furthermore, their algorithm requires prior knowledge of the observability structure indexes of the system under study. This may be an uneasy task for unknown multivariable systems.

In the last decades, Subspace State-Space IDentification (4SID) methods [11], which can estimate the system order and matrices from input-output data without prior knowledge of the plant under study, progressed significantly for identifying state-space models. In our previous work [12], we specified the subspace identification algorithms for lifted state-space models by a modified Numerical Subspace State-Space IDentification (N4SID) method. The modified N4SID method does not suffer from the problems of unguaranteed convergence and high sensitivity to the initial estimates, in contrast to the method in [10].

Subspace Predictive Control (SPC) method, initially introduced by Favoreel et al. [13], is synthesized by combining subspace identification methods with predictive control design. It is believed that control problems can receive the most benefit from SPC when the plant under study is linear time invariant (LTI) and difficult to be modeled from first principles. There are two basic steps in SPC. Firstly, a pair of multi-step ahead predictors are established from experimental data based on subspace identification methods. Then combining the predictors with optimizing an objective function, predictive control law is derived. Based on these two basic steps, Woodley et al. extended SPC to include \( \mathcal{H}_\infty \) specifications for the control design [14]. In addition, the subspace approach to state-space based GPC was investigated in [15] and this approach was applied to solid oxide fuel cells in [16]. Note that all the SPC methods mentioned above are limited to single-rate sampled-data systems.

As an exception, Wang et al. applied SPC to a special dual-rate system in which the output sampling period is \( m \) (\( m \) is a positive integer) times of the input updating period [17]. Note that the predictors in [17] did not tackle the causality constraints of lifted systems. However, Qin et al. claimed that the causal relations between the input and the output should be taken into consideration when subspace identification methods are performed [18]. Therefore, the
The proposed method in [17] cannot be extended to the whole scope of dual-rate systems. Although SPC is a kind of efficient data-driven predictive control method for LTI systems, the predictors in it cannot tackle the causality constraints of lifted systems. However, it was noted that predictors in SPC are high order models of the LTI plant under study [14]. Motivated by this fact, we propose a novel predictive control design for dual-rate systems. The proposed method is divided into three steps. Firstly, a lifted state-space model is identified from dual-rate sampled data by using the modified N4SID in [12]. Secondly, based on the identified lifted state-space model, we construct predictors which can predict the output of the dual-rate system under study in multi-step ahead. The predictors are similar to those in SPC except that they can handle the causality constraints of the lifted systems. Finally, combining the predictors with an objective function minimization, predictive control law is obtained for the dual-rates system under study.

The rest of this paper is organized as follows. The problem and preliminaries are stated in Section II. Section III introduces the lifted state-space models for dual-rate systems. In Section IV, the lifted state-space models are identified for dual-rate systems by the modified N4SID method. Section V discusses the predictive control design for dual-rate systems. Section VI presents an illustrative example.

II. PROBLEM STATEMENT

Consider a dual-rate system depicted in Fig. 1. Here \( u(k_1 T_1) \) is the input; ZOH is the zero order holder with period \( T_1 \); \( P_c \) represents an LTI continuous-time process with the following state-space representation:

\[
\begin{align*}
    x(t) &= A_c x(t) + B_c u(t) \\
    y(t) &= C x(t) + D u(t)
\end{align*}
\]

where \( x(t) \in \mathbb{R}^n \), \( u(t) \in \mathbb{R}^p \), \( y(t) \in \mathbb{R}^m \), \( A_c \), \( B_c \), \( C \) and \( D \) are the matrices of appropriate dimensions; \( ADc \) represents a sampler with period \( T_2 \). The output \( y(t) \) is sampled by a sampler \( ADC \) with period \( T_2 \). The sampled output is corrupted by a stochastic measurement noise \( v(k_2 T_2) \) which will be specified later. The measurement of the output is denoted by \( z(k_2 T_2) \).

\[
\begin{align*}
    u(k_1 T_1) &\rightarrow \text{ZOH} & u(t) &\rightarrow P_c & y(t) &\rightarrow \text{ADC} & y(k_2 T_2) &\rightarrow z(k_2 T_2)
\end{align*}
\]

Fig. 1. A dual-rate system

All the samplers and zero order holders are synchronized at time \( t = 0 \). For the dual-rate system, without loss of generality, it is assumed that the updating period \( T_1 \) and the sampling period \( T_2 \) satisfy \( T_1 = p T_b \) and \( T_2 = q T_b \) (\( p \) and \( q \) are coprime integers), where \( T_b \in \mathbb{R} \) and \( T_f = pq T_b \) are respectively the base period and the frame period [10].

It is well known that most system identification methods and predictive control designs cannot be directly applied to dual-rate systems. However, an isometric isomorphism from general dual-rate systems to LTI ones can be obtained by using the lifting technique [9]. The isometric isomorphism is called lifted system. Therefore, we can model dual-rate systems by identifying their lifted systems and then develop the predictive control strategies for dual-rate systems. Based on [9] and [10], the lifting technique and lifted state-space model will be introduced in the next section.

Although SPC is a kind of efficient data-driven predictive control methods, it will be shown that the predictors in SPC cannot tackle the causality constraints of the lifted systems in section V. Motivated by this, a new predictive control method is developed for dual-rate systems based on the identified lifted state-space models.

III. LIFTED STATE-SPACE MODEL

In this section, the lifting technique and the state-space representation of lifted systems will be introduced.

To obtain the lifted system for the dual-rate system in Fig. 1, lifting operators will be utilized. A \( q \)-fold lifting operator \( L_q \) maps \( u(k_1 T_1) \) to \( u(k T_f) \) (underline denotes lifting) as

\[
\begin{align*}
u(k T_f) = L_q u(k_1 T_1) = &\begin{bmatrix} u(k T_f) \\
    u(k T_f + T_1) \\
    \vdots \\
    u(k T_f + (q-1) T_1) \end{bmatrix}
\end{align*}
\]

where \( u(k T_f) \in \mathbb{R}^q \). And \( L_q^{-1} \) maps \( u \) back to \( u \). The lifting operator and the inverse lifting operator have the identities [9]

\[
L_q^{-1} L_q = I, \quad L_q L_q^{-1} = I.
\]

Similarly, the real output is given by a \( p \)-fold lifting operator as the following

\[
\begin{align*}
z(k T_f) = L_p y(k T_2) = &\begin{bmatrix} y(k T_f) \\
    y(k T_f + T_2) \\
    \vdots \\
    y(k T_f + (q-1) T_2) \end{bmatrix}
\end{align*}
\]

where \( y(k T_f) \in \mathbb{R}^m \). The lifted output measurement \( z(k T_f) \in \mathbb{R}^{qm} \) and the lifted noise \( y(k T_f) \in \mathbb{R}^{qm} \) can also be obtained by the \( p \)-fold lifting operator.

It is obvious that the system in Fig. 2 is equivalent to the dual-rate system in Fig. 1. Then the part in the rectangle is the lifted system.

\[
\begin{align*}
    \text{Lifted system} &\quad L_q \quad \text{dual-rate system} \quad L_p \quad \text{z(k T_f)}
\end{align*}
\]

Fig. 2. Lifted system

In order to obtain the state-space model of the lifted system in Fig. 2, the continuous-time process \( P_c \) is discretized by the base period \( T_b \) to get the discrete-time process \( P_d \). The state-space model of \( P_d \) is obtained as the following [2]

\[
\begin{align*}
x((k+1) T_b) &= A x(k T_b) + B u(k T_b) \\
    y(k T_b) &= C x(k T_b) + D u(k T_b)
\end{align*}
\]
where
\[ A = e^{At}, \quad B = \int_0^{T_b} e^{At}dtBc. \]

Then the following theorem gives the lifted state-space model:

**Theorem 1:** For the dual-rate system illustrated in Fig. 1, the coprimeness of the integers \( p \) and \( q \) implies that for every \( g \) \((0 \leq g \leq p-1)\), there exist the integers \( c_g \geq 0 \) and \( 0 \leq d_g < p \) so that \( gq = c_gp + d_g \). Then the lifted model is given by
\[
\begin{align*}
x((k+1)T_f) &= A_ix(kT_f) + B_iu(kT_f) \\
y(kT_f) &= C_ix(kT_f) + D_iu(kT_f)
\end{align*}
\]
where \( u(kT_f) \in R^p, y(kT_f) \in R^m, x(kT_f) \in R^m \) and the system matrices \([A_i, B_i, C_i, D_i]\) are given as follows
\[
A_i = A^{pq} \in R^{n \times n}, \\
B_i = \begin{bmatrix} B_{11} & B_{12} & \ldots & B_{1q} \end{bmatrix} \in R^{n \times qr}, \\
C_i = \begin{bmatrix} C_A \\ \vdots \\ C_A(p-1) \end{bmatrix} \in R^{m \times n}, \\
D_i = \begin{bmatrix} D_{10} & D_{11} & \ldots & D_{1q} \\
D_{20} & D_{21} & \ldots & D_{2q} \\
\vdots & \vdots & \ddots & \vdots \\
D_{(p-1)0} & D_{(p-1)1} & \ldots & D_{(p-1)q} \\
0 & 0 & \ldots & 0 \\
\end{bmatrix} \in R^{m \times QR}
\]
where
\[
B_s = \left( \sum_{k=0}^{p-1} A^{pq-kp}B \right) \in R^{n \times r}, \quad s = 1, 2, \ldots, q, \\
D_{gh} = C\left( \sum_{k=0}^{p-1} A^{pq-kp}B \right) \in R^{m \times r}, \\
D_{gc} = \left( \sum_{k=0}^{d_q-1} A^k \right)B + D \in R^{m \times r}.
\]

For the proof, the readers are referred to [10].

**Remark 1:** The block lower triangular matrix \( D_i \) corresponds to the causality constraints which means the output element \( y(kT_f + (a-1)T_f) \) \((a = 1, 2, \ldots, p)\) in \( y(kT_f) \) depends on the input element \( u(kT_f + (b-1)) \) \((b = 1, 2, \ldots, q)\) in \( u(kT_f) \) when \( kT_f + (a-1) \geq kT_f + (b-1) \). On the other hand, it is obvious that whatever the direct feedthrough matrix \( D \) of the continuous-time process equals to 0 or not, the lifted state-space direct feedthrough matrix will always be \( D_i \neq 0 \) inherently. Therefore, the causality constraints must be taken into consideration when identifying the lifted state-space model in (5).

Replacing the lifted output \( y(kT_f) \) by the lifted noise corrupted output measurement \( \tilde{y}(kT_f) \) and omitting \( T_f \), we have
\[
\begin{align*}
x(k+1) &= A_i x(k) + B_i u(k) \\
\tilde{y}(k) &= C_i x(k) + D_i u(k) + \nu(k)
\end{align*}
\]

**IV. IDENTIFICATION OF THE LIFTED STATE-SPACE MODEL**

In this section, we will identify the lifted state space models for dual-rate systems. In order to identify the lifted state-space model by the N4SID algorithm, we make assumptions based on [11] as follows

**Assumption 1:**
1. The state-space model in (1) is a minimal realization such that the state-space model in (4) is observable and controllable [2]. Meanwhile, the identifiability of the lifted model in (6) is ensured [10].
2. The system is operated in open loop such that the input and the noise are uncorrelated.
3. The noise term \( \nu(kT_f) \) in Fig. 1 is a stationary, zero mean white noise.
4. The input signal \( u(kT_f) \) in Fig. 1 is quasi-stationary and is persistently exciting of order \( 2qi \) where \( i \) will be define later.

Based on the assumptions above, the lifted state-space can be identified by the modified N4SID method in [12]. We briefly review the modified N4SID algorithm here.

The modified N4SID algorithm starts from defining the input and output block Hankel matrices as follows
\[
\begin{align*}
U_{0[i-1]} &= \begin{bmatrix} u(0) & u(1) & \ldots & u(j-1) \\
u(1) & u(2) & \ldots & u(j) \\
\vdots & \vdots & \ddots & \vdots \\
u(i-1) & u(i) & \ldots & u(i+j-1) \end{bmatrix}, \\
Z_{0[i-1]} &= \begin{bmatrix} z(0) & z(1) & \ldots & z(j-1) \\
z(1) & z(2) & \ldots & z(j) \\
\vdots & \vdots & \ddots & \vdots \\
z(i-1) & z(i) & \ldots & z(i+j-1) \end{bmatrix}.
\end{align*}
\]

\( U_{0[i-1]} \) and \( Z_{0[i-1]} \) can be defined in the similar way. \( i \) and \( j \) are user-defined indexes which are large enough. \( i \) should at least be larger than the maximum order of the lifted state-space model, i.e. \( i > n \), \( j \) is typically equal to \( N - 2i + 1 \) where \( N \) is the data length of all available data samples. In any case, \( j \) should be larger than \( 2i - 1 \).

Denote a state sequence as the following
\[
X_i := \begin{bmatrix} x(i) & x(i+1) & \ldots & x(i+j-1) \end{bmatrix}
\]
where \( X_i \in R^{n \times j} \), \( X_i \) and \( X_{i+1} \) can be estimated by performing the QR decomposition and the singular value decomposition (SVD) on the data block Hankel matrices. Meanwhile, the order of the lifted state-space model (6) can be determined by the number of the significant singular values when SVD is performed. For the details, the readers are referred to [11].

For the typical N4SID algorithm, once the estimated state sequences \( \hat{X}_i \) and \( \hat{X}_{i+1} \) are determined, the system matrices can be obtained by solving the following least squares (LS) problem:
\[
\begin{bmatrix} \hat{X}_{i+1} \\ \hat{Z}_{ij} \end{bmatrix} = \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix} \begin{bmatrix} \hat{X}_i \\ U_{ij} \end{bmatrix}.
\]
the causality constraints of the lifted state-space models. This
fact implies that before solving the LS problem (9), \( D_l \) should be
parameterized corresponding to the causality constraints.
The following proposition clarifies the problem.

**Proposition 1:** Rewrite the feedthrough term \( D_l \) as

\[
D_l = \begin{bmatrix}
D_{l1} & D_{l2} & \cdots & D_{lp} \\
D_{21} & D_{22} & \cdots & D_{2p} \\
\vdots & \vdots & \ddots & \vdots \\
D_{p1} & D_{p2} & \cdots & D_{pp}
\end{bmatrix}
\]

where \( D_{ab} \in \mathbb{R}^{p \times q} \) \((a = 1, 2, \ldots, p, \ b = 1, 2, \ldots, q)\). Then the causality constraints of the lifted state-space model (6) are ensured if and only if the subblock matrices \( D_{ab} \) in \( D_l \) satisfy

\[
D_{ab} = 0, \text{ for } (a - 1)q < (b - 1)p.
\]

For the proof, the readers are referred to [12]. After parameterizing \( D_l \) corresponding to Proposition 1, the matrices \( \{A_l, B_l, C_l, D_l\} \) in (6) can be obtained by solving the LS problem (9) subject to the constrained structure of \( D_l \).

Considering the causality constraints, the modified N4SID algorithm for dual-rate systems can be summarized as follows

**Step 1** Define the data Hankel matrices from the lifted input-output data set \( \{u(k), y(k)\} \);

**Step 2** Estimate the state sequences \( X_t \) and \( X_{t+1} \);

**Step 3** According to Proposition 1, parameterize \( D_l \) to be a lower block triangular matrix.

**Step 4** Obtain the lifted system matrices \( \{A_l, B_l, C_l, D_l\} \) in (6) by solving the LS problem (9) subject to the constrained structure of \( D_l \).

### V. PREDICTIVE CONTROL FOR DUAL-RATE SYSTEMS

Predictive control is not a particular controller design but more of a very ample scope of control methods. Fig. 3 characterizes predictive control briefly. In Fig. 3, \( t \) denotes the present time. \( N_l \)-step ahead predictors are used to predict the future output set \( \hat{y}(t+k) \) \((k = 1, 2, \cdots, N_l)\) based on the known input-output data up to time \( t \) and on the future input set \( u(t+k|t) \) \((k = 1, 2, \cdots, N_u)\). Note that \( N_l \) is the prediction horizon and \( N_u \) is the control horizon (\( N_u \) may be shorter than \( N_l \)) [5]. Then the input set \( u(t+k|t) \) \((k = 1, 2, \cdots, N_u)\) are obtained by optimizing a given objective function so that the output can track the reference trajectory \( r(t+k) \) \((k > 0)\) as close as possible.

![Fig. 3. Predictive control strategy.](image)

#### A. Subspace predictive control

SPC will be briefly reviewed here. In SPC [13], based on a set of data \( \{u(k), y(k)\} \) \((k = 0, 1, 2, \cdots)\) obtained from a multi-input multi-output (MIMO) LTI system, a pair of \( l \)-step ahead predictors \( \{L_m, L_n\} \) can be obtained by solving the following LS problem

\[
\min_{L_m, L_n} \| Y_f - \begin{bmatrix} L_w & L_u \end{bmatrix} \begin{bmatrix} W_p^* & U_f \end{bmatrix} \|_F^2
\]

where \( U_f, Y_f \) and \( W_p \) are data block Hankel matrices defined as follows

\[
U_f := U_{(2j-1)}, \quad Y_f := Y_{(2j-1)}, \quad W_p := \begin{bmatrix} U_p & Y_p \end{bmatrix} := \begin{bmatrix} U_{(2i-1)} \end{bmatrix} \begin{bmatrix} Y_{(2i-1)} \end{bmatrix}
\]

and \( \| \bullet \|_F \) denotes the Frobenius norm of \( \bullet \). Note that (10) is solved by QR decomposition in [13]. With \( \{L_m, L_n\} \), the output can be predicted \( l \) steps ahead as the following

\[
\hat{y}_f = L_m \begin{bmatrix} u_p & y_p \end{bmatrix} + L_n u_f
\]

where

\[
\hat{y}_f = \begin{bmatrix} \hat{y}(t+1) \cdots \hat{y}(t+\ell) \end{bmatrix},
\]

\[
u_f = \begin{bmatrix} u(t+1|t) \cdots u(t+\ell|t) \end{bmatrix},
\]

\[
y_p = \begin{bmatrix} y(t-1) \cdots y(t-\ell) \end{bmatrix},
\]

\[
u_p = \begin{bmatrix} u(t-1) \cdots u(t-\ell) \end{bmatrix}
\]

and \( t \) is the present time.

In SPC, the following objective function

\[
J = (\hat{y}_f - r_f)^T (\hat{y}_f - r_f) + u_f^T R u_f
\]

where \( r_f \) is the reference trajectory of appropriate dimension and \( R \) is user-defined weighting symmetric matrix will be minimized. The first term in (12) makes the output \( \hat{y}_f \) tracking the reference \( r_f \) and the second term keeps the input \( u_f \) at a reasonable level. Substitute \( \hat{y}_f \) in (12) by (11), we reach

\[
J = (L_m w_p + L_n u_f - r_f)^T (L_m w_p + L_n u_f - r_f) + u_f^T R u_f
\]

where

\[
w_p = \begin{bmatrix} u_p & y_p \end{bmatrix}.
\]

Then the optimal input \( u_f \) will be obtained by solving \( \frac{\partial J}{\partial u_f} = 0 \). Therefore, the optimal input \( u_f \) can be calculated as the following

\[
u_f = (L_m^T L_m + R)^{-1} L_m^T (r_f - L_n w_p).
\]

In SPC, only the first \( i_1 \) \((i_1 < i \) and is a user-defined parameter) steps in the calculated \( u_f \) are implemented to the plant under study. This is the so-called receding horizon principle [5].

Finally, SPC for LTI systems can be summarized as follows

**Step 1** Build the block Hankel matrices \( Y_f, U_f \) and \( W_p \) in (11) from the data set \( \{u(k), y(k)\} \);
Step 2 Solve the LS problem (10) to obtain the predictors $L_w$ and $L_u$;
Step 3 Repeat a) to b) with the newly obtained measurements of the output to obtain the control signal.
   a) Calculate $u_f$ based on (14);
   b) The first $t_1$ steps in the calculated $u_f$ are implemented.

B. Predictive control for dual-rate systems

Although SPC constructs multi-step ahead predictors for LTI systems from input-output data directly, it cannot be applied to the lifted systems. To elucidate this problem, the data set \{u(k), y(k)\} used in (10) and (11) are replaced by the lifted data set \{\hat{u}(k), \hat{y}(k)\} and the predictors in (11) are reformulated as follows

\[
L_w = \begin{bmatrix}
L_{w,1} & L_{w,2} & \ldots & L_{w,2i} \\
L_{w,1} & L_{w,2} & \ldots & L_{w,2i} \\
\vdots & \vdots & \ddots & \vdots \\
L_{w,1} & L_{w,2} & \ldots & L_{w,2i} \\
\end{bmatrix}, \quad
L_u = \begin{bmatrix}
L_{u,1} & L_{u,2} & \ldots & L_{u,2i} \\
L_{u,1} & L_{u,2} & \ldots & L_{u,2i} \\
\vdots & \vdots & \ddots & \vdots \\
L_{u,1} & L_{u,2} & \ldots & L_{u,2i} \\
\end{bmatrix}
\]

where $L_{w,a,b} \in \mathbb{R}^{pm \times qr}$ (a = 1, 2, · · · , i, b = 1, 2, · · · , i), $L_{u,a,b} \in \mathbb{R}^{pm \times pm}$ (a = 1, 2, · · · , i, b = i + 1, i + 2, · · · , 2i) and $L_{u,a,\bar{b}} \in \mathbb{R}^{pm \times qr}$ (a = 1, 2, · · · , i, \bar{b} = 1, 2, · · · , i). If $t$ is the present time, then according to (11), \(\hat{y}(t+k)\) which is the $k$th element of $u_f$ can be calculated as the following

\[
y(t+k) = \sum_{j=1}^{i} L_{w,a,j} u(t-i+j) + \sum_{j=2}^{i} L_{w,(2j-a),i} \hat{y}(t-i+j) + \sum_{j=1}^{k-1} L_{u,a,j} u(t+j) + \sum_{j=k}^{i} L_{u,a,j} \hat{y}(t+j).
\]

According to the explanation of the causality constraints in Remark 1, $L_{u,k}$ should be in a block lower triangular structure and $L_{u,j,4} = 0$ (j4 = k + 1, k + 2, · · · , i). This fact implies that the predictor $L_u$ should be a block lower triangular matrix. However, this cannot be ensured by solving the LS problem (10). As we claimed previously, the predictors cannot be directly applied to lifted systems. To clarify this problem, we propose a new method to construct the predictors \{\hat{L}_w, \hat{L}_u\} by the following proposition.

Proposition 2: $ip$-step ahead predictors \{\hat{L}_w, \hat{L}_u\} for the dual-rate system in Fig. 1 can be obtained from the lifted system matrices \{\hat{A}_i, \hat{B}_i, \hat{C}_i, \hat{D}_i\} as follows

\[
\hat{L}_w = \Gamma_i \begin{bmatrix} C_i & C_i \hat{A}_i & \ldots & \hat{A}_i \hat{A}_i + \hat{C}_i \hat{D}_i \end{bmatrix}, \quad \hat{L}_u = \hat{H}_i
\]

where

\[
\Gamma_i = \begin{bmatrix} C_i & C_i \hat{A}_i & \ldots & \hat{A}_i \hat{A}_i + \hat{C}_i \hat{D}_i \end{bmatrix} \in \mathbb{R}^{pmi \times n},
\]

\[
\Delta_i = \begin{bmatrix} \hat{A}_i^{-1} \hat{B}_i & \hat{A}_i^{-2} \hat{B}_i & \ldots & \hat{A}_i \hat{B}_i & \hat{B}_i \end{bmatrix} \in \mathbb{R}^{rs \times qr},
\]

\[
H_i = \begin{bmatrix} \hat{D}_1 & \hat{C}_1 \hat{B}_1 & \hat{G}_1 \hat{B}_1 & \hat{D}_1 & \cdot \cdot \cdot & \hat{D}_1 & \hat{C}_1 \hat{B}_1 & \hat{G}_1 \hat{B}_1 & \hat{D}_1 & \cdot \cdot \cdot & \hat{D}_1 & \hat{C}_1 \hat{B}_1 & \hat{G}_1 \hat{B}_1 & \hat{D}_1 & \cdot \cdot \cdot & \hat{D}_1 \end{bmatrix} \in \mathbb{R}^{pmi \times qr},
\]

and \(\cdot^+\) denotes the Moore-Penrose pseudo-inverse of \(\cdot\). Based on \{\hat{L}_w, \hat{L}_u\} in (15) and (16), the following prediction

\[
\hat{y}_f = L_w \begin{bmatrix} u_p \\ y_p \end{bmatrix} + L_u u_f
\]

(17)

where

\[
\hat{y}_f = \begin{bmatrix} \hat{y}(t+T_2)' & \hat{y}(t+2T_2)' & \cdots & \hat{y}(t+ipT_2)' \end{bmatrix}',
\]

\[
u_f = \begin{bmatrix} u(t+T_1)' & u(t+2T_1)' & \cdots & u(t+ipT_1)' \end{bmatrix}',
\]

\[
y_p = \begin{bmatrix} y(t-(ip+1)T_2)' & y(t-(ip+2)T_2)' & \cdots & y(t)' \end{bmatrix}',
\]

\[
u_p = \begin{bmatrix} u(t-(ip+1)T_1)' & u(t-(ip+2)T_1)' & \cdots & u(t)' \end{bmatrix}'.
\]

can be realized for the dual-rate system in Fig. 1.

Proof: By performing the iterations of the lifted state-space model (5), high order formulations of the lifted systems are obtained as follows

\[
Y_p = \Gamma X_0 + H U_p, \quad (18)
\]

\[
Y_f = \Gamma X_0 + H U_f, \quad (19)
\]

\[
X_i = \hat{A}_i X_0 + \Delta U_p. \quad (20)
\]

From (18), (19) and (20), $Y_f$ can be calculated as the following

\[
Y_f = \Gamma_i \left[ \begin{bmatrix} \Delta_i - \hat{A}_i \hat{A}_i \hat{H}_i & \hat{A}_i \hat{A}_i \hat{H}_i \end{bmatrix} \begin{bmatrix} U_p \\ Y_p \end{bmatrix} \right] + H U_f. \quad (21)
\]

Then (15), (16) and (17) can be derived from (21).

Remark 2: Because $L_u$ in (16) is of an appropriate block lower triangular structure, the predictors \{\hat{L}_w, \hat{L}_u\} obtained by Proposition 2 can handle the causality constraints of lifted systems.

Therefore, once the lifted system matrices \{\hat{A}_i, \hat{B}_i, \hat{C}_i, \hat{D}_i\} are identified, the $ip$-step ahead predictors \{\hat{L}_w, \hat{L}_u\} can be constructed to predict the output of the dual-rate system in Fig. 1. Combining with minimizing the objective function (12), the optimal predictive control signal $u_f$ for the dual-rate system under study can be obtained according to (14).

Finally, the data-driven predictive control design for dual-rate systems can be summarized as follows

Step 1 The lifted state-space model is identified for the dual-rate system in Fig. 1;
Step 2 According to (15) and (16), construct the predictors \{\hat{L}_w, \hat{L}_u\} based on the identified lifted system matrices \{\hat{A}_i, \hat{B}_i, \hat{C}_i, \hat{D}_i\};
Step 3 Repeat a) to b) with the newly obtained measurements of the output to obtain the control signal.

a) Derive the optimal control signal \( u_T \) by minimizing the objective function (12).

b) The first \( i \), where \( 1 \leq i \leq 10 \), steps of the optimal control signal \( u_T \) are implemented.

VI. NUMERICAL RESULTS

Consider a dual-rate system in which a continuous-time process \( P_c \) in Fig. 1 is as the following

\[
P(s) = \frac{1}{2s^2 + 3s + 1}
\]  (22)

The input updating period \( T_1 = 0.2s \) and the output sampling period \( T_2 = 0.3s \). Then we have the frame period \( T_f = 0.6s \) and the base period \( T_b = 0.1s \).

The lifted input vector and the lifted output vector are given as follows:

\[
u(0.6k_1) = \begin{bmatrix} u(0.6k) \\ u(0.6k + 0.2) \\ u(0.6k + 0.4) \end{bmatrix},
\]

\[
y(0.6k_2) = \begin{bmatrix} y(0.6k) \\ y(0.6k + 0.2) \\ y(0.6k + 0.3) \end{bmatrix}.
\]

Firstly, the lifted state space model \( \{A_l, B_l, C_l, D_l\} \) is identified from the lifted input-output data set \( \{u(0.6k_1), y(0.6k_2)\} \). Then based on the identified lifted state-space model for the dual-rate system, the 2i-step (i = 40) ahead predictors \( \{L_m, L_n\} \) are constructed according to Proposition 2 for the dual-rate system and the proposed predictive control design is performed. Similar to the simulation in [6], the predictive control simulation is performed with no noise included. The controlled output and the reference trajectory are depicted in Fig. VI. The numerical results indicate that the output controlled by the proposed predictive control method can track the reference trajectory very well.

VII. CONCLUSION

This paper presented a novel predictive control method for dual-rate systems based on lifted state-space model identified by the modified N4SID method. At first, the lifted state-space model for dual-rate systems was identified by the modified N4SID method. Then based on the identified lifted system matrices, a pair of predictors were established. Combining the predictors with an objective function minimization, the predictive control law was derived for dual-rate systems. To the best of our knowledge, this is the first work of developing a predictive control design in which the predictors can handle the causality constraints for dual-rate systems. Through numerical examples, we found that the proposed control method can control the output of a dual-rate system to track the reference trajectory very well. It is believed that the proposed predictive control method is applicable to the dual-rate systems where plentiful experimental data are available.

REFERENCES


