Decentralized robust control of interconnected uncertain nonlinear mechanical systems

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Abstract—This paper proposes a new method of decentralized robust control for large-scale interconnected uncertain nonlinear mechanical systems, by using disturbance observers (DOB). Rigorous stability analysis is given for the overall nonlinear system. Simulation results on a coupled double pendulum system are presented to confirm the established theoretical results.

I. INTRODUCTION

Many interconnected systems found in the real world such as industry manipulators and electric power systems are often composed of a set of subsystems. The centralized control for interconnected systems may be impractical due to a large amount of communications among the subsystems. A decentralized control system based only on local information is highly desirable. The fundamental uncertainties encountered in the decentralized controller design are the strength of the interactions among the subsystems. Many papers have been devoted to accounting for uncertain interconnections among the subsystems [1–15] in the last decades.

Most of the early works in decentralized control of large-scale systems are based on the assumptions that the interconnections are either bounded by constants or first-order polynomials in states. However, in practice, there do exist large-scale systems where the interconnections among the subsystems are of high-order, as discussed in [7], [8], [11]. The high-order interconnections can potentially destabilize an interconnected system if the decentralized controller does not explicitly account for these interconnections [7], [8].

In [7], [8], the adaptive decentralized control methods based on high gain stabilization technique was proposed. One major disadvantage with the adaptive high gain control schemes in these works is that the convergence rate of the adaptive system may be very slow because the number of parameters to be adaptively adjusted is usually very large, as pointed in [11]. To overcome this, in [11], a variable structure based decentralized control scheme was proposed, and numerical results showed that the method seemed to perform better than that in [7]. Although the theoretical analysis was based on the ideal switching function for sliding mode control and on a standard integral adaptive law to adjust the sliding mode control gain, the simulations were performed by using a smoothed switching function to reduce the inevitable chattering phenomena and an adaptive law with σ-modification. A more precise analysis may be necessary.

It should be pointed out here that differing from the most adaptive control techniques where the adaptive laws are adopted to estimate the unknown parameters of the uncertain model, in these works the adaptive laws are used to adaptively adjust the high control gains [7], [8] or switching function gains [11], instead of estimating uncertain system parameters. That is, the unknown interconnections are not canceled out actively. In [14], a decentralized adaptive controller was proposed, where the parameters of interconnections are adaptively estimated by using the desired trajectories of the other subsystems, so that the interconnections can be compensated by the estimated parameters. This approach, however, may exhibit unsatisfactory transient performance if there is no communication among the subsystems [15]. In [9], [10], the neural networks were introduced to estimate the local nonlinear functions of each subsystem. Recently, a decentralized controller has been proposed [12] and applied to multi-machine power systems [13], in which the lumped modeling error, disturbance and interconnections are estimated by the high-gain observer. However, it is assumed that the lumped uncertainty and its derivative are bounded with known constants over the domain of interest. This may not be practical in the real world.

As an alternative popular approach for compensating external disturbances and model mismatch, a DOB is often included into a motion controller. So far, many papers have been published on the DOB based motion controllers [16–18]. The DOB based motion controllers have been widely accepted in the industrial side, due to their simplicity and transparency of design. The key point of a DOB is to pass the external disturbances and model mismatch, lumped as an error term of the motion equation, through a low-pass filter and then to compensate the external disturbance and model mismatch by the output of the low-pass filter, which is viewed as the estimate of the lumped disturbance. Usually, the DOB based motion controllers are designed according to linear control theory, despite of the fact that the system under control may be nonlinear. Most recently, a novel robust nonlinear motion controller with DOB has been proposed, where the input-to-state stability (ISS) property [19] of the overall nonlinear control system is guaranteed [20].
This paper proposes a new method of decentralized robust control for large-scale interconnected uncertain nonlinear mechanical systems, by using DOBs. Rigorous stability analysis is given for the overall nonlinear system. The simulation results on a coupled double pendulum system are presented to confirm the established theoretical results.

II. PROBLEM STATEMENT

Consider a large-scale interconnected system which is composed of $N$ single input single output (SISO) nonlinear mechanical subsystems:

\[
\begin{align*}
\dot{x}_{i1} &= x_{i2} \\
\dot{x}_{i2} &= F_{i0}(x_i) + G_i(x_i)u_i + d_i(x,t)
\end{align*}
\]

(1)

for the $i$th ($i = 1, \cdots, N$) subsystem, $x_i = [x_{i1}, x_{i2}]^T$, $x_{i1}$ and $x_{i2}$ are the position and velocity respectively; $u_i$ is the control input; $G_i(x_i) = G_{i0}(x_i) + \Delta G_i(x_i)$ is an unknown function with known nominal function $G_{i0}(x_i)$ and uncertainty $\Delta G_i(x_i)$; $F_{i0}(x_i)$ is a known nominal function, and $d_i(x,t)$ is the uncertain disturbance term in which $x = [x_1^T, \cdots, x_N^T]^T$.

For simplicity and clarity of the basic idea, we confine our study on a large-scale interconnected system which is composed of $N$ SISO second-order nonlinear mechanical subsystems. The basic idea may be extended to the interconnected systems composed of multiple input multiple output (MIMO) subsystems with interconnections satisfying the matching condition [7], [8], [11], and to the systems which do no satisfy the matching condition [23].

We impose the following standing assumptions.

Assumption 1: $G_{i0}(x_i)$ and $G_i(x_i)$ are bounded away from zero with the same sign, for $x_i \in \Omega_i$, $\Omega_{i1}$ is the desired domain of operation. And there exist finite positive, but not necessarily known constants $M_{G_i}, M_{\Delta G_i} < \infty$ such that

\[
0 < \frac{G_{i0}(x_i)}{G_i(x_i)} \leq M_{G_i}, \quad \frac{\Delta G_i(x_i)}{G_i(x_i)} \leq M_{\Delta G_i}
\]

(2)

that is, the nominal function $G_{i0}(x_i)$ and the error function $\Delta G_i(x_i)$ do not grow in a higher order than $G_i(x_i)$ itself.

Assumption 2: The uncertainty term in each subsystem satisfies the following polynomial with known integer $p$ [7], [8], [11],

\[
d_i(x,t) \leq \sum_{j=1}^{N} d_{ik}v_{xi}, \quad v_{xi} = \sum_{k=0}^{p} \| x_i \|_2^k, \quad \| d_{ik} \| > 0
\]

(3)

Here, $\| \cdot \|_2$ stands for the Euclidean norm.

Assumption 3: The reference trajectory $x_{i1d}$ and its first-order and second-order derivatives, i.e., $\dot{x}_{i1d}$ and $\ddot{x}_{i1d}$ of each subsystem are bounded.

III. CONTROLLER DESIGN

Define the auxiliary error for each subsystem:

\[
\begin{align*}
r_i &= \dot{e}_{i1} + c_{i1}e_{i1} \\
&= (x_{i2} - \dot{x}_{i1d}) + c_{i1}(x_{i1} - x_{i1d}) \\
&= x_{i2} - \alpha_{i1}
\end{align*}
\]

(4)

where

\[
\alpha_{i1} = -c_{i1}e_{i1} + \dot{x}_{i1d}
\]

(5)

It is easy to see that if $r_i$ is made bounded, then $e_{i1}$ can be made sufficiently small provided a relatively large $c_{i1} > 0$.

We then have the following error subsystem:

\[
\dot{r}_i = F_{i0}(x_i) - \dot{\alpha}_{i1} + (G_{i0}(x_i) + \Delta G_i(x_i))u_i + d_i(x,t)
\]

(6)

According to Assumption 3, Assumption 2 can be rewritten as [9], [21], [22]

\[
d_i(x,t) \leq \sum_{j=1}^{N} d_{ijk}v_{jr}, \quad v_{jr} = \sum_{k=0}^{p} |r_j|^k, \quad \bar{d}_{ijk} > 0
\]

(7)

Taking the uncertain terms as the lumped disturbance $w_i$, we have

\[
w_i = \dot{r}_i - (F_{i0}(x_i) + G_{i0}(x_i))u_i - \dot{\alpha}_{i1}
\]

(8a)

\[
= d_i(x,t) + \Delta G_i(x_i)v_i
\]

(8b)

Since calculation of $\dot{r}_i$ is usually contaminated by high frequency noise, we have to pass (8a) through a low-pass filter $Q_i(s)$ to obtain the estimate of $w$ as $\hat{w}_i = Q_i(s)v_i$. This is the so called DOB studied extensively in the literature [16–18], [20].

To stabilize the subsystem, we design the following locally decentralized controller:

\[
u_i = u_{id} + u_{iw} + u_{ie}
\]

\[
u_{id} = -\frac{\alpha_{i20}}{G_{i0}(x_i)}, \quad u_{iw} = \frac{\bar{w}_i}{G_{i0}(x_i)}, \quad u_{ie} = \frac{-3}{\sum_{j=1}^{3} u_{idj}r_i}
\]

(9)

where

\[
\alpha_{i20} = -c_{i2}r_i + \alpha_{i1} - F_{i0}(x_i)
\]

\[
u_{id1} = \kappa_{i21}\bar{w}_i^2, \quad u_{id2} = \kappa_{i22}\bar{w}_i^2, \quad u_{id3} = \kappa_{i23}\bar{w}_i^2
\]

(10)

and $c_{i2}, \kappa_{i21}, \kappa_{i22}, \kappa_{i23} > 0$. Here, $u_{id}$ is a proportional feedback controller with model compensation; $u_{iw}$ is a compensating term by the disturbance observer’s output; $u_{idj}r_i$ is a nonlinear damping term [19] to counteract $\Delta G_i(x_i)$; $u_{id2r_i}$ is a nonlinear damping term to ensure boundedness of $r_i$ when the DOB is used; and $u_{id3r_i}$ is a nonlinear damping term to counteract the interconnection term $d_i(x,t)$. Notice that $u_{idj}(j = 1, 2, 3)$ are designed as time-varying control gains so that they grow at least as the same order as the corresponding terms to be counteracted.

IV. STABILITY ANALYSIS OF THE OVERALL SYSTEM

Based on the Chebyshev inequality

\[
\sum_{j=1}^{N} s_j \sum_{j=1}^{N} t_j \leq N \sum_{j=1}^{N} s_j t_j
\]

(11)

we first establish the following lemma [11], [21].
Lemma 1: The following inequality holds:
\[ \sum_{j=1}^{N} |r_j| \sum_{j=1}^{N} v_{jr} \leq N \sum_{j=1}^{N} |r_j| v_{jr} \]  
(12)

Applying the designed decentralized control input \( u_i \) to each subsystem, we have
\[ \dot{r}_i = -c_{i2} r_i + G_{i0}(\mathbf{x}_i) u_{ie} - \dot{w}_i + d_i(\mathbf{x}, t) + \Delta_{G_i}(\mathbf{x}_i) u_i \]
(13)
and hence
\[ \frac{d}{dt} \left( \frac{r_i^2}{2} \right) = -c_{i2} r_i^2 + G_{i0}(\mathbf{x}_i) u_{ie} r_i + d_i(\mathbf{x}, t) r_i + \Delta_{G_i}(\mathbf{x}_i) u_i r_i - \dot{w}_i r_i \]
(14)
where
\[ \eta_i = \frac{\Delta_{G_i}(\mathbf{x}_i)}{G_{i0}(\mathbf{x}_i)} \alpha_{i20} - \frac{G_i(\mathbf{x}_i)}{G_{i0}(\mathbf{x}_i)} \dot{w}_i \]
(15)
\[ D_i = \frac{G_i(\mathbf{x}_i)}{G_{i0}(\mathbf{x}_i)} \left( \alpha_{i20}^2 + \kappa_{22} |\dot{w}_i|^2 + \kappa_{23} v_{ie}^2 \right) \]
(16)

Define the Lyapunov function candidate as
\[ V = \frac{\| r \|^2}{2} = \sum_{i=1}^{N} \frac{r_i^2}{2} \]
(17)
where \( r = [r_1, \ldots, r_N]^T \).

Then, according to (7), 14~(16) and lemma 1, we have the following results.
\[ \dot{V} = - \sum_{i=1}^{N} (c_{i2} + D_i) r_i^2 + \sum_{i=1}^{N} \eta_i r_i + \sum_{i=1}^{N} d_i(\mathbf{x}, t) r_i \]
\[ \leq - \sum_{i=1}^{N} (c_{i2} + D_i) r_i^2 + \sum_{i=1}^{N} \eta_i r_i + \sum_{i=1}^{N} |r_i| |d_i(\mathbf{x}, t)| \]
\[ \leq - \sum_{i=1}^{N} (c_{i2} + D_i) r_i^2 + \sum_{i=1}^{N} \eta_i r_i + \sum_{i=1}^{N} |r_i| \sum_{j=1}^{N} d_{ij} v_{jr} \]
\[ \leq - \sum_{i=1}^{N} \frac{c_{i2}}{2} r_i^2 - \frac{1}{2} \sum_{i=1}^{N} \left( \frac{c_{i2}}{2} + D_i \right) r_i^2 - \frac{1}{2} \sum_{i=1}^{N} \left( \frac{c_{i2}}{2} + D_i \right) r_i^2 
+ \sum_{i=1}^{N} |\eta_i + N \max_{ij} (\bar{d}_{ij}) v_{ir}| |r_i| - \sum_{i=1}^{N} \delta_i^2 + \sum_{i=1}^{N} \delta_i^2 \]
\[ \leq - \sum_{i=1}^{N} \frac{c_{i2}}{2} r_i^2 - \frac{1}{2} \sum_{i=1}^{N} \left( \frac{c_{i2}}{2} + D_i \right) r_i^2 + \sum_{i=1}^{N} \delta_i^2 \]
\[ \leq - \min \left( \frac{c_{i2}}{2} \right) \| r \|^2 - \min \left( \frac{c_{i2}}{4} \right) \| r \|^2 + \sum_{i=1}^{N} \delta_i^2 \]
(18)
where
\[ \delta_i = \frac{\eta_i + N \max_{ij} (\bar{d}_{ij}) v_{ir}}{\sqrt{2} \left( \frac{c_{i2}}{2} + D_i \right)} \]
(19)
We can easily see by the inequalities in Assumptions 1 and 2 that each term in the numerator of \( \delta_i \) is counteracted by a nonlinear damping term in the denominator which grows at least as the same order as the corresponding term in the numerator, so that \( \delta_i \) is uniformly bounded.

Define
\[ \mu(t) = \sqrt{\frac{4}{\min_i (c_{i2})} \sum_{i=1}^{N} \delta_i^2} \]
(20)
Then we have
\[ \| r \| \geq \mu(t) \Rightarrow \frac{d}{dt} (\| r \|^2) \leq - \min_i (c_{i2}) \| r \|^2 \]
(21)
and hence \( \| r(t) \| \) is bounded as
\[ \| r(t) \| \leq \| r(0) \| e^{-\min_i (c_{i2}) t/2} + \sup_{0 \leq r \leq t} \mu(\tau) \]
(22)

Now we will see how the DOBs bring improvement. Keeping that all the internal signals are bounded in mind owing to (22), we rewrite (14).
\[ \frac{d}{dt} \left( \frac{r_i^2}{2} \right) = - (c_{i2} + D_{iw}) r_i^2 + w_i r_i - \dot{w}_i r_i \]
(23)
where
\[ D_{iw} = \kappa_{i21} \alpha_{i20}^2 + \kappa_{i22} \dot{w}_i^2 + \kappa_{i23} v_{iw}^2 \]
(24)

Notice that
\[ w_i - \dot{w}_i = \frac{\Delta_{G_i}(\mathbf{x}_i)}{G_{0i}(\mathbf{x}_i)} (-D_{iw} r_i + \alpha_{i20} + d_i(\mathbf{x}, t) - \frac{G_i(\mathbf{x}_i)}{G_{0i}(\mathbf{x}_i)} \dot{w}_i) \]
(25)

Recall the Lyapunov function candidate. Then by using (23) we have
\[ \dot{V} = - \sum_{i=1}^{N} (c_{i2} + D_{iw}) r_i^2 + \sum_{i=1}^{N} (w_i - \dot{w}_i) r_i \]
\[ \leq - \sum_{i=1}^{N} \frac{c_{i2}}{2} r_i^2 - \frac{1}{2} \sum_{i=1}^{N} \left( \frac{c_{i2}}{2} + D_{iw} \right) r_i^2 + \sum_{i=1}^{N} (w_i - \dot{w}_i) r_i \]
\[ - \frac{1}{2} \sum_{i=1}^{N} \left( \frac{c_{i2}}{2} + D_{iw} \right) r_i^2 - \sum_{i=1}^{N} \frac{(w_i - \dot{w}_i)^2}{2 \left( \frac{c_{i2}}{2} + D_{iw} \right)} \]
\[ + \sum_{i=1}^{N} \frac{(w_i - \dot{w}_i)^2}{2 \left( \frac{c_{i2}}{2} + D_{iw} \right)} \]
\[ \leq - \min \left( \frac{c_{i2}}{2} \right) \| r \|^2 - \min \left( \frac{c_{i2}}{4} \right) \| r \|^2 
+ \sum_{i=1}^{N} \frac{(w_i - \dot{w}_i)^2}{2 \left( \frac{c_{i2}}{2} + D_{iw} \right)} \]
(26)

Define
\[ \mu_w(t) = \sqrt{\frac{4}{\min_i (c_{i2})} \sum_{i=1}^{N} \frac{(w_i - \dot{w}_i)^2}{2 \left( \frac{c_{i2}}{2} + D_{iw} \right)}} \]
(27)
Then we have
\[ \| r \| \geq \mu_w(t) \Rightarrow \frac{d}{dt}(\| r \|^2) \leq -\min_i (c_i) \| r \|^2 \] (28)
and hence
\[ \| r(t) \| \leq \| r(0) \| e^{-\min_i (c_i) t/2} + \sup_{0 \leq \tau \leq t} \mu_w(\tau) \] (29)
Taking integration of (26), we have
\[ V(T) - V(0) \leq -\min_i \left( \frac{3c_i}{4} \right) \int_0^T \| r \|^2 dt \]
\[ + \int_0^T \sum_{i=1}^N \left( \frac{(\hat{w}_i - \bar{w}_i)^2}{2} + D_{i}\right) dt \]
\[ \text{such that} \]
\[ \lim_{T \to \infty} \frac{1}{T} \int_0^T \| r \|^2 dt \leq \frac{1}{3} \lim_{T \to \infty} \frac{1}{T} \int_0^T \mu_w(t) dt \] (30)
\[ \text{The results are summarized into the following theorem.} \]

**Theorem 1:** Let Assumptions 1–3 hold. All the internal signals are uniformly bounded and the following results hold:
1. The overall error system is ISS such that
\[ \| r(t) \| \leq \| r(0) \| e^{-\min_i (c_i) t/2} + \sup_{0 \leq \tau \leq t} \mu_w(\tau) \]
2. The mean square error satisfies
\[ \lim_{T \to \infty} \frac{1}{T} \int_0^T \| r \|^2 dt \leq \frac{1}{3} \lim_{T \to \infty} \frac{1}{T} \int_0^T \mu_w(t) dt \]

**Remark 1:** Investigation of \( \mu_w \) implies that owing to the compensation effect of DOB, \( \mu_w \) can be made very small at low-frequency domain. And the remained error terms \( \hat{w}_i - \bar{w}_i \) \((i = 1, \ldots, N)\) can be further reduced by increasing the control gains.

V. GUIDELINES FOR CONTROL PARAMETER DESIGN

The guidelines for design of the controller parameters are drawn here.

Since the nonlinear damping terms are nonlinear functions of some internal signals, it is recommendable to choose modest gains \( \kappa_{c_{i1}}, \kappa_{c_{i2}}, \kappa_{c_{i3}} \) to avoid noisy or large control efforts. Especially, since \( |\bar{w}_i|^2 \) may be large, its corresponding gain \( \kappa_{c_{i2}} \) should not be chosen large. The nonlinear damping terms are chosen to ensure boundedness of the control errors. It is not necessary to use large gains to suppress the control errors. The task to suppress the control errors can be performed by the DOBs with relatively broad pass-bands.

In contrast, the parameters of the nominal controllers \( c_{i1} \) and \( c_{i2} \) which also contribute to achieve small error signals can be chosen relatively large, without causing large amplitudes of the control inputs.

In this study, we adopt a simple second-order filter \( Q_i(s) \) of the DOB for each subsystem:
\[ Q_i(s) = \frac{1}{(1 + \lambda_i s)^2} \] (32)

where \( \lambda_i \) is the time-constant that determines the pass-band of \( Q_i(s) \). Usually, a small time-constant \( \lambda_i \) leads to a broad pass-band of the DOB, and hence achieves a small estimation error \( \hat{w} - \bar{w} \) of the DOB. However, too small a \( \lambda_i \) may make \( \bar{w} \) quite noisy, and hence a trade-off is required.

VI. SIMULATION EXAMPLE

Consider two inverted pendulums connected by a moving spring mounted on two carts (Fig. 1) [7], [8], [11], [12]. It is assumed that the pivot position of the moving spring is a function of time which can change along the full length \( l \) of the pendulums. The motion of the carts is specified as sinusoidal trajectories. The input to each pendulum is the torque \( u_i \) applied at the pivot point. The objective of the decentralized controller is to control each pendulum with only its own information so that each pendulum tracks its own desired reference trajectory while the connected spring and carts are moving.

Define \( x_i = [\theta_i, \dot{\theta}_i]^T = [x_{i1}, x_{i2}]^T, i = 1, 2 \). The dynamical equations of the coupled pendulums can be described as
\[ \dot{x}_1 = \left[ \begin{array}{c} \frac{g}{c_{cl}} \frac{k a(t)(a(t) - cl)}{c_{ml}^2} \frac{1}{x_{12}} \frac{1}{c_{ml}^2} \end{array} \right] u_1 + \left[ \begin{array}{c} \frac{0}{1} \end{array} \right] x_1 + \left[ \begin{array}{c} \frac{0}{0} \end{array} \right] x_2 \\
+ \left[ \begin{array}{c} \frac{0}{1} \end{array} \right] x_2 \] (33a)
\[ \dot{x}_2 = \left[ \begin{array}{c} \frac{g}{c_{cl}} \frac{k a(t)(a(t) - cl)}{c_{ml}^2} \frac{1}{x_{22}} \frac{1}{c_{ml}^2} \end{array} \right] u_2 + \left[ \begin{array}{c} \frac{0}{1} \end{array} \right] x_1 + \left[ \begin{array}{c} \frac{0}{0} \end{array} \right] x_2 \\
+ \left[ \begin{array}{c} \frac{0}{1} \end{array} \right] x_1 \] (33b)
where \( c = M/(M + m); \) \( k \) and \( g \) are spring and gravity constants; \( u_1 \) and \( u_2 \) are the control torques applied to the pendulums. For numerical simulations, we choose \( g = 9.8, \) \( l = 1, \) \( k = 1, \) \( M = m = 4. \) The motion of the carts is specified as sinusoidal trajectories, i.e., \( y_1 = \sin(\omega_1 t) \) and \( y_2 = L + \sin(\omega_2 t) \), where \( L \) is the natural length of the spring and \( \omega_1 \neq \omega_2. \) We select \( L = 2, \) \( \omega_1 = 2 \) and \( \omega_2 = 3. \) Also, we choose \( a(t) = \sin(5t). \)

Denote the nominal values of \( m \) and \( c \) as \( m_0 \) and \( c_0. \) We have \( m = m_0 + \Delta m, \) and \( c = c_0 + \Delta c. \) Then we rewrite the dynamical equations in the form of (1):

\[
F_{10}(x_1) = \frac{g}{c_0}x_{11}
\]
\[
G_{10}(x_1) = \frac{1}{c_0 m_0 l^2} - \frac{1}{c_0 m_0 l^2}
\]
\[
\Delta G_{1}(x_1) = \frac{1}{c m l^2} - \frac{1}{c m l^2}
\]
\[
d_1(x,t) = \left( \frac{g}{c} - \frac{g}{c_0} \right) x_{11} - \frac{m}{M} \sin(x_{11})x_{12}^2
\]
\[
+ \frac{ka(t)(a(t) - cl)}{cml^2}(x_{21} - x_{11})
\]
\[
- \frac{k(a(t) - cl)}{cml^2}(y_1 - y_2)
\]
\[
F_{20}(x_2) = \frac{g}{c_0}x_{21}
\]
\[
G_{20}(x_2) = \frac{1}{c_0 m_0 l^2} - \frac{1}{c_0 m_0 l^2}
\]
\[
\Delta G_{2}(x_2) = \frac{1}{c m l^2} - \frac{1}{c m l^2}
\]
\[
d_2(x,t) = \left( \frac{g}{c} - \frac{g}{c_0} \right) x_{21} - \frac{m}{M} \sin(x_{21})x_{22}^2
\]
\[
+ \frac{ka(t)(a(t) - cl)}{cml^2}(x_{11} - x_{21})
\]
\[
- \frac{k(a(t) - cl)}{cml^2}(y_2 - y_1)
\]

We can easily see that \( d_1(x,t) \) and \( d_2(x,t) \) are bounded by second-order polynomials of \( x_1 \) and \( x_2, \) that is, \( p = 2 \) in (3) \( [7], [8], [11], [12]. \)

The initial values are set as

\[ x_1 = [1.0, 0]^T, \quad x_2 = [1.0, 0]^T \]

The desired trajectories are given as low-pass rectangular waves:

\[ x_{id} = \frac{1}{(0.2s + 1)^2}y_{id} \quad i = 1, 2 \]

where the initial values are given as \( x_{1id}(0) = 1.0, x_{2id}(0) = 0. \) The command signals \( y_{id} \) and \( y_{2id} \) are rectangular waves. The amplitude of \( y_{id} \) changes between \( +1 \) and \( -1, \) and that of \( y_{2id} \) changes between \( +0.6 \) and \( -0.6. \)

The nominal physical parameters are set as \( m_0 = 1 \) and \( c_0 = 1 \) which are significantly different from their true values.

The controller gains and the DOB time-constants are given as follows.

\[ c_{i1} = 10, \quad c_{i2} = 10 \] \[ \kappa_{i21} = 1, \quad \kappa_{i22} = 1, \quad \kappa_{i23} = 1 \] \[ Q_i(s) = \frac{1}{(\lambda_i s + 1)^2}, \quad \lambda_i = 0.02 \]

The decentralized controller is discretized at a sampling period \( T = 0.5[ms]. \)

VII. CONCLUSIONS

In this paper, a theoretically guaranteed robust a decentralized robust control for large-scale interconnected uncertain nonlinear mechanical systems has been proposed. Rigorous stability analysis was performed as well. The theoretical results were verified through simulation studies on a benchmark system of coupled double pendulums.

REFERENCES


Fig. 2. Results by the nominal controller.

Fig. 3. Results by the proposed robust controller.