Fitness Landscape Analysis for Dynamic Resource Allocation in Multiuser OFDM Based Cognitive Radio Systems

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***Abstract*— Cognitive Radio (CR) is a promising technique for improving the spectrum efficiency in future wireless network. The downlink transmission in a multiuser Orthogonal Frequency Division Modulation (MU-OFDM) based CR system is investigated. Optimal allocating transmit power, bits and subcarriers among cognitive radio users (CRUs) can achieve high throughput while satisfying the given quality of services (QoS) requirement. The problem of dynamic resource allocation in multiuser OFDM based CR systems is a combinatorial optimization problem and is computationally complex. In order to solve the resource allocation problem efficiently, efficient and simple algorithms are needed. It has been shown that memetic algorithms (MAs) outperform other traditional algorithms for many combinatorial optimization problems. On the other hand, the performance of MAs is highly dependent on choice of local search and evolutionary operators. In order to achieve better performance, we need to choose appropriate local search method and evolutionary operators for a memetic algorithm. The local search and evolutionary operators selection should be based on the properties of a given problem. Fitness landscape is an important technique for analyzing the behavior of combinatorial optimization problems. We apply the fitness landscape to analyze the optimization problem proposed in this paper. Appropriate local search and evolutionary operators are derived for the proposed MA. Numerical experiments show that the performance of the proposed memetic algorithm is better than other existing algorithms.**

I. Introduction

With the fast development of wireless techniques, wireless applications and services are becoming more affordable for most people. The number of wireless applications, services and users is growing rapidly. However, as the available spectrum is scarce and specified by the government agencies, it is impossible to increase the spectrum arbitrarily for a wireless network. Even many different techniques have been proposed to improve the efficiency of spectrum utilization, the spectrum still turns to be scarce compared with the fast increasing number of wireless applications, services and users. For example, the cellular network has been developed from FDMA system to WCDMA system and the efficiency of spectrum utilization has been improved drastically. However, it is still hard to accommodate the continuously increasing number of subscribers. On the other hand, according to the FCC report [1], many of the frequency bands currently licensed for other services are grossly under-utilized.

Cognitive radio (CR) is a novel concept for improving the overall utilization of spectrum bands by allowing unlicensed secondary users (also referred to as CR users or CRUs) to access those frequency bands which are not currently being used by licensed primary users (PUs). Therefore, it can improve the efficiency of spectrum utilization and mitigate spectrum scarcity. However, when CRUs access to the PUs' frequency bands, the interfere to the PUs must satisfy given constraints. Therefore, CRUs have to sense the environment and rapidly adapt their transmission parameter values. Orthogonal frequency division multiplexing (OFDM) is a good modulation candidate for a CR system due to its flexibility in allocating resources among CRUs.

For a OFDM based wireless system, adaptive adjustment of the transmission parameters, such as transmit power, bits and subcarriers, according to the wireless environment can improve the performance. However, the resource allocation problem is a combinatorial optimization problem and the steps to find the optimal resource allocation among CRUs grows exponentially with the size of subcarriers. Even for the case of ignoring the mutual interference between PUs and CRUs, the problem of optimal allocation subcarriers, bits and transmit powers among users in a multiuser (MU) OFDM system is still computationally complex [2]. In order to reduce computational complexity, the resource allocation problem of a MU-OFDM system is solved into two steps by many suboptimal algorithms [3]-[6]: (1) determine the allocation of subcarriers to users. (2) determine the allocation of bits and transmit powers to users. Most of these algorithms are based on greedy approaches. When the variables are independent, a simple greedy algorithm can find the global optimal solution with low complexity. However, for the resource allocation problem in MU-OFDM systems, the variables are interdependent. In the worst case, the final solution obtained by these algorithms may be a local optimal solution far away from the global optimal solution. When considering the mutual interference between PUs and CRUs, the problem turns to be more complex.

Memetic algorithms (MAs) have been shown to outperform other traditional algorithms for many combinatorial problems [7]. Especially for many NP-hard problems such as travelling sales man problem (TSP) and quadratic assignment problem (QAP). MAs have been applied to combinatorial optimization problems widely. Normally, a genetic algorithm (GA) combined with a local search method is called a memetic algorithm. Compared with GAs, MAs are more effective and more efficient [8]. In this paper, we propose a memetic algorithm for the resource allocation problem in MU-OFDM based CR systems. However, the performance of a MA for a given problem is highly dependent on the selection of genetic operators and local search. Good local search and genetic operators will improve the results greatly. However, there is still little work on the choice of a good local search and genetic operators for a memetic algorithm. According to the “no free lunch” theorem, the selection of local search and genetic operators should be based on the given problem. Therefore, in order to choose appropriate local search and genetic operators, we need to analyze the resource allocation problem first. Fitness landscape, which was originally proposed for analyzing the evolutionary theory in [9], has been applied to understand the behavior of combinatorial optimization algorithms and predict their behaviors [10]-[12]. We apply fitness landscape to analyze the resource allocation problem in MU-OFDM based cognitive radio systems. Then we derive appropriate local search and genetic operators for the proposed memetic algorithm.

The paper is organized as follows: The system model and resource allocation problem is formulated in Section II. A SAMA algorithm for subcarrier allocation is proposed in Section III. Fitness landscape analysis on the resource allocation problem and the selection of local search and genetic operators are discussed in Section IV. For the bits allocation problem, a simple while efficient memetic algorithm is derived in Section V. The simulation is showed in Section VI and Section VII is the conclusion.

II. System Model

The system model used in this paper is the same as that in [13]. A description is provided below for the convenience of the reader.

We focus on the forward link in a multiuser OFDM CR system in which a base station (BS) transmits to one PU and K CRUs. The PU and CRUs occupy neighboring frequency bands as shown in Fig. 1.

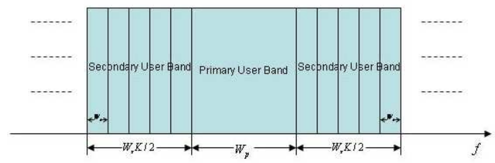

Fig. 1. Primary user band of width W_p and secondary user sub-bands, each of width W_s.

The PU band has a width of W_p Hz and has $N/2$ subcarriers, each occupying a band of width W_s Hz, on either side. The BS allocates subcarriers, subcarrier powers and bits to the CRUs dynamically. The channels from the BS to all users are modelled as slowly time-varying, i.e. they do not change appreciably between successive allocations. The BS is assumed to have perfect channel state information (CSI) for all users and subcarriers.

The power spectral density (PSD) of the n^{th} subcarrier signal is assumed to have the form [14]

$$\Phi_n(f) = P_n T_s \left(\frac{\sin \pi f T_s}{\pi f T_s} \right)^2, \tag{1}$$

where P_n denotes the subcarrier n transmit signal power and T_s is the symbol duration. The resulting interference power spilling into the PU band is given by

$$I_n(d_n, P_n) = \int_{d_n - W_p/2}^{d_n + W_p/2} |g_n|^2 \Phi_n(f) df = P_n IF_n \tag{2}$$

where g_n is the subcarrier n channel gain from the BS to the PU, d_n is the spectral distance between subcarrier n and the center frequency of the PU band and IF_n is the interference factor for subcarrier n.

The interference power introduced by the signal destined for the PU, hereafter referred to as the PU signal, into the band of subcarrier n at user k is

$$S_{nk}(d_n) = \int_{d_n - W_s/2}^{d_n + W_s/2} |h_{nk}|^2 \Phi_{RR}(e^{jw}) dw, \tag{3}$$

where h_{nk} is the subcarrier n gain from the BS to user k and $\Phi_{RR}(e^{jw})$ is the PSD of the PU signal.

Let P_{nk} denote the transmit power allocated to subcarrier n of user k. As discussed in [3], the maximum number of bits per symbol that can be transmitted on this subcarrier is

$$b_{nk} = \left\lfloor \log_2 \left(1 + \frac{|h_{nk}|^2 P_{nk}}{\Gamma(N_0 W_s + S_{nk})} \right) \right\rfloor, \tag{4}$$

where $\lfloor . \rfloor$ denotes the floor function, N_0 is the one-sided noise PSD and S_{nk} is given by (3). The term Γ indicates how close the system is operating to capacity and is set to 1 for convenience.

From (4), the additional signal power needed to transmit one extra bit to user k on subcarrier n can be expressed as:

$$\Delta P_{nk} = \frac{N_0 W_s + S_{nk}}{|h_{nk}|^2} 2^{b_{nk}}, \tag{5}$$

Using (2), we deduce that the additional interference power generated by such an additional signal power to the PU is

$$\Delta I_{nk} = \Delta P_{nk} IF_n. \tag{6}$$

Let $a_{nk} \in \{0, 1\}$ be a subcarrier allocation indicator function, i.e. $a_{nk} = 1$ if and only if subcarrier n is allocated to user k. To avoid excessive interference among CRUs, it is assumed that each subcarrier can be used for transmission to at most CRU at any given time.

The objective is to maximize the total CRU bit rate, R_s, subject to limits on the total CRU transmit power and PU tolerable interference power. More specifically, the optimization problem of interest is

$$\max R_s \triangleq W_s \sum_{k=1}^{K} \sum_{n=1}^{N} a_{nk} b_{nk} \tag{7}$$

subject to

$$a_{nk} \in \{0,1\},\ \forall n,k \tag{8}$$
$$\sum_{k=1}^{K} a_{nk} \leq 1,\ \forall n \tag{9}$$
$$P_{nk} \geq 0,\ \forall n,k \tag{10}$$
$$\sum_{k=1}^{K} \sum_{n=1}^{N} a_{nk} P_{nk} \leq P_{total}, \tag{11}$$
$$\sum_{k=1}^{K} \sum_{n=1}^{N} a_{nk} P_{nk} IF_n \leq I_{total}, \tag{12}$$

$$R_1 : R_2 : \cdots : R_K = \lambda_1 : \lambda_2 : \cdots : \lambda_K, \tag{13}$$

where P_{total} denotes the total CRU power limit and I_{total} is the maximum PU tolerable interference power, and

$$R_k = W_s \sum_{n=1}^{N} a_{nk} b_{nk} . \forall k = 1, \cdots, K \tag{14}$$

represents the total bit rate of k^{th} CRU. Inequality (9) reflects the condition that any given subcarrier can be allocated to at most one user. Inequalities (11) and (12) correspond to the power and interference constraints, respectively. (13) reflects the proportional fair among CRUs.

III. Proposed Algorithm for Subcarrier Allocation

Clearly, the objective function in equation (7) is a combinatorial optimization problem with two levels, (i.e., determine the subcarrier allocation indicator a_{nk} and transmit bits b_{nk}). The algorithm complexity of searching the optimal solution grows exponentially with the number of subcarriers. In order to reduce the algorithm complexity, we propose a simple algorithm, which is called subcarriers allocation for memetic algorithm (SAMA), to determine subcarrier allocation. The pseudo code listings of SAMA algorithm is showed in Fig. 2. Its algorithm complexity is $O(KN)$, where K denotes the number of CRUs and N represents the number of subcarriers. Firstly, we set a threshold to delete some worst subcarriers for all users. For the remnant $\hat{N}$ subcarriers, we assume that each user experiences a channel factor of

$$\Psi_k = \frac{1}{\hat{N}} \sum_{n=1}^{\hat{N}} \frac{|h_{nk}|^2}{\Gamma(N_0 W_s + S_{nk})},\ \forall k = 1, 2, \cdots, K \tag{15}$$

$$\overline{IF} = \frac{1}{\hat{N}} \sum_{n=1}^{\hat{N}} IF_n, \tag{16}$$

```
Algorithm SAMA:
for n=1 to number of subcarriers
    find Ψ(k,n) = max |h_nk|^2 / Γ(N_0 W_s + S_nk) ;
    based on Ψ(k,n), P_total and I_total ,
    calculate b_n to n^th subcarrier ;
    if b_n > 2
      the n^th subcarrier is available;
    else
      the n^th subcarrier isn't available;
    endif;
endfor;
initialize the number of subcarrier
allocated to k^th user;
m_k = 0 ∀k = 1,2,···,K;
calculate the b_k in equation (17);
for n=1 to number of available subcarriers
    find the k^th user such that
    m_k b_k / λ_k is the smallest;
    allocate n^th available subcarrier
    to k^th user;
endfor;
```

Fig. 2. Pseudo-code for Subcarrier Allocation Algorithm

on each channel, equal interference to PU and equal transmit power on each channel for all users. Therefore, the available bits loaded for k^{th} CRU on each channel can be expressed as

$$b_k = \min(\lfloor \log_2(1 + \frac{\Psi_k P_{total}}{\hat{N}}) \rfloor, \lfloor \log_2(1 + \frac{\Psi_k I_{total}}{\hat{N}\overline{IF}}) \rfloor). \quad \forall\ k = 1, 2, \cdots, K \tag{17}$$

Let k^{th} CRU be allocated m_k subcarriers. Then the objective is to find a set of m_k subcarriers $k = 1, 2, \cdots, K$ which satisfy

$$\max R_s = W_s \sum_{k=1}^{K} m_k b_k, \tag{18}$$

subject to

$$m_1 b_1 : m_2 b_2 : \cdots : m_K b_K = \lambda_1 : \lambda_2 : \cdots : \lambda_K, \tag{19}$$

$$P \leq P_{total}, \tag{20}$$

$$I \leq I_{total}, \tag{21}$$

where P is the total transmit power allocated to all subcarriers and I represents the total interference power to the PU. After subcarrier allocation, a bits allocation solution can be expressed as

$$b = \begin{bmatrix} b_1 & b_2 & \cdots & b_N \end{bmatrix}. \tag{22}$$

IV. Fitness Landscape Analysis for Bits Allocation

A. Bits Allocation Analysis

After applying SAMA algorithm to determine subcarrier allocation to CRUs, we need to determine the bits allocation to CRUs. From the bits allocation solution expression in (22), the bits allocation problem is still a combinatorial optimization problem. Let $\check{b}_n$ be the number of possible bits allocated to n^{th} subcarrier, then the steps to find the optimal bits allocation is $O(\prod_{n=1}^{N}\check{b}_n)$. Since $\check{b}_n \geq 2$ for real systems, $\prod_{n=1}^{N}\check{b}_n \geq 2^N$. Therefore, it is computational complex. In order to make the problem tractable, we need to find a simple while efficient algorithm to determine the bits allocation to CRUs.

B. Fitness Landscape Analysis

Though the notion of fitness landscape was first proposed in [9] for analyzing the gene interaction in biological evolution, it also was an important technique for analyzing the behavior of combinatorial optimization problem. Furthermore, it has been extended to analyze evolutionary algorithms. From [9], [12], each genotype has a "fitness" and the distribution of fitness values over the space of genotypes constitutes a fitness landscape. In our problem, we consider the set of solutions as a search space, the height of a point denotes the fitness of the solution associated with the point. Therefore, a heuristic algorithm can be considered as searching through the search space to find the highest peak of the landscape.

Normally, for a given combinatorial optimization problem, we can define a fitness landscape as $\Omega = (X, f, d)$, where the X is the set of solutions, f denotes the objective function $f : X \rightarrow \mathbb{R}$ and d represents the hamming distance of two solutions. Based on the measurement d, we also can construct the fitness landscape as a graph set: $G = (V, E)$, where V represents the set of solution (i.e. $V = E$), E denotes the set $E = \{(x, y) \in X \times X | d(x, y) = d_{\min}\}$ where $d_{\min}$ is the minimum distance between two points in the set X. Clearly, the minimum distance in our problem is $d_{\min} = 1$ and the maximum distance is $d_{\max} = N$. Accordingly, we can construct the neighborhood of a point x as $\mathcal{N}_k(x) = \{y \in X | d(x, y) \leq k\}$.

For different combinatorial optimization problems, the structures of their fitness landscapes are also different. There are several important properties for a fitness landscape. These properties have been proven to have great effects on the performance of a memetic algorithm. According to the No Free Lunch theorem, an algorithm has different performance on different problems. In order to obtain better results, we need to select appropriate algorithm and parameters for a given problem based on domain knowledge. Therefore, analyzing the structure of a fitness landscape of a given problem is necessary. In this paper, we focus on the following properties of a fitness landscape:

- the landscape ruggedness
- the number of local optima in the landscape
- the distribution of the peaks in the search space
- the number of iterations to reach a local optimum
- the structure of the basins of attraction of the local optima

For these properties, some are easy to be determined by statistical methods but others are difficult to determine. For example, it is very difficult to understand the properties of the number of local optima and the number of iteration to reach a local optimum by statistical methods. The NK model proposed in [12] is an important technique for analyzing these properties. In the NK model, N refers to the number of parts of a system. Each part makes a fitness contribution which depends upon that part and upon K other parts among the N. For example, suppose a solution vector $\mathbf{x} = [x_1, \cdots, x_N]^T$ to be a binary vector of length N, the fitness function can be expressed as

$$f(x) = \frac{1}{N}\sum_{i=1}^{N} f_i(x_i, x_{i1}, \cdots, x_{ik}), \tag{23}$$

where the fitness contribution f_i of locus i depends on the value of gene x_i and the values of K other genes $x_{i1}, \cdots, x_{ik}$. The function $f_i : \{0, 1\}^{K+1} \rightarrow \mathbb{R}$ assigns a uniformly distributed random number between 0 and 1 to each of its 2^{K+1} inputs.

In the following, we introduce some related statistical methods proposed to measure the properties of a fitness landscape.

Autocorrelation functions and random walk correlation functions have been proposed to measure the ruggedness of a fitness landscape in [11]. The autocorrelation function reflects the correlation of solutions with distance d in the search space. A fitness landscape is said to rugged if there is low correlation between neighboring points of the landscape, and a landscape is smooth if there is high correlation between neighboring points [12]. Therefore, the more rugged a landscape, the harder the problem for an algorithm. Let $X^2(d)$ be the set of all pairs of solutions in the search space with distance d. $X^2(d)$ can be expressed as

$$X^2(d) = \{(x, y) \in X \times X | d(x, y) = d\}, \tag{24}$$

and let $|X^2(d)|$ be the number of pairs in the set $X^2(d)$. The the autocorrelation function can be expressed as

$$\rho(d) = \frac{\mathrm{E}(f(x)f(y))_{d(x,y)=d} - \mathrm{E}^2(f)}{\mathrm{E}(f^2) - \mathrm{E}^2(f)}, \tag{25}$$

where $\mathrm{E}(\cdot)$ denotes the expectation function. In addition, the random walk function can be defined as

$$r(s) = \frac{\mathrm{E}(f(x_t)f(x_{t+s})) - \mathrm{E}^2(f)}{\mathrm{E}(f^2) - \mathrm{E}^2(f)}. \tag{26}$$

Based on autocorrelation function and random walk correlation function, the correlation length ℓ of the landscape is defined as

$$\ell = -\frac{1}{\ln(|r(1)|)} = -\frac{1}{\ln(|\rho(1)|)}, \tag{27}$$

for $r(1), \rho(1) \neq 0$. The correlation length directly reflects the ruggedness of a landscape: the lower the value for ℓ, the more rugged the landscape.

Fitness distance correlation (FDC), which was proposed in [10] as a measure for problem difficulty for genetic algorithms, is an important measure. The FDC coefficient is expressed as:

$$\varrho(f,d_{opt}) = \frac{cov(f,d_{opt})}{\sigma(f)\sigma(d_{opt})} = \frac{\mathrm{E}(fd_{opt}) - \mathrm{E}(f)\cdot\mathrm{E}(d_{opt})}{\sqrt{(\mathrm{E}(f^2)-\mathrm{E}^2(f))\cdot(\mathrm{E}(d_{opt}^2)-\mathrm{E}^2(d_{opt}))}}, \quad (28)$$

where d_{opt} represents the distance of a point to the nearest optimum. ϱ denotes the correlation of the fitness and distance of solutions to the nearest optimum in the search space. When $\varrho = -1.0$, it represents that the fitness and distance to the optimum are perfectly related. In this case, crossover based memetic algorithms can find a local optimum approximating to the global optimum closely. When $\varrho = 1.0$, it indicates that the fitness and distance to the optimum are not related at all. In this case, mutation based memetic algorithms are more preferable. However, there is a shortcoming for FDC as we need to know the global optimum before applying FDC. For most optimization problems, it is impossible to know the global optimum due to high complexity. We use a local optimum obtained by a simple memetic algorithm to approximate the global optimum when calculate the FDC of a fitness landscape. In addition, fitness distance analysis (FDA) is also an important technique for analyzing the correlation between fitness and distance to nearest optimum. FDA has been applied to analyze the fitness landscapes of combinatorial optimization problems [15], [12]. Based on FDA, we get the distribution of local optima in the search space and decide appropriate search method for the proposed memetic algorithm.

C. Local Search Analysis

Here we assume that subcarriers have been allocated to CRUs. we need to allocate bits among subcarriers. A bit allocation solution can be expressed as in (22). Accordingly, we define the fitness function to be

$$f(x) = \exp(M(\frac{\min R_k/\lambda_k}{\max R_k/\lambda_k} - 1))\sum_{n=1}^{N} b_n f(I)f(P), \quad (29)$$

where

$$f(I) = \begin{cases} 1 & \text{when } I \le I_{total} \\ \exp(-M(I/I_{total}-1)) & \text{otherwise} \end{cases}$$

$$f(P) = \begin{cases} 1 & \text{when } P \le P_{total} \\ \exp(-M(P/P_{total}-1)) & \text{otherwise.} \end{cases}$$

with I and P denoting the total interference to PU and total transmit powers among CRUs, respectively. M is an positive number large enough such that even small change can have a great effect on the fitness value.

Let ∇f_i be the fitness gain when adding or decreasing one bit to i^{th} subcarrier. As the distinct characteristics of wireless communication, the complexity of resource allocation algorithms can not be too high. In order to find an optimum,

```
Local−Search Procedure (x = [x_1  x_2  x_N] ∈ X)
repeat
   if f(I) < 1 or f(P) < 1
      find n = arg max ∇f_n ∀n = 1, ··· , N;
   else
      find k = arg max R_k/λ_k;
      find n = arg max ∇f_n , where n ∈ Θ_k;
   endif;
   update x_n;
until ∇f_i ≤ 0;
return x;
```

Fig. 3. Pseudo Code of Local Search Method

Instance	P_m	P_{total}	I_{total}	K
Instance 1	3	0.8	0.006	4
Instance 2	5	1.2	0.009	4
Instance 3	7	1.6	0.012	4
Instance 4	9	2.0	0.015	4
Instance 5	11	2.4	0.018	4
Instance 6	13	2.8	0.021	4

TABLE I
INSTANCES CONSTRUCTION

even a simple greedy local search needs to compare N fitness gain ∇f_i. Suppose Θ_k to be the set of subcarrier index corresponding to the subcarriers assigned to the k^{th} CRU. Based on the fitness definition in (29), we propose a simple yet efficient local search method for the proposed MA. In each search loop, when the total transmit power and interference satisfy the relevant constraints, the algorithm doesn't need to compare every subcarrier's fitness gain and thus is more efficient. The algorithm is based on (1-opt) method and the relevant pseudo code of the algorithm is showed in Fig 3. It is simpler than the original (1-opt) algorithm because it doesn't need to search the subcarriers completely in each loop.

In order to get more insight in the resource allocation problem, we consider 6 instances. These 6 instances are constructed as table I. When designing a memetic algorithm for a combinatorial problem, i.e., determining which local search and which genetic operators are best suited, we need to analyze its fitness landscape. Based on these instance, we use the fitness distance correlation to analyze the distribution of local optima in the search space. Initially, we produce 2000 local optima based on the greedy local search algorithm. In addition, we also estimate the correlation length based on equation (27). The local optima are obtained by the proposed local search method. The results are showed in table II. Where $\min d_{opt}$ denotes the minimum distance of the locally optimal solutions to the expected global optimum, $\bar{d}_{opt}$ is the average distance of the locally optimal solutions to the expected global optimum, $\bar{d}_{loc}$ represents the average distance between the local optima, N_d denotes the number of distinct local optima out of 2000 and ϱ is the fitness distance correlation coefficient.

According to the NK-landscape theory in [12], $K = N -$

Instance	$\min d_{opt}$	d_{opt}	d_{loc}	N_d	ϱ	ℓ
Instance 1	5	11.0145	10.8887	2000	-0.1447	2.3979
Instance 2	4	11.1970	11.1198	2000	-0.0992	2.0019
Instance 3	5	11.1620	10.5577	2000	-0.0112	2.2957
Instance 4	5	10.9370	10.7220	2000	-0.1026	2.2358
Instance 5	5	11.5070	11.6361	2000	-0.0746	2.7233
Instance 6	7	11.7155	10.9887	2000	-0.0820	2.8034

TABLE II

AVERAGE DISTANCE AND FITNESS DISTANCE CORRELATION OF LOCAL SEARCH

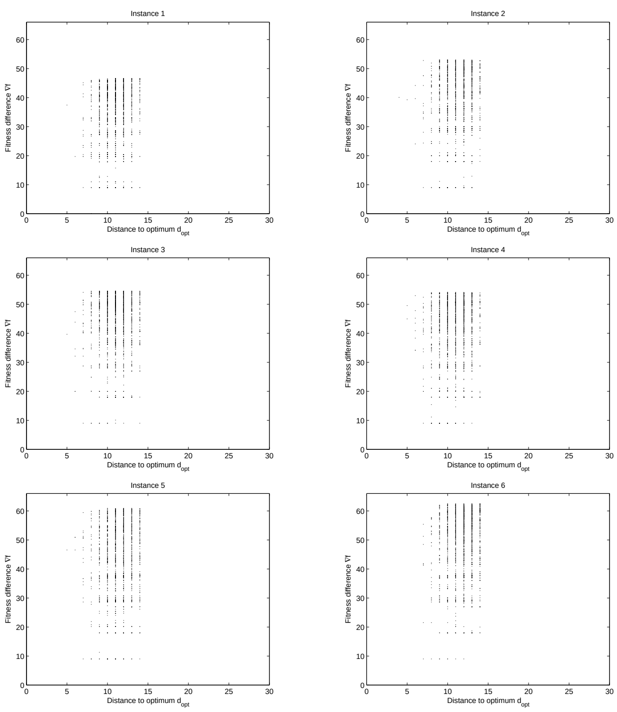

Fig. 4. Fitness Distance Plots for Local Search Method

1 for the bits allocation problem. There is low correlation between neighboring points of the landscape. Therefore, the fitness landscape will be rugged and the number of iteration to reach a local optimum will be small. From the table, since the $\varrho \ll -1$, there is low correlation between fitness and distance. Compared with the number of total subcarriers, ℓ is too small. Therefore, the fitness landscape is rugged. According to the statistical property of $\bar{d}_{opt}$, the local optima are distributed in a large range. The experiment results accord with the NK-landscape theory in [12].

The fitness distance plots for the six instances are shown in Figure. 4. The plots for these instances are similar and the local optima of each instance scatter in large range. These properties accord with the fitness distance correlation analysis in table II. Since the average distance of the population converges fast towards zero when crossover operator is exclusively used in a memetic algorithm. In this case, mutation based memetic algorithms are better than that of crossover based.

D. The Choice of Genetic Operators

For a memetic algorithm, we need to determine not only a good local search method, but also good genetic operators (crossover and mutation). Different experiments have showed that the effectiveness of these evolutionary operators highly depends on the distribution of local optimal in the search space. For the choice of local search method, we have applied FDC and FDA to analyze the problem. From the analysis above, we note that the fitness and distance to the global optimum are highly uncorrelated. Moreover, the distribution of local optimal scatters in a large range. In this case, crossover operator may have little effect on the performance of MAs. Therefore, mutation based MAs will be better in our problem.

V. PROPOSED MEMETIC ALGORITHM FOR BITS ALLOCATION

From the experiment results described above, the local optima are distributed in a large range and the fitness landscape is rugged. In this case, according to the NK-model theory in [12], it is hard to obtain a local optimum close to the global optimum by a suboptimal algorithm. On the other hand, MAs have been shown outperform other traditional algorithms, such as greedy algorithm and tabu algorithm, for combinatorial optimization problems. Based on the fitness landscape analysis on the bits allocation problem, we propose an efficient memetic algorithm for bits allocation of the optimization problem in (7). The pseudo code listings of our algorithm are showed in Fig. 5.

```
Algorithm MA:
  1) Initialize Population P; do
     P = Local_Search(P)
  2) for i=1 to Number_of_Generation do
         S = select_cm(P);
         for j=1 to #crossover do
             select i_a, i_b from S
             for crossover
             i_c = crossover(i_a, i_b);
             i_c = Local_Search(i_c);
         end
         add individual i_c to P;
         for k=1 to #mutation do
             select i_m from S for mutation
             i_m = mutation(i_m);
             i_m = Local_Search(i_m);
         end
         add individuals i_m to P;
         P = select(P);
     end
```

Fig. 5. Pseudo-code for the memetic algorithm

Let x_i be the i^{th} chromosome in a population.

$$\mathbf{x}_i = [x_{i1}, x_{x2}, \cdots, x_{iN}], \; i = 1, 2, \cdots, pop\_size \tag{30}$$

where pop_size denotes the population size. The initial integer solution vectors are randomly created within the region of admissible solutions. The original objective function of the optimization problem is evaluated as in (29).

Genetic Operations:

(1) Crossover: For each pair of parents $\mathbf{x}_1$ and $\mathbf{x}_2$,

$$\mathbf{x}_1 = [x_{11}, x_{12}, \cdots, x_{1p}, x_{1(p+1)}, \cdots, x_{1N}],$$
$$\mathbf{x}_2 = [x_{21}, x_{22}, \cdots, x_{2p}, x_{2(p+1)}, \cdots, x_{2N}],$$

we generate a random integer $p = 1, 2, \cdots, N-1$, using one cut point crossover operator to swap the two parents, to obtain two children

$$\mathbf{x}_1^{'} = [x_{11}, x_{12}, \cdots, x_{1p}, x_{2(p+1)}, \cdots, x_{2N}],$$
$$\mathbf{x}_2^{''} = [x_{21}, x_{22}, \cdots, x_{2p}, x_{1(p+1)}, \cdots, x_{1N}],$$

(2) Mutation: We substitute one bit of an individual randomly by an admissible integer for the selected position.

(3) Replacement Strategy: We select the better chromosomes among parent and offspring with fitness value. The number to be selected is pop_size and we let these chromosomes enter the next generation.

VI. Simulation

In this section, simulation results for the MA algorithm described in Section V are presented. Its performance is compared to that of the RC algorithm in [13] for a number of different scenarios.

Based on the discussion above, we propose a mutation based memetic algorithm for our problem. The parameters are set as: population size=40; generations=20; probability of crossover=0.05; probability of mutation=0.7.

The simulated system consists of one PU and K = 4 CRUs. The CRU band is 5 MHz wide and supports 16 subcarriers, each with a bandwidth, Ws, of 0.3125 MHz. The PU bandwidth is $Wp = Ws$ and the OFDM symbol duration is $Ts = 4\mu s$. Three cases of the bits rate requirements for users with $\lambda = \begin{bmatrix}1 & 1 & 1 & 1\end{bmatrix}, \begin{bmatrix}1 & 1 & 1 & 4\end{bmatrix}$, and $\begin{bmatrix}1 & 1 & 1 & 8\end{bmatrix}$ are considered. It is assumed that the subcarrier gains h_{nk} and g_k , for $n \in \{1, 2, \cdots, N\}, k \in \{1, 2, \cdots, K\}$ are outcomes of independent, identically distributed (i.i.d.) Rayleigh distributed random variables (rvs) with means equal to 1. The additive white Gaussian noise (AWGN) PSD, N_0, is set to 10^{-8} W/Hz. The PSD, $\Phi_{RR}(e^{jw})$, of the PU signal is assumed to be that of an elliptically filtered white noise process. The total CRU bit rate results are obtained by averaging over 1000 channel realizations.

Fig.6 shows the average total CRU bit rate, R_s as a function of the maximum tolerable interference power, I_{total}, with a PU signal power $P_m = 5$ W, $P_{total} = 1$ W and K = 4 for the three cases. As expected, R_s increases with I_{total}. Moreover, when the bit rate requirements for users are closer to uniform distribution, the total bit rate R_s is higher for different maximum tolerable interference power I_{total} because of the user diversity.

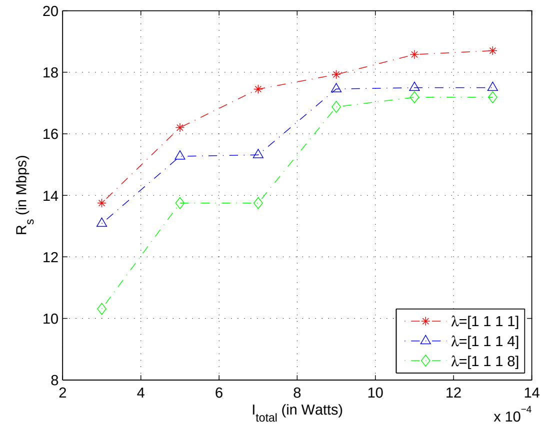

Fig. 6. Average total CRU bit rate, R_s, versus maximum tolerable interference, I_{total}, of the primary user with $P_{total} = 1\ W$ and $P_m = 5\ W$.

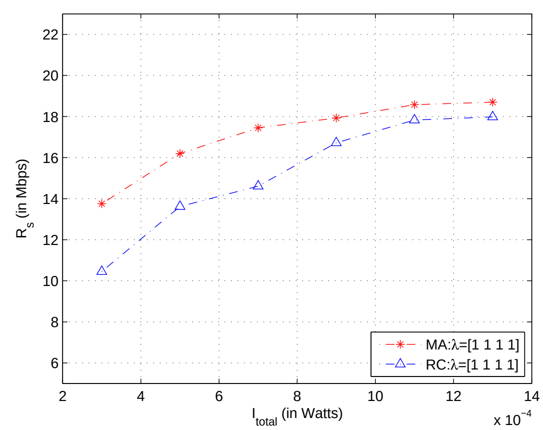

Fig. 7. Average total CRU bit rate, R_s, versus maximum tolerable interference, I_{total}, of the primary user with $P_{total} = 1\ W$ and $P_m = 5\ W$ in the case of $\lambda = [1\ 1\ 1\ 1]$.

Fig. 7 shows the average total CRU bit rate, R_s, as a function of the maximum tolerable interference power, I_{total}, with P_{total} = 1W, P_m = 5W in the case of bit rate requirements $\lambda = \begin{bmatrix}1 & 1 & 1 & 1\end{bmatrix}$. The bit rate obtained by the proposed MA is larger than that of RC algorithm in [13]. In the case of $I_{total} = 0.0003$W, the MA provides a 30% improvement in R_s.

Fig. 8 and 9 shows the average total CRU bit rate, R_s, as a function of the maximum tolerable interference power, I_{total}, with P_{total} = 1W, P_m = 5W in the case of bit rate requirements $\lambda = \begin{bmatrix}1 & 1 & 1 & 4\end{bmatrix}$ and $\lambda = \begin{bmatrix}1 & 1 & 1 & 8\end{bmatrix}$, respectively. The bit rate obtained by the proposed MA in these two cases are a little higher than that of RC algorithm.

VII. Conclusion

The resource allocation problem in a MU-OFDM based cognitive radio system is a combinatorial optimization problem and computational complex. In order to make the problem

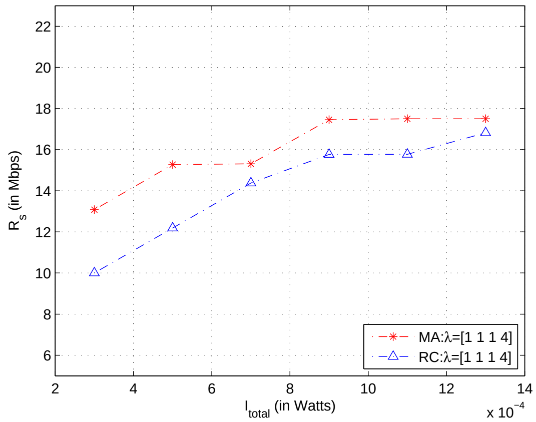

Fig. 8. Average total CRU bit rate, R_s, versus maximum tolerable interference, I_{total}, of the primary user with $P_{total} = 1\ W$ and $P_m = 5\ W$ in the case of $\lambda = [1\ 1\ 1\ 4]$.

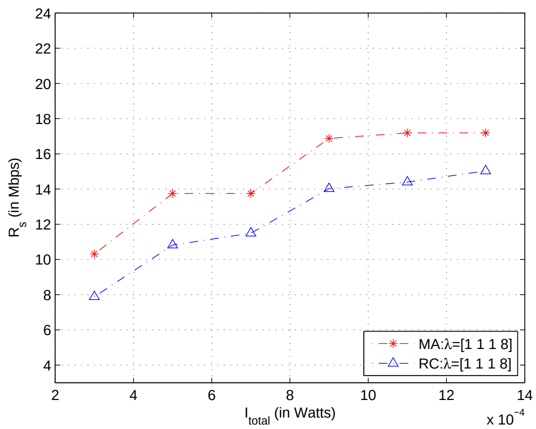

Fig. 9. Average total CRU bit rate, R_s, versus maximum tolerable interference, I_{total}, of the primary user with $P_{total} = 1\ W$ and $P_m = 5\ W$ in the case of $\lambda = [1\ 1\ 1\ 8]$.

tractable, we solve the problem into two steps. Firstly, we propose a simple algorithm SAMA to determine the subcarrier allocation. Then propose an efficient memetic algorithm to determine the bits allocation. On the other hand, the performance of MAs for a given problem is highly dependent on the selection of local search and genetic operators. In order to choose appropriate local search and genetic operators for the proposed MA, we apply fitness landscape to analyze the bits allocation problem. Experiment results show that fitness landscape of the bits allocation problem is rugged, local optima are distributed in a large range and the number of iteration to reach a local optimum is small. In this case, mutation operator will have more effect on the performance of proposed MA than that of crossover operator. We also propose a simple while efficient local search algorithm for the proposed MA. Compared to the existing algorithm in [13], simulation results show that the proposed subcarrier algorithm and memetic algorithm achieve better performance .

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