Application of Joint Source-Relay Scheduling to Cooperative Multiple Access Channels

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Abstract—In this paper, we propose a novel spectrally efficient cooperative transmission protocol for multiple access scenarios. Different to some existing cooperative multiple access schemes, the proposed scheme can exploit the availability of relays as an extra dimension to increase reception robustness. By carefully scheduling the multiple sources and relays’ transmission, a source with a poor connection to the destination can have higher priority to obtain better help from relays. As a result, the full diversity gain can be achievable for each user although only a fraction of the all relays is scheduled to help him. An achievable diversity-multiplexing tradeoff (DMT) is developed for the proposed transmission protocol to assist performance evaluation. With a large number of relays, the DMT achieved by the proposed scheme can approximate the optimal multiple-input single-input upper bound. Both analytical and numerical results show that the proposed protocol can outperform comparative schemes in most conditions.

I. INTRODUCTION

As an alternative of multiple-input multiple-output (MIMO) techniques, cooperative diversity offers an efficient and low-cost way to provide spatial diversity which is effective to combat multipath fading in wireless environments [1]. By encouraging single-antenna nodes to share their antennas cooperatively, a virtual antenna array can be formed and the robustness of reception can be improved without requiring extra hardware cost. Following initial studies on the basic scenario with one source-destination pair [2], [3], more recent studies for cooperative diversity have drifted to more complex and practical communication scenarios. In particular, the multiple access channel (MAC) scenario with multiple source nodes and one common destination has received a lot of attention.

MAC is an important communication scenario and has been recognized as one of the few fundamental building blocks for modern wireless networks [4], [5]. Its application can be found in wireless sensor networks, where a data fusion center collects data from multiple sensors, and in traditional cellular networks, where multiple users are communicating with a base station. In [6] a cooperative multiple access (CMA) transmission protocol was proposed and a similar work based on superimposed modulation can be found in [7]. In general, these existing CMA schemes only exploit cooperation among the source nodes and hence their achievable diversity gain is constrained by the number of the source nodes. Recall that in a wireless network, the number of active users is typically much smaller than the idle ones which can be utilized as relays. Hence to further enhance the system robustness, it is of interest to invite relay nodes into cooperation. However, different to source transmission, relay transmission is to repeat what have been transmitted in the networks, and hence it is desirable to suppress the bandwidth resource consumed by such relay transmission, which motivates the application of network coding to cooperative diversity to improve spectral efficiency of cooperative diversity [8], [9]. However, for the network coded cooperative scheme based on the decode-forward strategy, the computational complexity at the intermediate relay nodes is high due to the operations to decode and encode the received mixture.

In this paper, we propose a new relay-assisted cooperative transmission protocol for multiple access communication scenarios. By carefully scheduling the multiple sources and relays’ transmission, it is ensured that a source with poor connection to the destination can get a higher priority to obtain better help from relays. Hence the maximum diversity gain becomes achievable for each user, although only a fraction of the relays is scheduled to help one user. The non-orthogonal transmission strategy is also applied, where the source transmission and relay forwarding take place at the same time. In this way, the extra bandwidth resource consumed by relay transmission can be suppressed efficiently.

Diversity-multiplexing tradeoff has been widespread used to evaluate spectral efficiency of MIMO techniques. Since cooperative networks can be viewed as a special case of MIMO, it is desirable to utilize the diversity-multiplexing tradeoff as a tool for performance evaluation. Following the definition of the MAC outage events provided in [5], the outage probability achieved by the proposed transmission protocol is studied. Different to the schemes in [2], [6], the fact that the sources and relays are no longer random scheduled results in the difficulty that the addressed channel coefficients are ordered in a certain way. Order statistics has been applied to understand such implicit order structure and obtain closed-form expressions of the outage probability [10]. In specific, an achievable diversity-multiplexing tradeoff is developed for the proposed relay-assisted CMA. For a large number of relays, it can be shown that the DMT achieved by the proposed scheme can approximate the optimal tradeoff achieved by the classical multiple-input single-output scheme. Compared with the CMA schemes in [6], [7], the proposed scheme can achieve better...
DMT at low and intermediate multiplexing gain range, and eventually become the dominant one with a sufficient large number of relays. Monte-Carlo simulation results have been also provided to demonstrate the performance of the proposed transmission protocol.

II. DESCRIPTION FOR THE COOPERATIVE MULTIPLE ACCESS CHANNEL SCHEME

Consider a multiple access channel scenario with \( M \) source nodes, \( L \) relay nodes and one common destination. Each node is constrained by the half duplexing assumption. Time division duplexing is considered here due to its simplicity. All channels considered in this paper are assumed to be identical independent Raleigh fading.

A. Initialization

In this paper, the decode-forward (DF) strategy is used for relay transmission. It is assumed that each relay has the local channel state information (CSI) for the incoming and outgoing channels. Such CSI information can be obtained by asking the \( M \) source nodes and the common destination to broadcast pilot signals. In total, \( M + 1 \) time slots are required for such training signaling. Note that one time slot for training is unavoidable for the DF scenario with a single source-destination pair [2]. Given \( M \) source nodes, \( M \) time slots will be consumed also for the traditional scheme and the proposed transmission strategy only requires one extra time slot due to the broadcasting transmission by the destination.

Similar to [2], it is assumed that each relay can decode the \( m \)-th source information if \(| h_{mr} |^2 \geq \frac{2^{g_K}}{\rho} \), where \( h_{mr} \) is the coefficient for the channel between the \( m \)-th source and the relay \( r \), \( R \) denotes the targeted data rate and \( \rho \) denotes the signal-to-noise ratio. Among the \( L \) relay candidates, assume that there are \( K \) relays which can decode all source nodes’ information, \(| h_{mr} |^2 \geq \frac{2^{g_K}}{\rho} \), \( \forall m \in \{1, \ldots, M\} \). Through the controlling channel, the destination obtains the number of the qualified relays and determine the size of each data frame as \( N = QM \) and \( Q = \lceil \frac{K+1}{M} \rceil \).

B. Cooperative Transmission

Different to the CMA strategies in [6], [7], the \( M \) source nodes are no longer scheduled in the round-robin way. Instead, the source with the poorest connection to the destination will transmit first. In other words, the \( M \) source nodes are scheduled for transmission to satisfy

\[ h_1 \leq h_2 \leq \cdots \leq h_M. \]

where \( h_m \) denotes the coefficient for the channel between the \( m \)-th scheduled source and the destination. Accordingly, the \( K \) qualified relays are scheduled to satisfy

\[ g_1 \leq g_2 \leq \cdots \leq g_K. \]

where \( g_k \) denotes the coefficient for the channel between the destination and the relay scheduled at the \( (K - k + 1) \)-th time slot. The reason for such scheduling is to ensure a source with poorer connection to the destination will get better help from relay transmission. The details of the transmission protocol is following.

At the first time slot of each frame, the source with \( h_1 \) will be scheduled to transmit \( s_1(1) \). All \( K \) qualified relays decode the message and store it in their memory pool. The destination receives

\[ y(1) = h_1 s_1(1) + n(1). \]

At the second time slot, the relay with \( g_K \) (the best relay-destination channel) is scheduled to help the user with the poorest channel condition and forward \( s_1(1) \). Meantime, the source with \( h_2 \) is scheduled to transmit a new message \( s_2(1) \). The destination receives the superimposed observation as

\[ y(2) = h_2 s_2(1) + g_K s_1(1) + n(2). \]

Although the rest \((K-1)\) qualified relays also receive the mixture, they can decode \( s_2(1) \) since they have the prior information for \( s_1(1) \). Hence at the \( n \)-th time slot, the source with \( h_{n'} \) is scheduled to transmit a new message \( s_{n'}(\lceil \frac{n}{M} \rceil) \) where \( n' = (n \mod M) \). At the same time, for \( n \leq K \), the relay with \( g_{K-n+1} \) will be scheduled to transmit its previous observation. For \( n > K \), all qualified relays have been used and hence non-cooperative direct transmission will be adopted.

As a result, the signal model for one data frame can be written as

\[ y = Hs + n, \quad (1) \]

where \( y = \begin{bmatrix} y(1) & \cdots & y(N) \end{bmatrix}^T \), \( s = \begin{bmatrix} s_1(1) & \cdots & s_M(Q) \end{bmatrix}^T \), and the channel matrix is

\[
H = \begin{bmatrix}
1 & 0 & \cdots & \cdots & \cdots & 0 \\
0 & 1 & \cdots & \cdots & \cdots & 0 \\
\ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & 0 & \ddots & \ddots & \ddots & \ddots \\
0 & 0 & 0 & \ddots & \ddots & \ddots \\
0 & 0 & 0 & 0 & \ddots & \ddots \\
\end{bmatrix} N \times N
\]

An example for the signal model with \( K = 4 \) and \( M = 3 \) is shown as following

\[
\begin{bmatrix}
y(1) \\
y(2) \\
y(3) \\
y(4) \\
y(5) \\
y(6) \\
\end{bmatrix} =
\begin{bmatrix}
h_1 & 0 & 0 & 0 & 0 & 0 \\
g_4 & h_2 & 0 & 0 & 0 & 0 \\
0 & g_3 & h_3 & 0 & 0 & 0 \\
0 & 0 & g_2 & h_1 & 0 & 0 \\
0 & 0 & 0 & g_1 & h_2 & 0 \\
0 & 0 & 0 & 0 & h_3 & 0 \\
\end{bmatrix}
\begin{bmatrix}
s_1(1) \\
s_2(1) \\
s_3(1) \\
s_1(2) \\
s_2(2) \\
s_3(2) \\
\end{bmatrix}
\]

III. DIVERSITY-MULTIPLEXING TRADEOFF FOR THE PROPOSED CMA

The aim of this section is to provide analytical performance evaluation for the proposed cooperative protocol by using diversity-multiplexing tradeoff. To facilitate the development of analytical results, some approximations and assumptions will be first made. However, later in this paper, Monte-Carlo simulation will be carried out to demonstrate the performance of the proposed protocol without these assumptions.
A. Approximations and Assumptions

The following lemma provides the high SNR approximation for the relationship between the number of the qualified relays \( K \) and the total number of the relay candidates \( L \).

**Lemma 1:** At high SNR, all relay nodes can be selected as the qualified ones

\[
P(K = L) \rightarrow 1. \tag{2}
\]

**Proof:** Consider the set of the \( L \) source-relay channels \( \{h_{mr_1}, \ldots, h_{mr_L}\} \) associating with the \( m \)-th source node. According to the absolute value of \(|h_{mr}|^2\), we can have \( L \) ordered variables, \(|h_{mr_1}|^2 \leq \ldots \leq |h_{mr_L}|^2\). Denote \( k_m \) as the number of relays which can decode the \( m \)-th source’s information correctly and the probability of the event with \( k_m \) qualified relays can be expressed as

\[
P(k_m = k) = P(|h_{mr(L-k)}|^2 < \gamma, |h_{mr(L-k+1)}|^2 > \gamma), \tag{3}
\]

where \( \gamma = 2^{\rho - 1} \). Since Raleigh fading is addressed here, the variables \(|h_{mr}|^2\) i.i.d. exponential distributed. So by using order statistics [10], the outage probability can be obtained as

\[
P(k_m = k) = \frac{L!}{(L-k)!(k)!} \left[ 1 - e^{-\gamma} \right]^{L-k} \left[ e^{-\gamma} \right]^k. \tag{4}
\]

At high SNR, the use of exponential expansion can result in the following approximation

\[
P(k_m = k) \approx \frac{L!}{(L-k)!(k)!} \gamma^{L-k}. \tag{5}
\]

Obviously we have \( P(k_m = L) \rightarrow 1 \) and the probability for \( k \neq L \) is very small \( P(k_m = k) \rightarrow 0 \), when \( \rho \rightarrow \infty \). Following the same step, it is straightforward to show that at high SNR, all relays can decode the \( M \) sources’ information correctly and the lemma is proved.

As can be seen from the signal model (1), the structure of the channel matrix \( H \) is not regular, which will bring many difficulties to obtain the explicit expression of analytical results. In the following, it is assumed that the number of the relays plus one, \( L + 1 \), is the integral times of the number of sources \( M \), which can simplify the the channel matrix as

\[
H = \begin{bmatrix}
h_1 & 0 & 0 & 0 \\
g_L & h_2 & 0 & 0 \\
\vdots & \ddots & \ddots & \vdots \\
0 & 0 & g_1 & h_M
\end{bmatrix}_{N \times N}. \tag{6}
\]

Later in this paper, Monte-Carlo simulation results will be provided without assuming such a relationship between \( L \) and \( M \).

B. Achievable diversity-multiplexing tradeoff

First we recall the definition of the diversity gain and multiplexing gain as [11]

\[
d = \lim_{\rho \to \infty} \frac{\log[P_e(\rho)]}{\log \rho}, \quad \text{and} \quad r = \lim_{\rho \to \infty} \frac{R(\rho)}{\log \rho}. \tag{7}
\]

where \( P_e \) is the ML probability of detection error. The following theorem provides an achievable diversity-multiplexing tradeoff for the proposed cooperative protocols.

**Theorem 2:** Assume that all addressed channels are i.i.d. Raleigh fading and the number of the relay is \( L = QM - 1 \). The following diversity-multiplexing tradeoff is achievable.

\[
d(r) = (1 - r) + \left[ L - (L + M)r \right]^+, \tag{8}
\]

where \((x)^+\) denote \( \max\{x, 0\} \).

**Proof:** The details of the proof for this theorem will be provided in the next section.

For a fixed data rate, it can be shown that the proposed cooperative multiple access scheme can achieve the diversity gain \( L + 1 \). Since there are only \( L \) relays available to help each source’s transmission, \( L + 1 \) is the maximum diversity gain for the addressed scheme. Although each user only uses a fraction of the available relays, this full diversity gain is still achieved with the help of opportunistic scheduling. Rewrite the achievable diversity-multiplexing tradeoff as

\[
d(r) = \begin{cases} 
QM \left(1 - \frac{Q+1}{Q}r\right), & \text{if } 0 \leq r \leq \frac{L}{L+M} \\
1 - r, & \text{if } \frac{L}{L+M} < r \leq 1
\end{cases}.
\]

Recall that the optimal tradeoff for the traditional multiple-input single-input scheme can be expressed as

\[
d_{MISO}(r) = QM(1 - r), \quad \text{if } 0 \leq r \leq 1.
\]

For a large value of \( L \), the tradeoff achievable for the proposed transmission scheme can approximate the optimal MISO tradeoff, \( d(r) \sim d_{MISO}(r) \). Furthermore, the comparison of the two tradeoff provides us one guideline to improve the spectral efficiency of cooperative transmission. Constrained on a fixed number of the available relays \( L \), it is better to invite a small number of source nodes participating cooperation, which yield a large number of \( Q \) and reduce the performance gap between the proposed protocol and the optimal one. This conclusion is confirmed by the results shown in [12] which proves the fewer sources participating cooperation, the better performance we can achieve.

On the other hand, the diversity-multiplexing tradeoff achieved by the CMA schemes in [6], [7] can be written as

\[
d_{CMA}(r) = M(1 - r), \quad \text{if } 0 \leq r \leq 1.
\]

The advantage of the proposed relay-assisted CMA is its capability to exploit the relay nodes as an extra dimension and enhance the robustness of reception, whereas the diversity gain of the schemes in [6], [7] is constrained by the the number of the source nodes. Such a property is particularly important for the networks where the number of idle users is much larger than the active ones. However, for a large value of the multiplexing gain \( r \), the DMT achieved by the relay-assisted CMA is worse than the ones using the source nodes only in [6], [7]. One possible reason for such performance penalty is that the non-orthogonal transmission strategy is used in this paper. Consider that a source sends a new message \( s_t \) while the relay \( R_i \) is also transmitting. Because of the half duplex constraint,
the relay $R_i$ will miss this source message $s_i$, which makes it difficult for this relay to re-join the cooperative transmission later. On the other hand, the orthogonal transmission strategy is used in [6], [7], which means any node transmitting this time slot will not miss any concurrent transmission and hence can re-join in cooperation later without any difficulty.

C. Numerical Results

The analytical results developed in the previous section requires the approximation $K = L$ and the assumption $L = QM - 1$. In this section, we provide some numerical results based Monte-Carlo simulations and without relying on these assumptions.

In the first experiment, the outage performance of the proposed CMA scheme is compared with the superimposed modulation based CMA in [6], [7] as well as the non-cooperative scheme. The number of the source nodes is set as $M = 2$ and the number of relays is fixed as $L = 5$ (or $Q = 3$). In the next experiment, the performance of the scheme with varying number of relays will be studied. The targeted data rate is set as $R = 4$ and $R = 6$ bits per channel use (BPCU). By increasing the targeted data rate, the performance of all studied schemes is decreasing as shown in Fig. 1. And at all SNR and the targeted data rate, the proposed CMA scheme can achieve smaller outage probability compared with the two comparable schemes. The reason for this performance gain is that the comparable cooperative schemes can not exploit the available relays and their diversity gain is constrained by the number of sources.

In the second experiment, the outage probability achieved by the proposed scheme is shown in Fig. 2 as a function of SNR and with different number of the relay candidates $L$. The number of the source nodes is set as $M = 2$. As can be seen from the figure, the robustness of the multiple access transmission can be improved by increasing the number of the relays. As indicated by Theorem 2, a large number of the relays not only increases the diversity gain, but also improve the spectral efficiency of the proposed CMA scheme since the tradeoff achievable for the proposed scheme is approaching to the optimal MISO upper bound.

IV. PROOF FOR THEOREM 2

As discussed in [11], the ML probability can be tightly bounded by the outage probability at high SNR, and hence the outage probability will be focused in the following. Furthermore, $f(\rho)$ is said to be exponentially equal to $\rho^d$, denoted as $f(\rho) \approx \rho^d$, when

$$\lim_{\rho \to \infty} \frac{\log f(\rho)}{\log \rho} = d. \quad (9)$$

Since the multi-user scenario is considered; hence the outage event can be defined as following [5]

$$O_A \triangleq \bigcup_A O_A, \quad (10)$$

and the outage probability of the cooperative network can be written as

$$P(O) = P\left( \bigcup_A O_A \right) \leq \sum_A P(O_A), \quad (11)$$

where

$$O_A \triangleq \left\{ I(s_A; y|s_A^c, H = H) < Q \sum_{i \in A} R_i \right\}. \quad (12)$$

$A \subseteq \{1, \ldots, M\}$ and $|A|$ is denoted as the number of users in $A$. Note that the symmetric tradeoff is of interest in this paper, which means $|A|R = \sum_{i \in A} R_i$. From the definition, the mutual information for each outage event can be written as

$$I_A = \log \det \left[ I_N + \rho \mathbf{H} A \mathbf{H}^H \right]. \quad (13)$$
where $I_A \triangleq I(s_A; y|s_A^c, \mathbf{H} = \mathbf{H})$, and $\Gamma_A$ is formed by removing the columns of the identity matrix $I_N$ corresponding to the user in $\mathcal{A}$. Take $\mathcal{A} = \{i\}$ as an example. The $Q$ elements on the diagonal of the matrix $\Gamma_A$ are set as one, $\Gamma_A((q-1)Q + i, (q-1)Q + i) = 1$ for $q \in \{1, \ldots, Q\}$, and all other elements are zero.

According to the elements containing in the set $\mathcal{A}$, the outage events can be classified as the following three categories.

- The first category, denoted as $\mathcal{O}_1$, is caused by the set containing only one user, $|\mathcal{A}| = 1$. The following lemma provides an upper bound for the outage probability of this type of outage events.

**Lemma 3**: Assume that all addressed channels are i.i.d. Raleigh fading and the number of the relay is $L = QM - 1$. The outage probability for the event $\mathcal{O}_1$ can be upper bounded as

$$P(\mathcal{O}_1) \leq \rho^{-d_{\mathcal{O}_1}(r)},$$

where $d_{\mathcal{O}_1}(r) = 1 - r + (L - (L + M)r)^+$. 

**Proof**: See Appendix.

- The second type of the outage event, denoted as $\mathcal{O}_2$, is caused by the set $\mathcal{A}$ containing all $M$ users, $|\mathcal{A}| = M$, which is corresponding to the sum rate. The following lemma provides an upper bound for the outage probability associated with such an event.

**Lemma 4**: Assume that all addressed channels are i.i.d. Raleigh fading and the number of the relay is $L = QM - 1$. The outage probability for the event $\mathcal{O}_2$ can be upper bounded as

$$P(\mathcal{O}_2) \leq \rho^{-d_{\mathcal{O}_2}(r)},$$

where $d_{\mathcal{O}_2}(r) = M(1 - r) + (L - (L + 1)r)^+$. 

**Proof**: See [13].

- The third type, denoted as $\mathcal{O}_3$, is caused by the set $\mathcal{A}$ containing more than one users, but only an incomplete set of all $M$ users, $1 < |\mathcal{A}| < M$. Basically $\mathcal{O}_3$ includes all outage events excepted the ones in $\mathcal{O}_1$ and $\mathcal{O}_2$. To get a closed-form expression of the probability for $\mathcal{O}_3$ is difficult, and we only prove that the outage probability for $\mathcal{O}_3$ can be upper bounded by the one with $\mathcal{O}_1$. The following lemma will be needed for the development of $P(\mathcal{O}_3)$.

**Lemma 5**: Consider a $(P + 1) \times P$ complex-valued matrix, denoted as $\Theta_P$. Its $(m, m)$-th element is $h_m$ and its $(m + 1, m)$-th element is $s_{mn}$ for $m \in \{1, \ldots, P\}$. The rest elements of this matrix are zero. The following inequality holds

$$\det \{ I_P + \rho \Theta_P^H \Theta_P \} \geq \prod_{i=1}^{P} \rho |h_i|^2 + \prod_{j=1}^{P} (1 + \rho |g_j|^2)^2.$$

**Proof**: See [13].

By using this lemma, it can be shown that the outage probability of $\mathcal{O}_3$ is always smaller than the probability for $\mathcal{O}_1$.

**Lemma 6**: Assume that all addressed channels are i.i.d. Raleigh fading and the number of the relay is $L = QM - 1$. The outage probability for the event $\mathcal{O}_3$ can be upper bounded as

$$P(\mathcal{O}_3) \leq P(\mathcal{O}_1) \leq \rho^{-d_{\mathcal{O}_1}(r)}.$$

**Proof**: See [13].

Comparing the outage probability caused by the three types of events, the one associated with $\mathcal{O}_1$ is the dominant one. As the result, the overall outage probability can be upper bounded as

$$P(\mathcal{O}) \leq \sum_{\mathcal{A}} P(\mathcal{O}_\mathcal{A}) \leq \rho^{-d_{\mathcal{O}_1}(r)},$$

and Theorem 2 is proved.

**V. Conclusion**

In this paper, we have proposed a spectrally efficient cooperative transmission protocol for multiple access scenarios. By carefully scheduling the multiple sources and relays’ transmission, the full diversity gain can be achievable for each user although only a fraction of relays is utilized by each user. An achievable diversity-multiplexing tradeoff (DMT) was developed by applying order statistics. With a large number of relays, the DMT achieved by the proposed scheme can approach the optimal multiple-input single-input upper bound. Both analytical and numerical results demonstrated that the proposed protocol can outperform comparative schemes in most conditions.

**References**


