Approaching MISO Upper Bound: Design of New
Wireless Cooperative Transmission Protocols

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Abstract—While various cooperative protocols have been developed for the simple scenario with one source-destination pair, most of them still suffer a significant loss compared with the optimal multiple-input single-output (MISO) upper bound. The diversity-multiplexing tradeoff will be used as the criterion for performance evaluation. In this paper, we propose two new half-duplex decode-forward cooperative transmission protocols, whose performance can approach the optimal MISO bound, and achieve a better diversity-multiplexing tradeoff when compared with existing cooperative protocols, particularly for large multiplexing gains. Firstly, a simple protocol of cooperative transmission is devised by combining opportunistic strategies with non-orthogonal transmission. When the number of relays is large, the proposed opportunistic decode-forward cooperative protocol can approach the optimal MISO upper bound. Due to the relay-to-relay constraint, each relay can only be used once, which limits the achievable diversity gain. Such an observation motivates our second transmission protocol which can further push the performance of cooperative transmission close to the optimal upper bound. Secondly, a relaying protocol is proposed for a four-node network where two multiple-antenna relays alternately forward messages to the destination when they can successfully cancel the inter-relay interference using the zero forcing method. Monte-Carlo simulation has also been provided to demonstrate the performance of both protocols and comparable ones.

Index Terms—Cooperative transmission, diversity-multiplexing tradeoff, half-duplexing constraint, opportunistic decode-forward, relay-reuse decode-forward, zero forcing

I. INTRODUCTION

RECENTLY cooperative transmission has received a lot of attention due to its superior capability to combat multipath fading and enhance reception reliability. By allowing multiple single-antenna users to cooperate with each other, a virtual antenna array can be constructed. And hence spatial diversity can be obtained in a low-cost and effective way, which motivates its wide applications for various energy-constrained communication scenarios, such as wireless sensor networks. Diversity-multiplexing tradeoff (DMT) has been recognized as an effective information-theoretic criterion to evaluate the spectral efficiency of cooperative transmission protocols since cooperative networks can be viewed as a special case of multiple-input multiple-output (MIMO) systems [1], [2].

Related work. Due to the half duplexing constraint, the relay "transmit" phases are only a small portion of the frame period in [3]–[6]. For the classical orthogonal relaying transmission schemes in [3], [4], it has been shown that no diversity gain can be achieved for $\frac{1}{2} \leq r \leq 1$ where $r$ denotes the multiplexing gain. Motivated by such inefficiency, non-orthogonal transmission strategies have been introduced for cooperative networks [5], [6]. As shown in [6], the use of non-orthogonal transmission can ensure the achievable DMT better than that of direct transmission for all multiplexing gain. However, compared with the optimal MISO bound, the diversity gain achieved by the cooperative protocols in [5], [6] is only a small portion of the optimal one, particularly for $\frac{1}{2} \leq r \leq 1$. The cooperative scheme in [7] is one of the latest work to consider non-orthogonal strategies in the single-relay scenario, which can further enhance the DMT performance.

It was shown in [8] that the spectral efficiency of cooperative diversity can be improved by extending the relay "transmit" duration, which can obviously be realized by a spatial reuse of the relay-destination channels. To combat the half-duplexing constraint, the relay reuse can be implemented by arranging more than one relay to alternatively take turns to transmit messages received from the source in [8]–[15]. The upper bound of DMT of the cooperative scheme in [8] can be only achievable with the assumption that multiple relays are isolated from each other. For a more general scenario where the relay-to-relay link is considered, the works in [9]–[15] proposed to reuse relays by addressing the problem of inter-relay interference cancelation. The work in [9] proposed a scheme where inter-relay interference has been treated as a noise and two relays alternatively transmit messages received from the source using either amplify-and-forward (AF) or decode-and-forward (DF) scheme. As a result, such proposed methods cannot perform well for a strong inter-relay channel. The protocols in [10]–[12] utilize the concepts of dirty paper coding (DPC) and hybrid joint decoding to cancel inter-relay interference, which results in high computational complexity in comparison with suboptimal techniques such as linear precoding. The work in [13] makes some improvement to the system performance by making assumptions to the condition of inter-relay channels. The strategy in [14] is to partially subtract inter-relay interference, which can achieve the full diversity order by a simple subtraction operation, but could result in the accumulation of inter-relay interference after
several time slots at the destination terminal. The idea in [15] is one of the latest work to consider uplink transmission in the cellular system. This protocol named shifted successive DF relaying is based on successive relaying.

**Contribution.** The aim of this paper is to design a spectrally efficient cooperative transmission protocol with a better DMT, in particular at large multiplexing gain $\frac{1}{2} \leq r \leq 1$. In this paper, we consider two different DF relaying scenarios where the direct link from the source to the destination is included without the destination-source feedback, and a general inter-relay interference is contained, which is not assumed to be sufficiently strong or weak.

To enlarge the relay “transmit” phases, our first protocol is proposed for the first scenario with multiple single-antenna relays; the basic idea is to combine the opportunistic relaying strategy [16] with non-orthogonal transmission [5], [6], which yields the proposed opportunistic decode-forward protocol (ODF) with partial (the first hop) relay selection [17], [18]. To be more specific, a relay will participate in cooperative transmission only if it can decode source information correctly. While the source keeps transmitting a new message at each time slot, each qualified relay will take turns to forward its received messages to the destination. Similar protocols using the AF strategy can be found in [19], [20], where relay selection has not been considered and all relays will be invited for cooperation regardless of their channel conditions. Therefore the use of such protocols means that a relay with a bad connection to the source could be used, and hence bandwidth resource, such as time slots, will not be utilized efficiently. On the other hand, for the proposed protocol, relay selection has been applied in order to only schedule the relays which is helpful to improve reception reliability. The developed analytical result shows that the proposed protocol can approach the MISO upper bound by increasing the number of relays. Furthermore, such an achievable DMT is superior to the ones proposed in [6] and comparable to the upper bound developed in [8].

Each relay in the ODF scheme can only be used once due to the inter-relay interference limitation, and the number of relay “transmit” time slots is constrained by the relay number. To the best knowledge of authors’ knowledge, no existing approaches can effectively separate multiple unknown streams with single-antenna nodes, but many methods can be utilized to solve this problem for a multiple-antenna node. The joint maximum likelihood (ML) decoding can attain the highest diversity performance, but its high complexity makes it less attractive in comparison with the linear ones, such as zero forcing (ZF) and minimum mean square error (MMSE) strategies. Both of the two methods have the same performance at high signal-to-noise ratio (SNR), but the diversity analysis of ZF is more tractable. Such observations motivate our second protocol for the second scenario with two multiple-antenna DF relays which is suitable for cellular networks where Base-stations can be relays. Beyond the opportunistic relaying strategy and non-orthogonal transmission, ZF criterion and antenna selection are utilized at each relay. At both relays, the linear detection method based on ZF approaches has been used, which reduces the computational complexity compared to [15] which uses more complicated successful decoding. And each relay transmitter will choose the best antenna to forward messages to the destination, which can achieve full diversity gain while avoiding complicated synchronization and saving transmission power. The DMT achieved by the proposed relay-reuse protocol has been developed to show that the proposed scheme can achieve the diversity gain larger than ODF and can asymptotically achieve the optimal DMT of this system.

**Organization.** This paper is organized as follows. Section II describes two proposed cooperative transmission protocols for each scenario. Section III outlines the DMT performance and Monte-Carlo simulation results for each protocol. Section IV provides the development of each tradeoff, using several mathematical results in Appendix. Section V offers some concluding remarks.

## II. TWO COOPERATIVE TRANSMISSION PROTOCOLS

Consider a cooperative communication scenario with one source-destination (S, D) pair and multiple relays, where all nodes are under the half-duplex constraint. All wireless channels are assumed to be identical independent flat Rayleigh fading and quasi-static (i.e. constant for a sufficiently long time). S and D are equipped with single antenna, and different relay types are considered in different scenarios.

We will firstly describe the optimal relaying scheme for a virtual multiple-relay scenario. Then we will propose a simple protocol that can be utilized in the scenario with single-antenna relays, called *Opportunistic Decode-Forward Protocol* (ODF). While ODF can approach the optimal MISO bound, it still suffers some loss of diversity gains due to the inter-relay interference, which motivates our second protocol. Specifically we will focus on the scenario with multiple-antenna relays, and another protocol called *Relay-Reuse Decode-Forward* (RRDF) will be proposed to further improve the system performance.

### A. An Impractical Optimal Relaying Scheme

Prior to the descriptions of the proposed protocols, we firstly consider an impractical relaying scheme for the addressed communication scenario, which can be served as the motivation of our proposed protocols. Specifically the best relay with the strongest relay-destination channel is first selected from all the relays which can decode source information correctly. The frame length will be $P$ and at the $p$-th time slot, the best relay forwards the source message sent by the source at the $(p - 1)$-th time slot, and the source sends a new message at the same time. However, such a scheme with dual-hop relay selection is impractical due to the half-duplexing constraint. Specifically the best relay will miss the new message sent by the source at the $p$-th time slot due to the half-duplexing constraint, and hence it cannot forward such a message during the following time slot. Or in the other words, the best relay can only be used once. As a result, the length for the whole frame will be quite short, which reduces the achievable diversity-multiplexing tradeoff. Such an observation motivates our works to design practical relaying schemes with enhanced diversity-multiplexing tradeoff, which can be detailed in the two subsequent subsections.
B. Opportunistic Decode-Forward Protocol Description

For the first scenario, there are \( N \) relays equipped with a single antenna. The channel between the \( i \)th relay and the source is denoted as \( h_i \), where the source-destination channel is defined as \( g \). The channel between the \( i \)th relay and the destination is defined as \( q_i \).

The proposed protocol consists of two stages, initialization and data transmission respectively. During the initialization stage, the source will broadcast training information and each relay will estimate its local incoming channel coefficient \( h_i \). Based on such local channel information, each relay calculates its instantaneous mutual information \( \log[1 + \rho|h_i|^2] \), and decides whether it can decode source messages correctly\(^1\). Here \( \log(\cdot) \) is taken to base-2, and \( \rho \) denotes SNR. Only those relays with the mutual information larger than the target data rate will participate in cooperative transmission. It is assumed that the source has the access to the number of the qualified relays, denoted as \( K, 0 \leq K \leq N \). Such an assumption can be justified for time-division duplex systems where the source can have the access to the information for the one-hop source-relay channels. An alternative is to ask the relays to feedback one digit indicator, which is less challenging compared to feedback complex-value channel information. In total, \( K + 1 \) time slots are required for such an initialization stage.

For the data transmission stage, each data frame consists of \( K + 1 \) information\(^2\) bearing messages\(^3\) denoted as \( s = [s(1) \ldots s(K + 1)]^T \). The source keeps transmitting the message \( s(k) \) at each time slot. At the same time, the \( K \) qualified relays will take turns to forward the message transmitted by the source at the previous time interval to the destination. As in [5], [6], perfect time synchronization is assumed here for such non-orthogonal transmission. The details of the proposed protocol is described as following.

At the first time slot, the source broadcasts the message \( s(1) \) where the destination and all relays listen. Among \( N \) relaying candidates, assume that there are \( K \) relays whose incoming channels satisfy the condition \( \log[1 + \rho|h_i|^2] \geq R \). Here \( R \) denotes the targeted data rate. Hence these \( K \) relays can correctly decode the message \( s(1) \) and store it in their memory. At the \( k \)th time slot, the source transmits a new message \( s(k) \), and at the same time one of the \( K \) qualified relays will forward the previous message \( s(k-1) \). Hence the destination receives \( y(k) = gs(k) + y_{k-1}s(k-1) + n(k) \), and the rest qualified relays which have not transmitted will receive the mixture of two messages \( s(k) \) and \( s(k-1) \). However, these relays have the perfect knowledge of the message \( s(k-1) \) since these nodes have been listening for the previous time slots and it is assumed that their connection with the source is good enough for correct decoding. Hence the relays can decode the new message \( s(k) \) correctly and store it for the use of the next time slot. Transmission scheduling of the \( K \) qualified relays can be done by a simple round-robin scheduler or the distributed scheduling mechanism proposed in [4].

Note that the channel state information between relays is important for the proposed protocol. Two types of mechanisms can be used to obtain the channel information between relays. One is to apply first-order statistics-based channel estimation methods [21], which is to use superimposed training information for reliable channel estimation or equalization. It is interesting to point out that such training information has been provided by the transmission strategy since the relays have the priori information about the messages previously sent by the source. So first-order statistics-based algorithms can be implemented without extra consumption of bandwidth resource [22]. The other mechanism is to ask each of the \( K \) qualified relays to broadcast a training message prior to data transmission, which yields reduced computational complexity but causes about \( K \) extra time slots.

By stacking over \( (K + 1) \) time slots, the signal model for the proposed cooperative decode-forward transmission protocol can be written as \( y = \sqrt{E_s}H_Ks + n \), where \( y = [y(1) \ldots y(K + 1)] \) denotes the observation vector, \( n = [n(1) \ldots n(K + 1)] \) is the Gaussian additive noise vector, the channel matrix can be written as

\[
H_K = \begin{bmatrix}
g & 0 & \cdots & 0 
g_1 & g & \cdots & 0 
\vdots & \vdots & \ddots & \vdots 
0 & \cdots & g_K & g 
\end{bmatrix}_{(K+1) \times (K+1)}
\]

Hence conditioned on the fact that there are \( K \) qualified relays, the mutual information the proposed cooperative protocol can support is

\[
I_K = \frac{1}{(K + 1)} \log \det[I_{(K+1)} + \rho H_K H_K^H],
\]

where the factor \( \frac{1}{(K + 1)} \) is due to the fact that communication happens in \( (K + 1) \) successive time slots. Note that the achievable rate of \( I_K \) is conditioned on the use of random Gaussian codes of infinite length. Apparently, this ODF protocol can also be extended to a multiple-antenna relays scenario straightforwardly. As shown in the next section, the gap between the diversity-multiplexing tradeoff achieved by ODF and the optimal MISO upper bound could be still large if there are only a small number of relays. Therefore it is of interest to study how to approach the upper bound given a limited number of relays, which motivates the following proposed cooperative protocol.

C. Relay-Reuse Decode-Forward Protocol Description

For the second scenario, we assume that there are only two relays for simplicity, which are denoted as \( R_1 \) and \( R_2 \). Each relay is equipped with \( M_r \) antennas, where \( M_r \geq 2 \), and every antenna at each relay is marked with a distinct index number from 1 to \( M_r \). To reduce complexity, only the best antenna of each relay is used to transmit messages during each frame, and the indexes for the used antennas are denoted as \( r_1 \) for \( R_1 \), \( r_2 \) for \( R_2 \), where \( r_1, r_2 \in \{1, 2, \ldots, M_r\} \). As shown in

\( \text{\footnote{1}It is assumed that the source message can be decoded successfully by a relay if its instantaneous mutual information is larger than the targeted data rate. Similar assumptions have been used in [3].} \)

\( \text{\footnote{2}After a initialization, the length of one data frame is fixed by the source as \( K + 1 \) and doesn’t change until the next initialization. For the quasi-static channels, the number of time slots between two contiguous initializations is sufficiently large.} \)

\( \text{\footnote{3}Each message could be more than one symbol, or even a huge data packet. But to simplify the performance analysis, we assume that each message contains only one symbol.} \)
Fig. 1, denote the channel vector from single-antenna $S$ to multiple-antenna $R_i$, as a column vector $h_{R_i}$, where $i = 1, 2$, and the source-destination channel is defined as $g$. The channel coefficient vector from $R_i$ to single-antenna $D$ is denoted as a column vector $g_i = (g_{i,1}, \cdots, g_{i,M_r})^T$. The inter-relay channel vector from the $m$th antenna of $R_i$ to the other relay is described as a column vector $h_{i,m}$.

![Diagram](a) When $S$ and $R_1$ transmit signals. (b) When $S$ and $R_2$ transmit signals.

Fig. 1. The second system model with two multiple-antennas relays, where $M_r = 2$.

The proposed protocol consists of two stages for initialization and transmission respectively. After the initialization stage, each relay is assumed to have the access to the local channel state information (CSI). The details about how to obtain the required CSI will be provided at the end of this section. Depending on the channel condition, the number of the relays we can use is different and accordingly the frame length\(^4\) will be also different. The criterion for the relay selection is $\log(1 + \rho |h_{R_i}|^2) > R_0$ where $| \cdot |$ denotes the Euclidean norm of a vector, and $R_0$ is the number of bits containing in each codeword transmitted by the source. Note that $R_0$ is not the data rate. The data rate for a particular situation will be equal to $\frac{R_0 \times N_x}{N_t}$, where $N_x$ is the number of the transmitted codewords and $N_t$ is the number of the used time slots. Apparently the average data rate calculated from $R_0$ will be equal to the expected data rate $R$. We will provide more discussions about the relationship between the two rates later in this paper.

The flow chart of RRDF is shown in Fig.2. For the data transmission stage, during the first time slot, the source broadcasts the message $s(1)$ and the other nodes listen. Denote $A_j$ as the situation with $j$ qualified relays which can decode $s(1)$ correctly, where $j = 0, 1, 2$. After the first time slot, different transmission strategies will be used dependent on the channel condition. For the situation $A_0$, we can only adopt direct link for transmission. For the situation $A_1$, the qualified relay will transmit $s(1)$ during the second time slot while the source remains silent, which means the data rate per channel use is $R_0/2$.

When both relays can decode the source messages, different local incoming source-relay/inter-relay channel condition causes different numbers of successful relays which can correctly decode source message mixed by inter-relay interference using zero-forcing criterion. Denote $E_k$ as the event with $k$ successful relays, $k = 0, 1, 2$. A relay will use ZF decoding only if it can satisfy the required ZF condition $\gamma^{(i)}_2 \geq 2R_0 - 1$, where the definition of $\gamma^{(i)}_2$ and operation details will be provided in Appendix. For the event $E_0$, during the second time slot, $R_1$ transmits $s(1)$ while $S$ keeps transmitting new message $s(2)$, and $R_2$ listens. During the third time slot, $R_2$ transmits $s(2)$ while $S$ and $R_1$ remain silent. The transmission rate for such an event is $\frac{3}{2}R_0$. For the event $E_1$, consider that $R_1$ is the only successful one to cancel inter-relay interference without loss of generality. The same transmission scheduling as $E_0$ is adopted during the first two time slots. The difference is that, during the third time slot, $R_2$ transmits $s(2)$ while $S$ keeps transmitting a new message $s(3)$ and $R_1$ separates the messages from the source and the other relay using ZF decoding. So during the fourth time slot, $R_1$ will be reused once to transmit the message $s(3)$ while $R_2$ and $S$ remain silent. Then one frame terminates due to the fact that $R_2$ is not able to be reused. The transmission rate for such an event is $\frac{3}{2}R_0$. For the event $E_2$, both relays can be reused infinite times until the channel state varies, which means that the frame length $L$ can be arbitrary large as long as the channel state does not vary. Under this condition, $S$ will keep transmitting a new message $s(n)(n \geq 2)$ at each time slot, so $L$ codewords will be transmitted during each frame and the transmission rate for such an event is $R_0$. At the same time, the two relays take turns to forward the source messages to the destination. It can be easily evaluated the mutual information for each situation, which is provided in TABLE I, where $L \geq 3$ and is assumed to be a odd number.

The required CSI can be obtained by requesting the source and the destination to broadcast training signals. Firstly, the source will broadcast training information and each relay will estimate its local incoming channel coefficient vector $h_{R_i}$, where $i = 1, 2$. Then each relay will calculate its instantaneous mutual information $\log(1 + \rho |h_{R_i}|^2)$, and decide whether it can
decode source messages correctly. Only those relays whose mutual information is larger than $R_0$ will participate in the cooperative transmission. Secondly, the destination will broadcast training information and each qualified relay will estimate the relay-destination channel gain vector $g_i$. Based on the calculation of such channel information, $R_i$ will know which element in $g_i$ has the largest magnitude, and then determine the transmittal antenna index as $r_i = \arg \max_{r=1, \ldots, M_c} |g_{i,r}|^2$. Furthermore, the inter-relay channel coefficient vector $h_{i,r_i}$ can be obtained by asking both relays additionally broadcast training singles in turns using the best antenna. With the knowledge of such source-relay/inter-relay channel information, each relay can calculate whether it can satisfy the ZF condition. Finally, each relay broadcasts a two-bit digit indicator to inform the source whether it is qualified and able to successfully cancel the inter-relay interference, so that the source can ascertain the system situation and determine the number of transmission time slots in one data frame. Provided that there are more than two relays, a practical way to implement the proposed protocol is to first select the best two among all the relays, which can further reduce the system overhead consumed by the required CSI assumptions.

### III. Diversity-Multiplexing Tradeoff and Numerical Result

In this section, the achievable DMT and numerical results of each proposed protocol will be given. Recall the diversity and multiplexing gain which can be defined as [1]

$$d \triangleq - \lim_{\rho \to -\infty} \frac{\log[P_e(\rho)]}{\log \rho} \quad \text{and} \quad r \triangleq \lim_{\rho \to -\infty} \frac{R(\rho)}{\log \rho},$$  

(3)

where $P_e$ is the ML probability of detection error and $R$ is the average data rate per channel use. Following the similar steps in [1], [2], the outage probability will be studied since the ML error probability can be tightly bounded by the outage probability at high SNR.

### A. Performance of ODF in the First Scenario

The following theorem provides the DMT for the proposed transmission protocol.

**Theorem 1**: The achievable DMT for the ODF protocol in the first scenario can be written as

$$d_{\text{ODF}}(r) = (1 - r) + (N - (N + 1)r)^+. \quad (4)$$

*Proof: refer to Section IV-A.*

#### Two Remarks for Theorem 1:

1. **Comparison with the schemes in [6]:** Recall that the DMT achieved by the non-orthogonal amplify-forward (NAF) scheme with $N$ relays is $d_{\text{NAF}}(r) = (1 - r) + N(1 - 2r)^+$. For the case of $N = 1$, the DMT achieved by the two schemes is the exactly same. For the case of $N \geq 1$, the proposed scheme outperforms the NAF scheme $d_{\text{ODF}}(r) \geq d_{\text{NAF}}(r)$, for all $0 \leq r \leq 1$. Such performance gain is due to the fact that for the proposed scheme no intermission is needed between two contiguous relay “transmit” time slots.

Recall that the expression for the DMT achieved by the dynamic decode-forward (DDF) scheme [6] with $N$ relays can be shown as

$$d_{\text{DDF}}(r) = \begin{cases} 
\frac{(N + 1)(1 - r)}{r} & \text{if } \frac{1}{N + 1} \geq r \geq 0, \\
1 + \frac{N(1 - 2r)}{1 - r} & \text{if } \frac{1}{2} \geq r \geq \frac{1}{N + 1}, \\
\frac{1 - r}{r} & \text{if } 1 \geq r \geq \frac{1}{2}.
\end{cases}$$

(5)

With some algebraic manipulations, it can be shown $d_{\text{ODF}}(r) \geq d_{\text{DDF}}(r)$, for $\frac{1}{N + 1} \leq r \leq \frac{N + 1}{N}$. Provided that $N \geq 1$, we can have the following approximation $\frac{1}{N + 1} \approx 0$ and $\frac{N + 1}{N} \approx 1$, and hence the range $\frac{1}{N + 1} \leq r \leq \frac{N}{N + 1}$ becomes the dominated one. Such superior performance for the large number of $N$ is important for many application, such as density sensor networks or traditional cellular networks where many idle users sit around the active one.

2. **Comparison with the schemes in [8]:** Recall that the upper bound of DMT for the $N$-relay $M$-slot sequential amplify-forward (SAF) scheme can be expressed as $d_{\text{SAF}}(r) = (1 - r) + N \left(1 - \frac{M}{M + r} \right)^+$. Hence conditioned on $M \leq N + 1$, it can be shown that $d_{\text{ODF}}(r) \geq d_{\text{SAF}}(r)$ for all $0 \leq r \leq 1$. Notice that the upper bound of the DMT of SAF is only achievable with the assumption of that relays are isolated with each other, which could be difficult in practice. Hence the proposed ODF scheme could be more easily implemented compared with the SAF scheme.

To facilitate performance evaluation, the DMT achieved by various cooperative schemes is shown in Fig. 3. As can be

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### TABLE I

<table>
<thead>
<tr>
<th>Each Situation</th>
<th>Mutual Information</th>
<th>Channel Vector or Channel Matrix</th>
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<tbody>
<tr>
<td>Situation $A_0$</td>
<td>$I_{A_0} = \log(1 + \rho</td>
<td>g</td>
</tr>
<tr>
<td>Situation $A_1$</td>
<td>$I_{A_1} = \frac{1}{2} \log \det[I_2 + \rho h_{A_1} h_{A_1}^H]$</td>
<td>$h_{A_1} = (g, g_{r_1}, g_{r_2})^T$</td>
</tr>
<tr>
<td>Situation $A_2$ Event $E_0$</td>
<td>$I_{A_2, E_0} = \frac{1}{2} \log \det[I_2 + \rho h_{A_2, E_0} h_{A_2, E_0}^H]$</td>
<td>$H_{A_2, E_0} = \begin{pmatrix} g &amp; 0 &amp; 0 \ g_{r_1} &amp; g &amp; 0 \ 0 &amp; 0 &amp; g_{r_2} \end{pmatrix}$</td>
</tr>
<tr>
<td>Situation $A_2$ Event $E_1$</td>
<td>$I_{A_2, E_1} = \frac{1}{2} \log \det[I_2 + \rho h_{A_2, E_1} h_{A_2, E_1}^H]$</td>
<td>$H_{A_2, E_1} = \begin{pmatrix} g &amp; 0 &amp; \cdots &amp; 0 \ g_{r_1} &amp; g &amp; \cdots &amp; 0 \ \vdots &amp; \vdots &amp; \ddots &amp; \vdots \ 0 &amp; 0 &amp; g_{r_2} &amp; g \end{pmatrix}$</td>
</tr>
<tr>
<td>Situation $A_2$ Event $E_2$</td>
<td>$I_{A_2, E_2} = \frac{1}{2} \log \det[I_2 + \rho h_{A_2, E_2} h_{A_2, E_2}^H]$</td>
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seen from the figure, provided that the number of relays is sufficiently large, the proposed scheme can outperform both the NAF and DDF scheme for all multiplexing gain, which confirms the previous discussion. Consistent to our analytical discussion, the DMT achievable for the proposed scheme is better than the upper bound of the DMT for the SAF scheme, conditioned on $M \leq N + 1$. For the case of $M > N + 1$, the SAF scheme will outperform the proposed protocol, but the DMT of the SAF is only achievable with the strong assumption of isolated relays. Furthermore, for a sufficiently large $N$, the DMT achieved by ODF will be very close to the optimal MISO bound.

For the numerical result, the performance of the proposed cooperative protocol will be evaluated by using Monte-Carlo simulations. Consider the communication scenario with one source-destination pair and $N$ single-antenna relays, the targeted average data rate is set as $R = 2 \text{ bits per channel use (BPCU)}$ or $R = 4 \text{ BPCU}$. In addition to direct transmission, two existing cooperative schemes will be used for comparison, the NAF and DDF schemes in [6] respectively.

For the first experiment, the number of relays is fixed as $N = 4$. As can be seen from Fig. 4, the proposed protocol achieves the smallest outage probability among the four transmission schemes. For a fixed data rate and sufficiently large SNR, the slope of the curves for the three cooperative schemes becomes similar, which is due to the fact that all cooperative schemes can achieve the full diversity gain $N + 1$. However, a constant gain of error performance can be observed between the proposed ODF scheme and the other two schemes when the all curves have the same slope. Such a performance gap can be further enlarged by increasing the targeted data rate. For the second experiment, we study the performance of the proposed cooperative scheme as a function of the number of relays $N$. As shown by Fig. 5, the performance of the proposed scheme can be continuously improved by increasing the number of relays. The slope of the curves for the outage probability is also increasing with more relays available since more diversity gain can be achieved.

### B. Performance of RRDF in the Second Scenario

The following theorem provides the achievable DMT for the proposed transmission protocol.

**Theorem 2**: When the frame length $L \geq 3$ and is assumed to be odd, and the number of relay antennas $M_r \geq 2$,
the achievable DMT for the proposed RRDF protocol in the second scenario can be shown as

\[ d_{RRDF}^*(r) = (1 - r) + 2M_r \left( 1 - \frac{L}{L-1}r \right)^+ \]  \hspace{1cm} (6)

**Proof:** refer to Section IV-B.

In order to compare the performance of the two proposed protocols, we provide the following lemma for the proposed ODF protocol in the scenario with multiple-antenna relays.

**Lemma 3:** The achievable DMT for the ODF protocol in the second scenario can be expressed as

\[ d_{ODF2} = (1 - r) + M_r (2 - 3r)^+ \]  \hspace{1cm} (7)

This lemma can be easily obtained from the proof of Theorem 1 and Theorem 2, so that we omit the proof details here for space limitation.

**Two Remarks for Theorem 2 and Lemma 3:**

1. From Lemma 3 one can see that when \( r \geq \frac{2}{3} \), the diversity gain of this protocol is identical to non-cooperative protocol. This is due to fact that at most only two thirds symbols can be retransmitted in a frame constrained by the small relay number.

2. **Comparison with the two schemes in the second scenario:**

As shown in Theorem 2, for the case of \( L = 3 \), the DMT achieved by the two schemes is exactly same. For the case of \( L \geq 3 \), RRDF outperforms the ODF scheme \( d_{RRDF}^*(r) \geq d_{ODF2} \), for all \( 0 \leq r \leq 1 \). Moreover, for the flat Rayleigh fading and quasi-static channels, \( L \) can be designed largely enough to satisfy \( \frac{L}{L-1} \approx 1 \), such that the optimal system DMT \( (2M_r + 1)(1 - r) \) can be asymptotically achieved.

The DMT achieved by the two cooperative schemes is shown in Fig. 6. As can be seen from this figure, RRDF scheme outperforms ODF for all \( 0 \leq r \leq 1 \), and the superiority of RRDF becomes more obvious by increasing \( M_r \) or \( L \). For a sufficiently large \( L \), the DMT achieved by RRDF will be very close to the optimal MISO bound.

For the numerical result, the performance of the proposed cooperative protocol will be evaluated by using Monte-Carlo simulations. The targeted average data rate is also set as \( R = 2 \) BPCU or \( R = 4 \) BPCU. For this communication scenario with one source-destination pair and two \( M_r \)-antenna relays, in addition to direct transmission, ODF scheme will be also used for comparison with the RRDF protocol. The number of antennas at each relay is fixed as \( M_r = 2 \), and the frame length \( L \) in Situation A2 Event E2 is designed as a odd number 11.

From Fig. 7, one can see that the RRDF protocol performs worse at low SNR. This phenomenon is due to the fact that the performance of the RRDF protocol is much more sensitive to the source-relay/inter-relay channel condition at low SNR. The outage event is prone to occur at each relay at low SNR, which results in some degradation to the performance. But by observing the outage probability plot in Fig. 7, it is obvious that RRDF achieves the smallest outage probability among the three transmission schemes when increasing SNR. Moreover, the performance gain between RRDF and the others can be further enlarged at high SNR by increasing the targeted average data rate.

IV. PROOF OF THEOREM 1 AND THEOREM 2

In this section, the DMT performance of each protocol is characterized by the outage behavior based on outage events and outage probabilities. The whole proof process of Theorem 1 is given in the first subsection, but only the main proof steps of Theorem 2 can be seen in the second subsection. The proof details of two complicated lemmas in the second subsection are put into Appendix.

A. Proof of Theorem 1

From Section II-B, the outage event for the proposed cooperative ODF protocol can be defined as \( O \triangleq \bigcup_K O_K \), where \( O_K \triangleq \{ I_K < R \} \). Denote \( E_K \) as the event that there are \( K \) qualified relays, and hence the outage probability can be expressed as

\[ P(O) = \sum_{K=0}^{N} P(O_K)P(E_K). \] \hspace{1cm} (8)

Define \( z_n = |g_n|^2 \) and \( v_n = |h_n|^2 \) which are exponentially distributed. Without loss of generality, consider that the \( N \) relays have been ordered according to their incoming
channel quality, i.e., $v(1) \leq \cdots \leq v(N)$. The probability for the event with $K$ qualified relays can be expressed as $P(E_K) = P(v_i(N-K) \leq \varepsilon, v_i(N-K+1) \geq \varepsilon)$, where $\varepsilon = \frac{\rho x}{n - 1}$.

Applying order statistics [23], the joint probability density function (PDF) of $\{v_i(K), v_i(K+1)\}$ can be obtained as

$$f(v_i(N-K), v_i(N-K+1)) = \frac{N!}{(N-K-1)!(K)!} \left[1 - e^{-\varepsilon}\right]^{N-K} e^{-\varepsilon} K^K \rho^{-K(N-K)(1-r)} \times f(v_i(N-K), f(v_i(N-K+1))]^{N-K-1}$$

(9)

where $f(x)$ denotes the PDF $f(x) = e^{-x}(x > 0)$ and $F(x)$ is the cumulative distribution $F(x) = 1 - e^{-x}$. Now the probability to have $K$ qualified relays $P(E_K)$ can be calculated as

$$P(E_K) = \frac{N!}{(N-K)!K!} \left[1 - e^{-\varepsilon}\right]^{N-K} e^{-\varepsilon} K^K \rho^{-K(N-K)(1-r)}$$

(10)

where $\approx$ is used to denote exponential equality [1], i.e., $f(\rho) \approx \rho^n$ to denote $\lim_{\rho \to \infty} \frac{\log f(\rho)}{\log \rho} = n$ ($\leq$ and $\geq$ are similarly defined). On the other hand, from (2), the outage probability for the event with $K$ qualified relays can be expressed as

$$P(O_K) = P(I_K < R) = P \left( \det[I_{K+1} + \rho H_K H_K^H] < 2^{(K+1)R} \right)$$

(11)

Define $D_n = \det[I_n + \rho H_{n-1} H_{n-1}^H]$, where $H_{n-1}$ is the $n \times n$ top-left submatrix from $H_K$. According to [24], the determinant of the tridiagonal matrix $D_n$ can be obtained iteratively as $D_n = [1 + \rho x + \rho z_{n-1}]D_{n-1} - \rho^2 x z_{n-1} D_{n-2}$, where $x = |g|^2$. Then we can obtain the following inequality

$$\det[I_{K+1} + \rho H_K H_K^H] \geq (1 + \rho x)^{K+1} + \prod_{i=1}^{K} \rho z_i$$

(12)

whose proof can be accomplished by following similar steps in [8]. By using the inequality, the outage probability can be upper bounded as

$$P(O_K) \leq P \left( \prod_{i=1}^{K} \rho z_i < 2^{(K+1)R} \right)$$

(13)

$$\leq P \left( (1 + \rho x) < 2^R \right) P \left( \prod_{i=1}^{K} \rho z_i < 2^{(K+1)R} \right).$$

To find the probability $P \left( \prod_{i=1}^{K} \rho z_i < 2^{(K+1)R} \right)$, the PDF of the variable $z = \prod_{i=1}^{K} \rho z_i$ can be approximated as $f_z(z) = \mathcal{E}_{z_1, \ldots, z_K} \left\{ \delta (z - \prod_{i=1}^{K} \rho z_i) \right\}$, where $\delta(\cdot)$ is the Dirac delta function and the expectation is evaluated using the joint density of $\{z_1, \ldots, z_K\}$. And then the desirable outage probability can be obtained from the integration of the developed PDF. Following the similar steps in [25], the outage probability can be approximated as

$$P \left( \prod_{i=1}^{K} \rho z_i < 2^{(K+1)R} \right) = 2^{-\lambda K} R$$

(14)

where $(x)^+ = \max\{x, 0\}$. The mathematical details have been omitted here due to space limitation. Then substitute the above equation into (13), the upper bound of $P(O_K)$ can be expressed as

$$P(O_K) \leq 2^{-N(1-r)\cdot(K+1)r}$$

(15)

Now by combining (10) and (15), the overall outage probability of the ODF protocol in the first scenario can be obtained as

$$P(O) = \sum_{K=0}^{N} \rho^{(1-r)(N-K+1)+[K-(K+1)r]} \geq \rho^{d_{ODF}^1(r)}$$

(16)

where $d_{ODF}^1(r) = (1 - r) + (N - (N + 1)r)$ and the last relationship is due to the fact that the outage probability with $K = N$ is the dominant factor. So far, Theorem 1 has been proved.

**B. Proof of Theorem 2**

According to Section II-C, the outage event for the proposed RRDF protocol can be defined as $O = \bigcup_{j=0}^{2} O_{A_j}$, where $O_{A_j} \triangleq I_{A_j} < R_j$. Moreover, the outage event $O_{A_2}$ can be divided by $E_k$ as $O_{A_2} \triangleq \bigcup_{k=0}^{2} O_{A_2, E_k}$, where $O_{A_2, E_k} \triangleq I_{A_2, E_k} < \frac{k+2}{k+1} R_0$ and $O_{A_1, E_2} \triangleq I_{A_1, E_2} < R_0$. Therefore, the outage probability of the system can be expressed as

$$P(O) = \sum_{j=0}^{2} P(O_{A_j} P(A_j).$$

(17)

where the third term on the right side can be further expressed as

$$P(O_{A_2}) = \sum_{k=0}^{2} P(O_{A_2, E_k} P(E_k)$$

(18)
Before the analysis of the conditional outage probabilities, we firstly determine how many relays will be qualified in the first time-slot. At this time, \( S \) is broadcasting message \( s(1) \) and the other nodes are listening. Furthermore, the node pair \((S, R_i)\) can be recognized as a SIMO system, where \( i = 1, 2 \). Let \( B \) represent the outage event at \( R_i \). From [1], one can see that \( P(B) = \rho^{-M_i(1-r_0)} \), where \( r_0 \triangleq \lim_{\rho \to \infty} R_0(\rho) \). Hence it’s easy to express the probability of each situation as

\[
P(A_j) = \left( \frac{2}{j} \right) [1 - P(B)]^j [P(B)]^{2-j} \rho^{-2(2-j)M_i(1-r_0)}
\]

where \( j = 0, 1, 2 \). With the knowledge of the qualified relays number, each term on the right side of Eq. (17) can be calculated in the subsequent sections.

1) Situation \( A_0 \): Recall the mutual information \( I_{A_0} \) in TABLE I, and \( P(O_{A_0}) = \rho^{-(1-r_0)} \) can be obtained from [1]. So that the first term can be expressed as

\[
P(O_{A_0})P(A_0) = \rho^{-(2M_r+1)(1-r_0)}. \tag{20}
\]

2) Situation \( A_1 \): The mutual information \( I_{A_1} \) is provided in TABLE I. Define \( g_{i, max} = g_{i,r_1} \) for simplicity, where \( |g_{i,max}|^2 \triangleq \max\{|g_{i,1}|^2, \cdots, |g_{i,M_r}|^2\} \). Then the outage probability at the destination in this situation can be written as

\[
P(O_{A_1}) = P(I_{A_1} < \frac{1}{2}R_0) \leq P \left( |g|^2 < \frac{2R_0}{\rho} - 1 \right)
	\times P \left( |g_{i,m}|^2 < \frac{2R_0}{\rho}, \forall m \in \{1, \cdots, M_r\} \right)
\]

\[
\leq \rho^{-(M_r+1)(1-r_0)}. \tag{21}
\]

By combining (21) with (19), the second term can be shown as

\[
P(O_{A_1})P(A_1) \leq \rho^{-(2M_r+1)(1-r_0)}. \tag{22}
\]

3) Situation \( A_2 \): Define the event that \( R_1 \) fails to decode the source message in the third time slot as \( C \). Before addressing each probability of Event \( E_k \) which represents the number of relays with the ability of successful inter-relay interference cancelation, the probability of Event \( C \) needs to be determined in the following lemma.

Lemma 4: When the ZF decoding is utilized at each relay, the probability of Event \( C \) which shows that \( R_1 \) can’t satisfy the ZF condition in the third time slot can be revealed as \( P(C) = \rho^{-(M_r-1)(1-r_0)} \).

Proof: see Appendix.

Obviously \( R_2 \) has the same property as \( R_1 \). Hence the probability of Event \( E_k \) can be obtained as

\[
P(E_k) = \binom{2}{k} [1 - P(C)]^k [P(C)]^{2-k} \rho^{-2-k(M_r-1)(1-r_0)}.
\]

Besides, the conditional probability \( P(O_{A_2}, E_k) \) can be revealed in the subsequent lemma.

Lemma 5: By assuming quasi static and frequency non-selective Rayleigh fading channels, each conditional probability of the outage event at destination \( A_2 \) Event \( E_k \) can be revealed as

\[
P(O_{A_2}, E_k) \leq \begin{cases} 
\rho^{-2(2M_r+1)(1-r_0)} & \text{if } k = 0 \\
\rho^{-2(M_r+1)(1-r_0)} & \text{if } k = 1 \\
\rho^{-[(1-r_0)+2M_r(1-\frac{1}{\rho}r_0)])} & \text{if } k = 2
\end{cases}. \tag{24}
\]

Proof: see Appendix.

Now, by integrating (23) and (24), the last item, \( i.e. \ P(O_{A_2})P(A_2) \), can be upper bounded as

\[
P(O_{A_2})P(A_2) \leq \rho^{-\min\{(4M_r-1)(1-r_0), \frac{2}{3}M_r(1-r_0), 2M_r(1-\frac{1}{\rho}r_0)\} + 2M_r(1-\frac{1}{\rho}r_0)} \]

\[
\leq \rho^{-[(1-r_0)+2M_r(1-\frac{1}{\rho}r_0)]}. \tag{25}
\]

Dependent on the definition of the variable-rate strategy in Eq. (34) of [3] and considering every situation \( A_j \) and event \( E_k \), the expected transmission rate \( R \ BPCU \) can be expressed as

\[
R = P(O_0)R_0 + P(A_0)\frac{R_0}{2} + P(A_2, E_0)\frac{2R_0}{3} + P(A_2, E_1)\frac{3R_0}{4} + P(A_2, E_2)R_0. \tag{27}
\]

The mapping criterion from \( R \) to \( R_0 \) is also given in [3]. It’s not difficult to prove the inequality \( P(E_k) + P(A_2) \rightarrow \infty \leq P(A_2, E_k) \leq P(E_k) \), so \( P(A_2, E_k) \approx P(E_k) \) can be obtained when \( k = 1, 2 \) in large-SNR region. From (19) and (23), and substitute \( R = r \log \rho, R_0 = r_0 \log \rho, P(A_2, E_0) = P(A_2) - P(A_2, E_1) - P(A_2, E_2) \) into (27), \( r \) can be revealed as

\[
r \approx r_0 \left[ P_1^2 + P_1(1-P_1) + \frac{2}{3}(1-P_1)^2 + \frac{1}{6}P_2(1-P_2) + \frac{1}{3}(1-P_2)^2 \right], \quad (28)
\]

where \( P_1 = \rho^{-M_r(1-r_0)}, P_2 = \rho^{-(M_r-1)(1-r_0)} \). Following the analysis in Claim 3 of [3] and the similar steps in [15], Theorem 2 can be proved by substituting \( r_0 = r \) into (26).

V. CONCLUSION

In this regular paper, two new forms of cooperative transmission protocol have been proposed. ODF is constructed by combining opportunistic strategies with non-orthogonal transmission. Beyond the two strategies, RRDF also consists zero-forcing and antenna selection criterion at each relay. An achievable diversity-multiplexing tradeoff was developed to evaluate the spectral efficiency of the proposed schemes. For the scenario with \( N \) single-antenna relays, when \( N \to \infty \), the DMT achievable for ODF can approach the optimal MISO upper bound. Compared with some existing protocols, the proposed scheme achieves a better DMT, particular at \( \frac{1}{2} \leq r \leq \frac{N}{N+1} \). For the second scenario with two multiple-antenna relays, the proposed RRDF protocol can achieve the
optimal DMT when the frame length $L$ is sufficiently large. Monte-Carlo simulation results demonstrate that both the proposed protocols can achieve better error performance than the comparative schemes in most simulation conditions. One promising future topic is to further improve the performance of ODF or RRDF by applying the dynamic decoding strategy as in [6]. Furthermore, following the similar steps in [6, 25], it will be interesting to study the application of the proposed protocol to the multiple access and broadcasting scenarios.

APPENDIX

A. Proof of Lemma 4:

In the situation $A_2$, let’s focus on the third time slot, the $r_{3}$th antenna of $R_3$ broadcasts the message $s(2)$ while $S$ keeps transmitting new message $s(3)$. The signal vector received by $R_1$ is
\[ r_{R1}(3) = \sqrt{E_2}H_{R1}s_3 + w_1, \]
where $r_{R1}(3), w_1 \in C^{M_r}, H_{R1} = [h_{2,r_1}, h_{R1}], s_3 = [s(2), s(3)]^T$ and $w_1$ is the noise vector. Such a signal model can be recognized as a virtual MIMO system with two transmit and $M_r$ receive antennas, as the channel matrix $H_{R1}$ with dimension $M_r \times 2$ and $H_{R1} \sim CN(0, I_2 \otimes \Sigma)$, where $\Sigma$ is the covariance matrix with dimension $M_r \times M_r$. The zero-forcing decoding is used at the receiver to decompose the SNR into two parallel streams. According to [26], the SNR of the second stream at the relay receiver which is related to the objective message $s(3)$ can be computed as
\[ \gamma_{2} = \rho h_{R1}^H Q \Lambda Q h_{R1}, \]
where $\Lambda$ is the eigen value and $Q \Lambda Q^H = H_{M_r} - h_{2,r_2} h_{2,r_2}^H h_{2,r_2}^{-1} h_{2,r_2}^H$. (The SNR $\gamma_{2}$ at $R_2$ in the fourth time slot is similarly defined.) Moreover, when $h_{R1} \sim CN(0, \Sigma)$, where $\Sigma = \sigma^2 I_{M_r}$, and $\sigma^2$ is the variance of each channel coefficient, according to [27], $\gamma_{2}$ is a chi-square variable with the PDF
\[ f_{\gamma_{2}}(\gamma_{2}) = \left(\frac{\gamma_{2}}{\rho^2}\right)^{M_r-1} \exp\left(-\frac{\gamma_{2}}{\rho^2}\right). \]

With the knowledge of $\gamma_{2}$’s distribution and based on the similar analysis of [27], the probability of the event $C$ can be expressed as
\[ P(C) = 1 - P(\gamma_{2} \geq 2^{R_0} - 1) \approx \frac{(2^{R_0} - 1)^{M_r - 1}}{M_r - 1} \left(\frac{\rho^2}{M_r - 1}\right)^{M_r - 1} \approx \rho^{-\frac{1}{2}(1 - r_0)} . \]

B. Proof of Lemma 5:

In the situation $A_2$, each conditional probability in Lemma 5 can be calculated as follows.

1) Event $E_0$: When $E_0$ happens, the mutual information $I_{A_2,E_0}$ is given in TABLE I. By following the similar steps in [8], we can obtain the following inequality
\[ \det[I_3 + \rho H_{A_2,E_0} H_{A_2,E_0}^H] \geq (1 + \rho |g|^2) + \rho^2 |g_{1,max}|^2 |g_{2,max}|^2 . \]

By using the conclusions of (13) and (14), the conditional probability $P(O_{A_2,E_0})$ can be revealed as
\[ P(O_{A_2,E_0}) = P(I_{A_2,E_0} < \frac{2}{3} R_0) \leq \left[ \frac{2R_0 - 1}{\rho^2} \right] \prod_{m=1}^{M_r} P \left( |g_{1,m}|^2 |g_{2,m}|^2 < \frac{2^2 R_0}{\rho^2} \right) \leq \rho^{-\frac{1}{2}(2M_r+1)(1 - r_0)} . \]

2) Event $E_1$: The mutual information in this event is provided in TABLE I. By following the similar analysis of Event $E_0$, the conditional probability $P(O_{A_2,E_1})$ can be shown as
\[ P(O_{A_2,E_1}) = P(I_{A_2,E_1} < \frac{3}{4} R_0) \leq \left[ \frac{2R_0 - 1}{\rho^3} \right] \prod_{m=1}^{M_r} P \left( |g_{1,m}|^2 |g_{2,m}|^2 < \frac{2^3 R_0}{\rho^3} \right) . \]

Furthermore, the probability $P\left(|g_{1,m}|^2 |g_{2,m}|^2 < \frac{2^3 R_0}{\rho^3}\right)$ can be revealed in the following corollary.

Corollary 1: Define $X_1 = |g_{1,m}|^2$ and $X_2 = |g_{2,m}|^2$ which are subjected to the exponential distribution, it can be shown that \[ P\left(X_1 X_2 < \frac{2^3 R_0}{\rho^3}\right) = \rho^{-\frac{1}{2}(1 - r_0)} . \]

Proof: refer to the next section.

Combining (34) with Corollary 1 concludes the proof of the second term of Lemma 5.

3) Event $E_2$: The mutual information $I_{A_2,E_2}$ is given in TABLE I. According to the conclusions of (13) and (14) and by following the similar analysis of Event $E_0$, $P(O_{A_2,E_2})$ can be expressed as
\[ P(O_{A_2,E_2}) \leq \left[ \frac{2R_0 - 1}{\rho^3} \right] \prod_{m=1}^{M_r} P \left( |g_{1,m}|^2 |g_{2,m}|^2 < \frac{2^2 R_0}{\rho^3} \right) \leq \rho^{-\frac{1}{2}(1 - r_0)} . \]

So far, Lemma 5 has been proved.

C. Proof of Corollary 1:

Firstly, another useful corollary which will be used later in this subsection can be shown as follows.

Corollary 2: If a continuous function $\tilde{F}(v)$ is monotonic and bounded for $v \in (b_f, +\infty)$, where $b_f$ is a constant, the limitation $v_{\infty} = +\infty \tilde{F}(v)$ must exist.

Proof: It is assumed that $\tilde{F}(v)$ is monotonic decreasing and lower bounded without loss of generality. According to the infimum principle, the infimum of $\tilde{F}(v)$ must exist for $v \in (b_f, +\infty)$, which can be denoted as $C_f$. It means that $\tilde{F}(v) \geq C_f$ for any $v > b_f$ and $\exists \alpha > 0$ to ensure that $\tilde{F}(v_0) - \alpha < C_f$ for $\forall \alpha > 0$. So that $|\tilde{F}(v) - C_f| < \alpha$ is obtained for any $v > v_0$ due to monotonicity. Hence the limitation $v_{\infty} = +\infty \tilde{F}(v)$ must be equal to $C_f$. If $\tilde{F}(v)$ is upper bounded and monotonic increasing, the limitation can be proved to be existed in the similar steps.
Assume that $X_1$ and $X_2$ are independently exponentially distributed with the variance $(\delta_i)^2$. Let $a_0 = 2^{3/3}e^{-\rho^2} = \rho^{-3(1-\rho)}$, $D = \{(x_1, x_2) \in \mathbb{R}^2 \mid x_1^2 + x_2^2 < a_0\}$, and $f_{x_1,x_2}(x_1, x_2) = f_{x_1}(x_1)f_{x_2}(x_2)$ which is the joint density of $\{x_1, x_2\}$, where $f_{x_i}(x_i)$ denotes $x_i$’s PDF $f_{x_i} = \frac{1}{\pi}e^{-\pi x_i^2}$, $i = 1, 2$. Then the probability $P (X_1^2 + X_2^2 < 2^{3/3}e^{-\rho^2})$ can be expressed as

$$
\Phi_a = P (X_1^2 + X_2^2 < a_0) = \int_0^{\infty} f_{x_1,x_2}(x_1, x_2)dx_1dx_2
$$

$$
= \int_0^{+\infty} e^{-x} (1 - e^{-\sqrt{a_0/(6\pi x)^2}})dx
$$

$$
= \sum_{i=1}^{\infty} \frac{(-1)^{i+1}}{i!} (\sqrt{a}) \int_0^{+\infty} e^{-x} x^{i-2}dx, \quad (36)
$$

where $a = a_0/\delta_i$. Now, we firstly define the function $\Phi_a(u)$ as

$$
\Phi_a(u) = \sum_{i=1}^{\infty} \frac{(-1)^{i+1}}{i!} (\sqrt{a})^i B_i(u), \quad (37)
$$

where $B_i(u) = \int_0^{+\infty} \frac{e^{-x}}{x^{i-2}}dx$. Then the improper integral $\Phi_a$ can be calculated as: $\Phi_a = \lim_{u \to 0^+} \Phi_a(u)$.

Moreover, $B_i(u)$ can be calculated for a odd or even $i$ respectively as follows.

- **Odd $i$:** From [28] (Eq. 3.381.6) and [29], Whittaker Function can be used as following,

$$
B_i(u) = u^{i/2}e^{-u/2}W_{i/4,-(i-2)/4}(u)
$$

$$
= u^{i/2}e^{-u/2} \left[ \frac{\Gamma(\frac{i+2}{2})}{\Gamma\left(\frac{i}{2}\right)} M_{i/4,-(i-2)/4}(u) \right]
$$

$$
+ \frac{\Gamma\left(\frac{i+2}{2}\right)}{\Gamma(1)} M_{-i/4,(i-2)/4}(u), \quad (38)
$$

where the gamma function can be shown as

$$
\Gamma(z) = \begin{cases} \sqrt{\pi}, & \text{if } z = \frac{1}{2}, \\
\frac{(2i-1)!}{2^{i-1}\pi^{i/2}} \sqrt{\pi}, & \text{if } z = \frac{1}{2}, i = 3, 5, \cdots, \\
\frac{(-1)^{i/2}i!}{(2i-2)!} \sqrt{\pi}, & \text{if } z = -\frac{i}{2}, i = 3, 5, \cdots
\end{cases}
$$

(39)

and the confluent hypergeometric function is defined as

$$
M_{k,m}(u) = u^{1/2+m}e^{-u/2} \left[ 1 + \frac{\frac{1}{2} + m - k}{1/(2m+1)} u \right]
$$

$$
+ \frac{\left(\frac{1}{2} + m - k\right)}{2!(2m+1)(2m+2)} u^2 + \cdots. \quad (40)
$$

Now that, when $i$ is odd, $B_i(u)$ can be obtained in (41) shown at the top of the next page, where $a(1) \to 0$ as $u \to 0^+$.

- **Even $i$:** From [28] (Eq. 3.351.4), Exponential Integral Function can be used as following,

$$
B_i(u) = (-1)^{i/2} \frac{Ei(-u)}{(i/2)^{i/2}} + e^{-u}
$$

$$
\times \sum_{n=0}^{i/2-2} \frac{(-1)^n u^{(i-2-2n)/2}}{(i/2-1)(i/2-2)\cdots(2-1-i/2-n)!}, \quad (42)
$$

where $i \geq 4$, and Exponential Integral Function can be shown as $Ei(-u) = \ln(u) + \sum_{k=1}^{\infty} (-u)^k/k!$. Now that, when $i$ is even, $B_i(u)$ can be obtained in (43) shown at the top of the next page.

By substituting $B_i(u)$ into (37) and rearranging the infinite series, $\Phi_a(u)$ can be rewritten as

$$
\Phi_a(u) = \sum_{i=1}^{\infty} \frac{(-1)^{i+1}}{i!} \Gamma\left(-\frac{i-2}{2}\right) \left(\sqrt{a}\right)^i
$$

$$
+ e^{-u} \sum_{n=0}^{\infty} \phi_n(u) + o(1), \quad (44)
$$

where,

$$
\phi_n(u) = \frac{(-1)^n a^{n+1}}{n!(2n+2)!} \ln(u)
$$

$$
+ \sum_{i=2n+3}^{\infty} \frac{(-1)^{i+n+1} 2^{(i-n)/2}}{i! (i-2) (i-4) \cdots (i-2-2n)} \phi_n(u). \quad (45)
$$

Let $v = \sqrt{a}u^{-1/2}$, $\phi_n(v)$ can be expressed as

$$
\phi_n(v) = \frac{(-1)^n a^{n+1} \ln a}{n!(2n+2)!} + a^{n+1} F_n(v), \quad (46)
$$

where,

$$
F_n(v) = \frac{2(-1)^{n+1}}{n!(2n+2)!} \ln(v)
$$

$$
+ 2^{n+1} \sum_{i=2n+3}^{\infty} \frac{(-1)^{i+n+1} v^{i-2-2n}}{i! (i-2) (i-4) \cdots (i-2-2n)}. \quad (47)
$$

Furthermore, the limitation $\lim_{v \to +\infty} F_n(v)$ can be prove to be exist as follows.

1. **When $n = 0$:** From (47) can be written as $F_0(v) = -\ln(v) + 2 \sum_{n=3}^{\infty} (-1)^{n+1} v^{n-2}$. The differentiation of $F_0(v)$ can be obtained as

$$
\frac{dF_0(u)}{du} = \frac{-1}{v} + 2 \sum_{n=3}^{\infty} \frac{(-1)^n v^{n-3}}{d!}
$$

$$
= -2 v^{-1} \left[-1 + v + e^{-v}\right]. \quad (48)
$$

Note that $\exists b_0 > 0$ to make sure that $e^{-v} < v$ and $\frac{dF_0(u)}{du} < 0$ for any $v > b_0$, such as $b_0 = 1$. Then we integrate $\frac{dF_0(u)}{du}$ from $b_0$ to $v$, $F_0(v)$ can be expressed as

$$
F_0(v) = -v^{-2} + 2v^{-1} - 2 \int_{b_0}^{v} \frac{e^{-t}}{t^2}dt + C_{b_0}, \quad (49)
$$

where $v > b_0$ and $C_{b_0}$ is a constant. And the upper bound of $|F_0(v)|$ can be expressed as

$$
|F_0(v)| < |v^{-2}| + |2v^{-1}| + 2 \int_{b_0}^{v} \frac{1}{t^2}dt + |C_{b_0}|
$$

$$
< |C_{b_0}| + 4b_0^{-1} + b_0^{-2}. \quad (50)
$$

Hence $F_0(v)$ is bounded and monotonic decreasing when $v \in (b_0, +\infty)$. From Corollary 2, one can see that the limitation $\lim_{v \to +\infty} F_0(v)$ exists.

2. **When $n \geq 0$:** the recurrence relation between $F_{n+1}(v)$ and $F_n(v)$ can be revealed as

$$
\frac{dF_{n+1}(v)}{dv} = \frac{2}{v^2} \left[-F_n(v) + \frac{2(-1)^{n+1} \ln(v)}{n!(2n+2)!}ight]
$$

$$
+ \frac{(-1)^{n+1} v^n}{(2n+3)!(2n+1)!}. \quad (51)
$$
\[ \begin{align*}
B_i(u) &= e^{-u} \left( \frac{\Gamma\left(\frac{1}{2}\right) + o(1)}{\Gamma(-i-2/2) + \sum_{n=0}^{\infty} \frac{(-i)^{n+1} u^{-(i-2-n)/2}}{(i-2)(i-4) \cdots (i-2-n)} + o(1)} \right) \\
&= e^{-u} \left( \frac{-\ln(u) + o(1)}{(i-2)^{1/4} \ln(u) + \sum_{n=0}^{\infty} \frac{(-1)^{n+1} u^{-(i-2-n)/2}}{(i-2)(i-4) \cdots (i-2-n)} + o(1)} \right) \quad \text{if } i = 2
\end{align*} \]

If \( \lim_{v \to +\infty} F_n(v) = C_n \), where \( C_n \) is a constant, \( \exists b_{n+1} > 0 \) to ensure that \( F_n(v), \ln(v) < v < \frac{dF_{n+1}(v)}{dv} \) is always positive (or always negative) for any \( v > b_{n+1} \).

Then we integrate \( \frac{dF_{n+1}(v)}{dv} \) from \( b_{n+1} \) to \( v \), \( F_0(v) \), \( F_{n+1}(v) \) can be expressed as

\[ F_{n+1}(v) = \int_{b_{n+1}}^{v} \left[ -F_n(t) + \frac{2(-1)^{n+1} \ln(t)}{n!(2n+2)!} \right] dt - \frac{(-1)^{n+1} v^{2n+2}}{(2n+3)! \ln(v) + 2n+1} + C_{b_{n+1}}, \quad (52) \]

where \( v > b_{n+1} \) and \( C_{b_{n+1}} \) is a constant. Hence \( F_{n+1}(v) \) is monotonic and bounded when \( v \in (b_{n+1}, +\infty) \).

From Corollary 2, one can see that the limitation \( \lim_{v \to +\infty} F_{n+1}(v) \) exists.

Then, by recalling (46), we have

\[ \lim_{u \to 0^+} \phi_{n}(u) = \lim_{v \to +\infty} \phi_n(v) = \frac{(-1)^n a^{n+1} \ln(a)}{n!(2n+2)!} + C_n a^{n+1}. \]

By recalling (44), \( \Phi_a \) can be expressed as

\[ \Phi_a = \lim_{u \to 0^+} \Phi_{n+1}(u) = \sum_{i=1, i \text{ is odd}}^{\infty} \frac{(-1)^{i+1}}{i!} \Gamma\left(\frac{i}{2}\right) (\sqrt{a})^i + \sum_{n=0}^{\infty} \frac{(-1)^n a^{n+1} \ln(a)}{n!(2n+2)!} + \sum_{n=0}^{\infty} C_n a^{n+1}. \]

For a sufficiently large \( \rho \), \( \Phi_a \) can be approximately calculated as: \( \Phi_a \approx \sqrt{\pi a} \rho^{-\frac{1}{2}} (1 - \rho) \).
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