Robust $H_{\infty}$ finite-time control of switched stochastic systems with time delay

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Abstract: In this paper, we consider the problem of robust $H_{\infty}$ finite-time control for a class of switched stochastic systems with time delay. Firstly, the concept of finite-time boundedness is extended to switched stochastic systems. Then, based on the average dwell time method, some sufficient conditions under which switched stochastic systems are finite-time stochastically bounded are given. Moreover, finite-time $H_{\infty}$ performance is investigated, and robust $H_{\infty}$ controller is designed to guarantee that the corresponding closed-loop system is finite-time stochastically bounded with $H_{\infty}$ performance. All conditions are formulated in terms of a set of linear matrix inequalities (LMIs). Finally, a numerical example is given to illustrate the effectiveness of the proposed approach.

Key Words: Switched stochastic systems; time delay; $H_{\infty}$ control; finite-time control; average dwell time

1 Introduction

Switched systems are a class of hybrid dynamical systems composed of a finite number of continuous-time subsystems or discrete-time subsystems and a switching rule specifying which subsystem is to be actuated. Many real-world systems, such as networked control systems [1], aircraft systems [2] and robot systems [3], are inherently multimodal. Mathematically, these systems are usually modeled as switched systems. The past decades has witnessed the growing interest in the field of switched systems, and many useful results on stability analysis and control synthesis for such systems are reported [4-6]. It is noted that the time-delay phenomenon is frequently encountered in engineering and social systems, and the existence of which may cause instability or undesirable system performance in feedback systems. Therefore, many research efforts have been devoted to the study of switched time-delay systems over the past years [7-8].

It is well known that stochastic disturbance exists in many actual operation, stochastic systems have attracted considerable attention during the past several decades. Early results can be found in [9], and the $H_{\infty}$ control problem of stochastic systems with time delay is investigated in [10]. Study on $H_{\infty}/H_{\infty}$ control of stochastic system is developed in [11]. Stability analysis on stochastic system with multiple delays is proposed in [12]. Moreover, stability analysis and control synthesis of stochastic switched systems are addressed in [13]. Asynchronous switching and $H_{\infty}$ control of stochastic switched systems are discussed in [14]. The research on stability of stochastic delay systems are presented in [15].

Most of the existing literature on stability analysis of switched systems has focused on global asymptotic stability and exponential stability over an infinite time interval. In many engineering systems, we are interested in that whether the state trajectories of the system are bounded over a short time. The concept of finite-time stability (short time stability) was introduced by Dorato to investigate the transient performance of a system [16]. Recently, based on linear matrix inequality(LMI) technique, many useful results on finite-time stability analysis and stabilization have appeared [17-20]. Furthermore, finite-time boundedness and stabilization of switched linear systems have been studied in [18]. The problems of finite-time boundedness and $L_{2}$-gain of switched systems with time delay have been investigated in [19]. However, to the best of our knowledge, the issue of robust $H_{\infty}$ finite-time control for switched stochastic systems with time delay has not yet been addressed, and this constitutes the main motivation of the present study.

The remainder of the paper is organized as follows. In section 2, problem statement and some useful lemmas are given. In section 3, based on the average dwell time method, sufficient conditions of finite-time stochastic boundedness (FTSB) and finite-time $H_{\infty}$ performance for switched stochastic systems with time delay are proposed. Then, a robust $H_{\infty}$ controller is designed. In section 4, a numerical example is given to illustrate the effectiveness of the proposed approach. Finally, concluding remarks are provided in section 5.

Notations: In this paper, the superscript “$T$” denotes the transpose, and the symmetric terms in a matrices are denoted by * . The notation $X \succ Y$ ($X \succeq Y$ ) means that matrix $X-Y$ is positive definite (positive semi-definite, respectively). $R^{n}$ denotes the $n$ dimensional Euclidean space. $\|y(t)\|$ denotes the Euclidean norm. $\lambda_{\max}(P)$ denotes the maximum eigenvalue of matrix $P$ . $I$ is an identity matrix with appropriate dimension. $\text{diag}\{a_{i}\}$ denotes diagonal matrix with the diagonal elements $a_{i}, i = 1, 2, \cdots, n$. 

*This work is supported by the National Natural Science Foundation of China under Grant No. 60974027 and NUST Research Funding (2011YBXM26).
2 Problem formulation and preliminaries

Consider the following switched stochastic system with time-delay
\[
\dot{x}(t)=\begin{bmatrix} \bar{A}(\sigma(t)) & \bar{B}(\sigma(t)) \\ \bar{C}(\sigma(t)) & \bar{D}(\sigma(t)) \end{bmatrix}x(t)+G(t)\nu(t)+\bar{H}(\sigma(t))\sigma(t)dt+D_{\sigma(t)}x(t)dh(t),
\]
where \( x(t) \in \mathbb{R}^n \) is the state vector, \( \sigma(t) \in \mathbb{R}^m \) is the switching signal, \( u(t) \in \mathbb{R}^p \) is the control input, \( \nu(t) \in \mathbb{R}^q \) is the disturbance input, \( A_{\sigma} \), \( B_{\sigma} \), \( C_{\sigma} \), \( D_{\sigma} \) are uncertain real matrices with appropriate dimensions, and \( H_{\sigma} \) is the switching number of \( \sigma(t) \). The switching signal \( \sigma(t) \) is piecewise constant and right continuous. The following unforced nominal counterpart of system (1)
\[
\dot{x}(t)=\begin{bmatrix} \bar{A}(\sigma(t)) & \bar{B}(\sigma(t)) \\ \bar{C}(\sigma(t)) & \bar{D}(\sigma(t)) \end{bmatrix}x(t)+\bar{G}(\sigma(t))\sigma(t)dt+D_{\sigma(t)}x(t)dh(t),
\]
where \( \bar{A}(\sigma(t)), \bar{B}(\sigma(t)), \bar{C}(\sigma(t)), \bar{D}(\sigma(t)) \) are uncertain real matrices with appropriate dimensions, is known to be exponentially stable.

Lemma 1 Let \( U, V, W \) and \( X \) be constant matrices of appropriate dimensions with \( X=X^T \), then for all \( V^TV \leq I \), \( X+UV+WV^TW^T<0 \), if and only if there exist scalars \( \varepsilon>0 \) such that
\[
X+\varepsilon U^TU+\varepsilon^{-1}W^TW<0
\]

3 Main results

3.1 Finite-time stochastic boundedness (FTSB) analysis

In this subsection, we are interested in FTSB analysis of switched systems with time delay. Consider the following unforced nominal counterpart of system (1)
\[
\dot{x}(t)=\begin{bmatrix} \bar{A}(\sigma(t)) & \bar{B}(\sigma(t)) \\ \bar{C}(\sigma(t)) & \bar{D}(\sigma(t)) \end{bmatrix}x(t)+\bar{G}(\sigma(t))\sigma(t)dt+D_{\sigma(t)}x(t)dh(t),
\]
where \( \bar{A}(\sigma(t)), \bar{B}(\sigma(t)), \bar{C}(\sigma(t)), \bar{D}(\sigma(t)) \) are uncertain real matrices with appropriate dimensions, \( \bar{G}(\sigma(t)) \) is unknown time-varying matrix that satisfies
\[
F^T(t)\bar{G}(t)F(t)\leq I,
\]
Definition 2[21] For any \( \bar{G}(t) \), the switching number of \( \sigma(t) \) on an interval \( (T_i, T_{i+1}) \).

Definition 3[21] For any scalars \( \lambda > 0 \) and positive matrix \( R>0 \), system (1) is said to be finite-time stochastically bounded with respect to \((c_i, c_j, T, R, \nu, \sigma)\).

The following lemma plays an important role in our later development.

Lemma 1[22] Let \( U, V, W \) and \( X \) be constant matrices of appropriate dimensions with \( X=X^T \), then for all \( V^TV \leq I \), \( X+UV+WV^TW^T<0 \), if and only if there exist scalars \( \varepsilon>0 \) such that
\[
X+\varepsilon U^TU+\varepsilon^{-1}W^TW<0
\]

Proof Choose the following piecewise Lyapunov functional candidate for system (11)
the form of each $V_i(t,x(t)) \ (i \in \mathbb{N})$ is given by
\[ V_i(t,x(t)) = V_{i,1}(t,x(t)) + V_{i,2}(t,x(t)) \]
where $V_{i,j}(t,x(t)) = x^T(t)P_i x(t)$,
\[ V_{2,j}(t,x(t)) = \int_{t-j}^{t} x^T(s)Q_j x(s)\, ds \]
For the sake of simplicity, $V_i(t,x(t))$ is written as $V_i(t)$ in this paper. According to Ito formula, along the trajectory of system (11), we have
\[ dV_i(t) = \mathcal{L}V_i(t)dt + 2x^T(t)P_i D_t x(t)\, dw(t) \]
where
\[ \mathcal{L}V_i(t) = 2x^T(t)P_i A x(t) + B x(t-h(t)) + G x(t) + x^T(t)Q x(t) \]
\[ = \xi^T(t)\Theta_i \xi(t) \]
where $\xi(t) = \left[ x^T(t) \quad x^T(t-h(t)) \quad \nu^T(t) \right]^T$,
\[ \Theta_i = \begin{bmatrix} A_i^T P_i + P_i A_i + Q_i + D_i^T P_i D_i & P_i A_i & -Q_i & \ast & \ast \end{bmatrix} \]
Using Schur complement, we can obtain from (15) that
\[ \mathcal{L}V_i(t) < \alpha V_i(t) + \beta J_i(t) \]
Moreover, we have
\[ \mathcal{L}V_i(t) < \alpha V_i(t) + \beta J_i(t) \]
Noticing that
\[ d(e^{-\alpha t}V_i(t)) = -\alpha e^{-\alpha t}V_i(t)dt + e^{-\alpha t}dV_i(t) \]
\[ = e^{-\alpha t}\mathcal{L}V_i(t) + 2e^{-\alpha t}x^T(t)P_i D_t x(t)\, dw(t) \]
\[ < \beta e^{-\alpha t}J_i(t)dt + 2e^{-\alpha t}x^T(t)P_i D_t x(t)\, dw(t) \]
Integrating both sides of (23) from $t_j$ to $t$, and taking expectations, we have
\[ E[V_i(t)] < e^{\alpha(t-t_j)}E[V_i(t_j)] + \int_{t_j}^{t} e^{\alpha(t-s)}\beta J_i(s)\, ds \]
According to (17)-(19), we have
\[ E[V_i(t)] \leq \mu E[V_i(t_j)] \quad \forall i, j \in \mathbb{N} \]
For any $t \in [t_j,t_{j+1})$, using Ito formula, from (24)-(25), it is not difficult to get that
\[ E[V_i(t_j)] \leq e^{\alpha(t-t_j)}E[V_i(t_j)] + \int_{t_j}^{t} e^{\alpha(t-s)}\beta J_i(s)\, ds \]
Then, we have
\[ E[V_i(t)] < e^{\alpha(t-t_j)}E[V_i(t_j)] + \mu E[V_i(t_j)] + \int_{0}^{t} e^{\alpha(t-s)}\beta J_i(s)\, ds \]
\[ + \mu E[V_i(t_j)] \int_{0}^{t} e^{\alpha(t-s)}\beta J_i(s)\, ds + \ldots + \int_{0}^{t} e^{\alpha(t-s)}\beta J_i(s)\, ds \]
\[ = e^{\alpha t}E[V_i(t)] + e^{\alpha t} \sum_{i=0}^{n-1} e^{\alpha(t-s)}\beta J_i(s)\, ds \]
\[ \leq e^{\alpha t}E[V_i(t)] + e^{\alpha t} \sum_{i=0}^{n-1} e^{\alpha(t-s)}\beta \lambda_m(s)\nu(s)\, ds \]
\[ = e^{\alpha t}E[V_i(t)] + e^{\alpha t} \sum_{i=0}^{n-1} e^{\alpha(t-s)}\beta \lambda_m(s)\nu(s)\, ds \]
Noticing that $N_i(0,T) \leq T/T_n$, we have
\[ E\left[ x^T(t)R(t) \right] < \frac{(\lambda \hat{\lambda} + \lambda \hat{\lambda} h_i) + \lambda \hat{\lambda} \beta \hat{\psi}}{\hat{\lambda}} e^{\alpha t} \frac{T}{T_n} \]
When $\mu = 1$, \[ E\left[ x^T(t)R(t) \right] < c_2 e^{\alpha t} \frac{T}{T_n} = c_2 \]
When $\mu > 1$, we have
\[ \ln(\lambda \hat{\psi}) - \ln[(\lambda \hat{\lambda} + \lambda \hat{\lambda} h_i) + \lambda \hat{\lambda} \beta \hat{\psi}] - \alpha T > 0 \]
and (16) is equivalent to
\[ T < \frac{\ln \mu}{\ln(\lambda \hat{\psi}) - \ln[(\lambda \hat{\lambda} + \lambda \hat{\lambda} h_i) + \lambda \hat{\lambda} \beta \hat{\psi}] - \alpha T} \]
Substituting (28) to (26), we have
\[ E[x^T(t)R(t)] < \frac{(\lambda \hat{\lambda} + \lambda \hat{\lambda} h_i) + \lambda \hat{\lambda} \beta \hat{\psi}}{\hat{\lambda}} e^{\alpha t} e^{\frac{\ln(\lambda \hat{\psi}) - \ln[(\lambda \hat{\lambda} + \lambda \hat{\lambda} h_i) + \lambda \hat{\lambda} \beta \hat{\psi}] - \alpha T}{\hat{\lambda}}} = c_2 \]
The proof is completed.

3.2 $H_{\infty}$ performance analysis

In this subsection, we focus on the $H_{\infty}$ performance analysis of the switched stochastic systems with time delay. Consider the following switched stochastic system
\[ dx(t) = \begin{bmatrix} A_{m(i)} x(t) + A_{m(i)} x(t-h(t)) + B_{m(i)} u(t) \\ + G_{m(i)}(t) \end{bmatrix} dt + D_{m(i)} x(t) \, dw(t) \]
\[ x(t) = \varphi(t), \quad t \in [-h,0] \]
where $m(i)$ is the current model index and $\varphi(t)$ is the initial condition.

Theorem 2 For any scalars $\alpha, \beta, \gamma > 0$, $h_i < 1$, if there exist symmetric positive definite matrices $P_i, Q_i > 0$ such that the following inequalities
\[ \lambda_i R < P_i < \lambda_i I, \quad Q_i < \lambda_i I \]
\[ \lambda_i c_2 \quad (\lambda_i + \lambda_i h_i) c_1 + \gamma \psi > e^{\alpha t} \]
\[ \begin{bmatrix} A_i^T P_i + P_i A_i + Q_i - \alpha P_i & P_i A_i & -Q_i & \ast & \ast \\ \ast & -1(h_i)Q_i & \ast & \ast & \ast \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ \ast & -\gamma I & 0 & 0 & 0 \end{bmatrix} < 0 \]
hold for all $i \in \mathbb{N}$, and the average dwell time satisfies
\[ T_n > T'_n = \max\left\{ t \left| \frac{\ln \mu}{\ln(\lambda_i \hat{\psi}) - \ln[(\lambda_i \hat{\lambda} + \lambda_i \hat{\lambda} h_i) + \lambda_i \hat{\lambda} \beta \hat{\psi}] - \alpha T} \right. \right\}, \]
where $\mu \geq 1$ satisfies
\[ P_i \leq \mu P_i, \quad Q_i \leq \mu Q_i, \quad \forall i, j \in \mathbb{N}. \]
Then system (30) is finite-time stochastically bounded with weighted $H_{\infty}$ performance $\gamma$ with respect to $(c_1, c_2, T, R, \psi, \sigma)$.

Proof It is an obvious fact that (34) implies the following inequality
By Theorem 1, we can obtain that system (30) is finite-time stochastically bounded with respect to $(c_1, c_2, T, R, \psi, \sigma)$. Now we are in a position to prove that system (30) have weighted $H_\infty$ performance $\gamma$ with respect to $(c_1, c_2, T, R, \psi, \sigma)$.

Choose the same Lyapunov functional as in (18) for the system (11), following the proof line of Theorem 1, we have

$$\dot{V}(t) < \alpha V(t) + \Gamma(t),$$

where $\Gamma(t) = \gamma^2 \dot{V}(t) + z^T(t)z(t)$.

Moreover, we can get that

$$\dot{V}(t) < \alpha V(t) + \Gamma(t).$$

Noticing that

$$d(e^{-\alpha t}V(t)) = -e^{-\alpha t}V(t)dt + e^{-\alpha t}dV(t)$$

and

$$d(e^{-\alpha t}V(t))dt = 2e^{-\alpha t}x^T(t)P_{22}x(t)dt$$

we have

$$\dot{V}(t) = \alpha V(t) + \Gamma(t).$$

Integrating both sides of (40) and taking expectations, we have

$$E\{V(t)\} < e^{\alpha t - \gamma^2}V(0) + \int_{t-\gamma^2}^{t} e^{\alpha t - \gamma^2} \Gamma(s)ds.$$  

For any $t \in [t_i, t_{i+1})$, using Itô formula, we can obtain that

$$\dot{V}(t) = e^{\alpha t - \gamma^2}V(t) + \int_{t}^{t+\gamma^2} e^{\alpha t'} \dot{V}(t')dt'$$

Moreover

$$\dot{V}(t) = \alpha V(t) + \int_{t}^{t+\gamma^2} \Gamma(t)dt'$$

Integrating both sides of (40) with respect to $12(, ,,,)$ and taking expectations, we have

$$\dot{V}(t) = \alpha V(t) + \int_{t}^{t+\gamma^2} \Gamma(t)dt'$$

Under the zero initial condition, we have

$$0 \leq E\{V(t)\} < \int_{0}^{t} e^{\alpha t - \gamma^2} \Gamma(s)ds.$$  

Moreover

$$\int_{0}^{t} e^{\alpha t - \gamma^2} \Gamma(s)ds < \int_{0}^{t} \mu_2 \mu_4 \gamma^2(s)\dot{V}(s)ds$$

Multiplying both sides of (43) by $\mu_4 \dot{V}(s)$ leads to

$$\int_{0}^{t} e^{\alpha t - \gamma^2} \Gamma(s)ds < \int_{0}^{t} \mu_2 \mu_4 \gamma^2(s)\dot{V}(s)ds$$

Noticing that $N_2(0,s) \leq \frac{8}{T_\mu}$ and $T_\mu \leq \frac{\ln \mu}{\alpha}$, we have

$$\int_{0}^{t} e^{\alpha t - \gamma^2} \Gamma(s)ds < \int_{0}^{t} \mu_2 \mu_4 \gamma^2(s)\dot{V}(s)ds$$

Let $t = T$, multiplying both sides of (45) by $e^{-\alpha t}$, we have

$$\int_{0}^{T} e^{-\alpha t} \dot{V}(s)ds < \int_{0}^{T} e^{-\alpha t} \gamma^2(s)\dot{V}(s)ds < \int_{0}^{T} \Gamma(s)ds$$

Setting $\lambda = 2\alpha$, according to Definition 3, we know that Theorem 2 holds. The proof is completed.

### 3.3 Robust $H_\infty$ finite-time control

Consider system (1), under the state feedback controller $u(t) = K_{\sigma(t)}x(t)$, $t \in [0, T]$, the corresponding closed-loop system can be described as

$$\dot{x}(t) = (\hat{A}_{\sigma(t)} + B_{\sigma(t)}K_{\sigma(t)})x(t) + \hat{B}_{\sigma(t)}x(t-h(t))$$

$$+ G_{\sigma(t)}\varphi(t)$$

$$x(t) = \varphi(t), t \in [-h, 0]$$

$$z(t) = M_{\sigma(t)}x(t)$$

Theorem 3 Consider system (1), for any scalars $\alpha, \beta, \gamma > 0$, $h_2 < 1$, if there exist matrix $Z$, and symmetric positive definite matrices $X_1, \dot{X}_1 < 0$ such that

$$\lambda_1 \dot{X}_1 < X_1 < \lambda_1 \dot{X}_1,$$

$X_1 \dot{X}_1 \leq \lambda_1 I,$

$$\lambda_2 \dot{X}_1 > X_1,$$

$$X_1 \dot{X}_1 \leq \lambda_1 I,$$

Then, under the designed controller $u(t) = K_{\sigma(t)}x(t)$, the corresponding closed-loop system (47) is finite-time stochastically bounded with weighted $H_\infty$ performance $\gamma$ with respect to $(c_1, c_2, T, R, \psi, \sigma)$. Moreover, the control gain $K_i = Z_i X_i^{-1}, \forall i \in N$.

Proof By Theorem 2, system (47) is finite-time stochastically bounded with weighted $H_\infty$ performance $\gamma$ with respect to $(c_1, c_2, T, R, \psi, \sigma)$ if the following inequality is satisfied

$$\begin{bmatrix}
A_{11} & PA_0 & PG & DP & M_1
\end{bmatrix}$$

$$\begin{bmatrix}
\begin{array}{cccc}
\alpha & -1 & 0 & 0 \\
\gamma^2I & 0 & 0 & 0 \\
\gamma & -P_{11} & 0 & 0 \\
\gamma & -P_{11} & -I & 0 \\
\end{array}
\end{bmatrix} < 0$$

where $A_{11} = (\hat{A} + B_{\sigma(t)}K)^T P + P(\hat{A} + B_{\sigma(t)}K) - \alpha P + Q$.

Using $\Omega = diag\{P_{11}, \gamma^2I, \gamma, -I\}$ to pre- and post-multiply the left side of (54) leads to
where $\dot{T}_{11} = X_i(\dot{A} + B K_i) + (\dot{A} + B K_i) X_i - \alpha X_i + Y_i$, $T_{22} = -(1-h_2)Y_i$, $X_i = P_i^{-1}, Y_i = P_i^{-1} Q P_i^{-1}$.

Noticing (5), we have

$$\dot{T}_i = T_i - \Delta T_i,$$

where

$$T_{i1} = X_i(A + B K_i) + (A + B K_i) X_i - \alpha X_i + Y_i$$

$$T_{i2} = -(1-h_2)Y_i.$$

By Lemma 1, we have

$$\Delta T_i \leq \varepsilon_i$$

Using Schur complement, we can obtain from (51) that the inequality (55) holds. Denote $X_i = P_i^{-1}$ and $Y_i = P_i^{-1} Q P_i^{-1}$, we can get that (48), (49), (53) are equivalent to (31), (32), (36), respectively. The proof is completed.

### 4 Numerical example

In this section, a numerical example is presented to illustrate the effectiveness of the proposed results. Consider system (1) with the following parameters

$$\dot{X}_i = \begin{bmatrix} 2.3 & 0.8 \\ 1.2 & 3.5 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad B_i = \begin{bmatrix} 2 & 3 \\ 5 & 2 \end{bmatrix},$$

$$D_i = \begin{bmatrix} -0.4 & 0 \\ 0 & -0.6 \end{bmatrix}, \quad M_i = \begin{bmatrix} 0.6 & 0.3 \\ 0.1 & 0.1 \end{bmatrix},$$

$$E_{11} = \begin{bmatrix} 0 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad E_{21} = \begin{bmatrix} 0 & 0 \\ -0.1 & 0 \end{bmatrix}, \quad E_{31} = \begin{bmatrix} 0 & -0.1 \\ 0 & 0 \end{bmatrix},$$

$$H_i = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad A_i = \begin{bmatrix} 2.8 & 1.7 \\ 0.6 & 1.9 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} -0.5 & 0.3 \\ 0 & -0.4 \end{bmatrix},$$

$$B_i = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}, \quad D_i = \begin{bmatrix} 0 & 0 \\ 0 & -0.1 \end{bmatrix}, \quad M_i = \begin{bmatrix} 0.7 & 0.4 \end{bmatrix},$$

$$E_{22} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad E_{12} = \begin{bmatrix} 0.1 & 0 \end{bmatrix}, \quad E_{32} = \begin{bmatrix} 0 \end{bmatrix},$$

$$H_i = \begin{bmatrix} 0.1 \end{bmatrix}, \quad H_i = \begin{bmatrix} 0.2 \end{bmatrix}, \quad H_i = \begin{bmatrix} 0 \end{bmatrix},$$

$$F_i = \begin{bmatrix} \sin t \\ \cos t \end{bmatrix}, \quad F_i = \begin{bmatrix} \cos t \\ \sin t \end{bmatrix},$$

the disturbance input $v(t) = \begin{bmatrix} 1.2e^{-0.5t} \\ 1.6e^{-0.7t} \end{bmatrix}$.

The values of $c_1, c_2, T, \psi, R$ are given as follows:

$c_1 = 1$, $c_2 = 8$, $T = 3$, $\psi = 2$, $R = I$.

Choosing $\alpha = 0.1$, $h(t) = 0.5 + 0.2 \sin t$, $c_i = \varepsilon_i = 1$, $\gamma = 1$, we have $h_2 = 0.2$, $h_2 = 0.5$. Then, solving the LMIs in Theorem 3, we have

$$X_i = \begin{bmatrix} 0.7373 & -0.0786 \\ -0.0786 & 0.8400 \end{bmatrix}, \quad Y_i = \begin{bmatrix} 1.3564 \\ 0 \end{bmatrix} 1.3564,$$

$$Z_i = \begin{bmatrix} 0.2306 & -2.0030 \\ -2.0030 & 0.7387 \end{bmatrix}, \quad X_i = \begin{bmatrix} 0.7194 & -0.0615 \\ -0.0615 & 0.8476 \end{bmatrix}.$$

$$Y_i = \begin{bmatrix} 1.3905 \\ 1.3905 \end{bmatrix}, \quad Z_i = \begin{bmatrix} -1.9526 & 3.9247 \\ 3.9247 & -6.6231 \end{bmatrix}.$$

Because of $K_i = X_i^{-1}$, $i = \{1, 2\}$, we have

$$K_i = \begin{bmatrix} 0.0591 & -2.3790 \\ -2.3790 & 4.4604 \end{bmatrix}, \quad K_i = \begin{bmatrix} 2.3736 & 4.4604 \\ 4.4604 & 1.0388 \end{bmatrix} -7.7382.'$$

Then, we can obtain that $\mu = 1.0492$ and $T_i = 0.4803$.

According to the Theorem 3, we know that under the average dwell time $T_i > T_i$, the designed controller can guarantee that system (1) is finite-time stochastically bounded with weighted $H_\infty$ performance $\gamma$ with respect to $(1,8,3,1,2,\sigma)$.

The simulation results are shown in Figs.1-3, where $x(t) = [0,0]^T$ for $t \in [-0.5,0]$, and $x(0) = [0.6,-0.8]^T$, $T_i = 0.5$. Fig.1 shows the switching signal of the switched system with the average dwell time $T_i = 0.5$. Fig.2 and Fig.3 present the state trajectories of the closed-loop switched system. The simulation results illustrate the effectiveness of the proposed approach.
5 Conclusions

In this paper, the problem of robust finite-time $H_{\infty}$ control for switched stochastic systems with time delay has been investigated. Based on the average dwell time method, sufficient conditions are presented to guarantee that the switched stochastic delay system is finite-time stochastically bounded. Moreover, finite-time $H_{\infty}$ performance is discussed and robust finite-time $H_{\infty}$ controller is designed to guarantee that the corresponding closed-loop system is finite-time stochastically bounded with $H_{\infty}$ performance. Finally, a numerical example is provided to illustrate the effectiveness of the proposed approach.

References


