Revised multi-segment goal programming: Percentage goal programming

Ching-Ter Chang, Huang-Mu Chen, Zheng-Yun Zhuang*

The Graduate Institute of Business Management, Chang Gung University, (333) 259 Wen-Hwa 1st Road, Kwei-Shan, Tao-Yuan, Taiwan

A R T I C L E   I N F O

Article history:
Received 16 February 2012
Received in revised form 24 August 2012
Accepted 27 August 2012
Available online 12 September 2012

Keywords:
Goal programming
Multi-segment
Multi-choice
Percentage
Coefficient
In-between selection

A B S T R A C T

The multi-segment goal programming (MSGP) model is an extension model of GP wherein the core thinking is inherited from the multi-choice goal programming (MCGP) model. In this paper, we recommend certain points of the MSGP model and offer a Revised MSGP Model as an aid to burdened decision makers who cannot expect an either-or selection of coefficients in practice. The proposed model takes into account a scenario in which the selection of all possible coefficients pertaining to each decision variable in the MSGP model can be an in-between selection instead of an exclusive-or selection. We hope this study can fill in a possible gap that might exist when applying the MSGP model, and can offer an extension model for practitioners when they use this model to solve related decision problems.

© 2012 Elsevier Ltd. All rights reserved.

1. Introduction

Goal programming (GP) has gained wide popularity since it was proposed by Charnes and Cooper (1961). Many extension models with descendants have been derived in the past decades (Aouni & Kettani, 2001). These include weighted GP, fuzzy GP, lexicographical GP, min–max GP, integer GP and multi-choice GP (MCGP) (Chang, 2007; Lin, 2004; Romero, 2001; Tamiz, Jones, & El-Darzi, 1995; Tamiz, Jones, & Romero, 1998; Zimmermann, 1978). Other decision-making process-related or decision maker (DM)’s preference incorporated models have also been proposed, such as interactive GP (Dyer, 1972), the promethee method with GP (Martel & Aouni, 1990) and GP with utility functions (Chang, 2011). Also, many integrated GP models such as the Fuzzy MCGP (Bankian-Tabrizi, Shahanaghi, & Jabalameli, 2012) have been proposed, while some GP mathematicians have become passionate about the equivalence construal of the models (Mohamed, 1997).

Among the models mentioned above, the MCGP model (Chang, 2007, 2008) is a GP model that allows the right-hand-side (RHS) of each goal to be varied among two or more aspiration levels. With MCGP, a DM can consider multiple levels of aspired target values for each goal. This model is particularly helpful for DMs who are not that certain about their own aspired levels of goals, and is especially helpful for the internal control of enterprise general managers who would not like to see their sales potential under-estimated or cost over-estimated. Following the core spirit of MCGP, Multi-segment GP (MSGP) has been proposed by Liao (2009). In the MSGP model, the coefficient on the left-hand-side (LHS) allows the DMs to set multiple segments of a coefficient on the LHS for decision variables.

As Liao (2009) has mentioned, “if only two-segment aspiration levels exist in each market, this is a case of a multi-objective decision making (MODM) problem with an either-selection”. In real cases of pricing issues in marketing, there is a common problem faced by DMs, especially when price discrimination (PD) policy is executed. Consider a simple case in which we would like to sell a product to two groups of customers, VIP customers (VIPC) and normal customers (NC). In this case, the product could be marked with two different prices. The MSGP model is then very suitable for being applied to a vendor who sells many types of product to either VIP or NC at the same time. However, as Liao (2009) has mentioned, the model is an ‘either-selection’. This implies that the capability of the MSGP model is restricted to the PD problem, due to the fact that the uncertain but possible coefficients pertaining to each decision variable in the MSGP model have an exclusive-or relationship.

To show this problem, take the following example from Liao (2009),

\[ g_1 : (3 \text{ or } 6)x_1 + 2x_2 + x_3 = 115 \]  
\[ g_2 : 4x_1 + (5 \text{ or } 9)x_2 + 2x_3 = 80 \]  
\[ g_3 : 3.5x_1 + 5x_2 + (7 \text{ or } 10)x_3 = 110 \]  
\[ x_2 + x_3 \geq 9, \ x_2 \geq 5, \ x_1 + x_2 + x_3 \geq 21 \]

* Corresponding author. Tel.: +886 3 2118800x3713/3694; fax: +886 3 2118212.
E-mail addresses: chingter@mail.cgu.edu.tw (C.-T. Chang), jckychen@gmail.com (H.-M. Chen), waynemcgwire@yahoo.com, d9940001@stmail.cgu.edu.tw (Z.-Y. Zhuang).

0360-8352/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.cie.2012.08.005
P1 can be expressed by the MSGP model as follows:
\[
\begin{align*}
\min & \quad z = d_1^+ + d_1^- + d_2^+ + d_2^- + d_3^+ + d_3^- \\
\text{s.t.} & \quad (3b_1 + 6(1 - b_1))x_1 + 2x_2 + x_3 - d_1^+ + d_1^- = 115 \\
& \quad 4x_1 + (5b_2 + 9(1 - b_2))x_2 + x_3 - d_2^+ + d_2^- = 80 \\
& \quad 3.5x_1 + 5x_2 + (7b_3 + 10(1 - b_3))x_3 - d_3^+ + d_3^- = 110 \\
& \quad d_i^+ - d_i^- \geq 0, \quad i = 1, 2, 3
\end{align*}
\]
where \(b_1, b_2, \) and \(b_3\) are binary variables; \(d_i^+\) and \(d_i^-\) are the positive and negative deviation variables, respectively.

The solution obtained by Liao (2009) is as follows:
\[
(x_1, x_2, x_3, b_1, b_2, b_3) = (11.54, 5, 4.46, 0, 1, 0)
\]

As the result, for \(g_1\), the coefficient 6 for decision variable \(x_1\) takes effect, while the coefficient 5 for decision variable \(x_2\) takes effect for \(g_2\), and the coefficient 10 for \(x_3\) takes effect for \(g_3\).

This study proposes an extension model of MSGP, called the Revised MSGP Model (RMSGP), wherein all possible coefficients of each decision variable, \(x_i\), are not alone. In order to deal with this concern, a concept of mixture percentage \((s_i)\) of the possible coefficients is introduced. With P1 for example, such a concept represents the percentage of goods to be sold to one customer group, as opposed to another. That is, taking the two uncertain coefficients \((i.e., 3 \text{ or } 6)\) for \(x_1\) in \(g_1\), of P1, the proposed model can determine a mixture percentage, \(\tau_i\) \((\text{the solution for the variable } \tau_i)\) that represents how the two coefficients act together when being solved, and \(\tau_i x_i\) and \((1 - \tau_i) x_i\) indicate the exact quantities to be sold to the VPC and NC, respectively. This is especially useful for marketing decisions, where it is necessary to decide a proper product portfolio in advance.

A percentage GP approach (%GP) is also proposed to solve problems modeled using the RMSGP. This approach is capable of solving an uncertain decision variable coefficient that is continuous between both ends. In fact, uncertainties of objective function coefficients, of constraint’s RHS values and of constraint’s LHS coefficients, are the three main categories of uncertainties in linear programming (LP) models. These uncertainties can be represented by interval numbers or fuzzy numbers (Tong, 1994). The uncertain coefficient modeled by %GP is somehow akin to the interval number concept of the uncertain constraint’s LHS coefficients category. Nevertheless, most existing approaches solve problems with coefficients that have intrinsic uncertainty. While the application scenario of the MSGP/RMSGP is not intrinsically uncertain, it requires the solution of a “non-intrinsic, in-between uncertainty”, wherein the two ends of the interval are known and fixed prior to modeling. This raises the need for a %GP approach that appropriates the scenario of the RMSGP.

In summary, by furnishing the possible flaw of MSGP, this study not only provides supplements to the trends of MCGP and MSGP research, but also improves to the field of GP models. The practical use of the proposed model also can be expected.

The remainder of this paper is organized as follows: Section 2 describes the details from the prototype of MSGP to the proposed RMSGP model as well as the %GP approach. A brief example with contextual settings is also given in the section to show how the proposed model can assist the decision making process in marketing considering product portfolio and PD. By taking an exemplar from the previous MSGP study, Section 3 demonstrates the solution process of RMSGP by %GP and compares the result against that of traditional MSGP (solved by mixed binary GP). Section 4 encompasses the implications and contributions of the proposed model, and Section 5 offers concluding.

2. Modeling the percentage GP

2.1. The Revised MSGP Model

Akin to any GP model, the simultaneous equations of the MSGP model can be represented in a matrix form as follows:

\[
\text{(MSGP)} \min \quad (+) \quad (D^+ + D^-)
\]
\[
CX - D^+ + D^- = A
\]

where \(X\) is the \(n \times 1\) decision vector, \(C\) is a \(k \times n\) matrix of coefficients wherein each element \(c_{ij}\) contains possible coefficients semantically linked with the exclusive-or (XOR) operator; \(D^+\) and \(D^-\) are \(n \times 1\) vectors of surplus and slack deviations; \((+)\) is the direct sum operator taking all the elements of the \(n \times 1\) vector as summands, and \(A\) is the \(k \times 1\) vector that describes the aspiration levels of each of the \(k\) goals.

Thus, P1 from the MSGP study (Liao, 2009) can be written as follows:

\[
\begin{align*}
(P2) & \quad \text{(Goal)} \\
& \quad \begin{bmatrix} 3 \text{ XOR } 6 \end{bmatrix} \begin{bmatrix} 2 \end{bmatrix} 1 \\
& \quad \begin{bmatrix} 4 \end{bmatrix} \begin{bmatrix} 5 \text{ XOR } 9 \end{bmatrix} 2 \\
& \quad \begin{bmatrix} \frac{7}{2} \end{bmatrix} \begin{bmatrix} 5 \end{bmatrix} \begin{bmatrix} 7 \text{ XOR } 10 \end{bmatrix} 115 \\
& \quad \begin{bmatrix} 80 \end{bmatrix} \begin{bmatrix} 110 \end{bmatrix}
\end{align*}
\]

For the determination of an appropriate coefficient value in-between an interval (e.g. in-between 3 and 6), coefficients transformation from discrete numbers to interval numbers is needed (we adhere to the interval coefficient expression in Tong’s research (1994)):}

\[
\begin{align*}
(P3) & \quad \begin{bmatrix} [3, 6] \end{bmatrix} \begin{bmatrix} 2 \end{bmatrix} 1 \\
& \quad \begin{bmatrix} 4 \end{bmatrix} \begin{bmatrix} [5, 9] \end{bmatrix} 2 \\
& \quad \begin{bmatrix} \frac{7}{2} \end{bmatrix} \begin{bmatrix} 5 \end{bmatrix} \begin{bmatrix} [7, 10] \end{bmatrix} 115 \\
& \quad \begin{bmatrix} 80 \end{bmatrix} \begin{bmatrix} 110 \end{bmatrix}
\end{align*}
\]

These simultaneous equations can be re-written as follows:

\[
\begin{align*}
(P4) & \quad \begin{bmatrix} \xi_{11}(3, 6) \end{bmatrix} \begin{bmatrix} \xi_{12}(2, 2) \end{bmatrix} 1 \\
& \quad \begin{bmatrix} \xi_{21}(4, 4) \end{bmatrix} \begin{bmatrix} \xi_{22}(5, 9) \end{bmatrix} 2 \\
& \quad \begin{bmatrix} \xi_{31}(\frac{7}{2}) \end{bmatrix} \begin{bmatrix} \xi_{32}(5, 5) \end{bmatrix} \begin{bmatrix} \xi_{33}(7, 10) \end{bmatrix} 115 \\
& \quad \begin{bmatrix} x_1 \end{bmatrix} \begin{bmatrix} x_2 \end{bmatrix} \begin{bmatrix} x_3 \end{bmatrix} \begin{bmatrix} 80 \end{bmatrix} \begin{bmatrix} 110 \end{bmatrix}
\end{align*}
\]

where \(\xi_{31}(3, 6)\) is a “function” in terms of an upper bound of the coefficient 6 and a lower bound of the coefficient 3 (the output value of \(\xi_{11}(3, 6) \in [3, 6]\)) \(\xi_{22}(2, 2)\) is a function that produces a coefficient value of 2; other coefficients in terms of \(\xi_{ij}(a_{ij}, b_{ij})\) functions can be recognized correspondingly, in which \(i\) and \(j\) are indices for the coefficient function of the \(j\)th decision variable \((x_j)\) in the \(i\)th goal criteria.

Thus, the coefficient matrix \(C\) in the MSGP model can be no longer a matrix in which the elements \((c_{ij})\) are simply or-linked values. It becomes another matrix \(\Theta\) with function elements \(\xi_{ij}(a_{ij}, b_{ij})\), each of which is a ‘coefficient function’. Therefore, the model can be re-defined as follows:

\[
\text{(Revised MSGP)} \min \quad (+) \quad (D^+ + D^-)
\]
\[
\Theta X - D^+ + D^- = A
\]

where \(\Theta\) is a \(k \times n\) matrix of coefficients wherein each element \(\xi_{ij}(a_{ij}, b_{ij})\) is a ‘coefficient function’ in terms of its upper and lower bounds \(a_{ij}\) and \(b_{ij}\); other variables are defined as in (MSGP).
2.2. The %GP approach for solving the RMSGP model

Before proposing the %GP approach to solve the RMSGP model, there is a brief review of the relevant literature.

2.2.1. Interval coefficients

There are three main uncertainties in LP, namely, the objective function coefficient uncertainty, the constraint’s RHS uncertainty and the constraint’s LHS coefficients uncertainty. Tong [1994] categorized fuzzy-related LP into two types: interval number LP (INLP) and fuzzy number LP (FNLP). An INLP problem can be expressed in a general form as:

$$\min Z = \sum_{j=1}^{n} [c_j, d_j]x_j$$

$$\sum_{j=1}^{n} [a_{ij}, b_{ij}]x_j \geq [c_i, f_i], \forall i$$

As indicated, the above constraints, looped by i, can be expanded into $2n + 1$ inequalities. The author suggested a wise and reduced method to separate the model into two classical LPs that contain maximum value range inequalities and minimal value range inequalities, to be solved individually, for the determination of a more informative optimal interval solution of the INLP model.

As can be seen from the study, although the interval coefficient (Bitran, 1980) in the constraints’ LHS, represented by an interval number, is similar to the expression of the RMSGP, the interval number is an “unknown number” that falls into an approximate range, bounded by $a_i$ and $b_i$. However, it is not a function; in the example for INLP, the DM has “known ranges of coefficients” for the inherently unknown coefficients (i.e., this implies an “intrinsic uncertainty”) and the intention is still to solve the decision vector which contains $x_i$ while the problem cases for RMSGP have fixed, known ends of the interval range.

Some points are addressed in the study. (i) Information: the solution does not imply one certain solution with one certain minimized target, but implies ranges of decision variables and the objective function (i.e., it does not suggest a deterministic value for either each decision variable or the objective value). (ii) Types of LP uncertainties: the objective function coefficients, the constraint RHS values and the constraint LHS coefficients can be intervals, although they are not necessarily intervals.

Inuiuchi and Sakawa (1995) used the minimax and maximin decision theories to solve LP problems with interval objective function coefficients. They solved such problems using the proposed method of a “relaxation procedure”. In this procedure, a minimax regret solution can be obtained when the reference solution set is possibly optimal. The article is influential in its methodological contribution to solving the problem with a “relaxation procedure” and the repetitive use of the simplex method.

The main application of the abovementioned study is to the objective function with interval coefficients, not on the constraint LHS coefficients, as the RMSGP addresses.

Chinneck and Ramadan (2000) pointed out the insufficiency of hit-and-miss approaches to the variation of the uncertain coefficients, because of their inability to uncover the complete range of possible optimum objective function values. They proposed an approximate model with a new approach to find the best optimum and the worst optimum, in which four algorithmic tools are used to support two types of LPIC (LP with interval coefficients) problems.

In the paper, the authors clearly conclude that there are three kinds of “specific values to be chosen”, in LP. In addition, the application scenario in the study uses values of coefficients that are “approximately known” or “general estimates” (i.e., this again implies intrinsic uncertainty). This leads to the key assumption of this study that “any unknown coefficient can be expressed as an interval (a lower- and upper-bounded range of real numbers)”.

The above statement demonstrates the difference in the application scenarios between the abovementioned study and MSGP/RMSGP. In the application scenario of the RMSGP, the coefficient is not an “approximately known, general estimate”, but a value to be decided, within a known range with two known ends, which must be expressed as an interval. That is, the application scenario of previous studies is that the DM can say “the coefficient of $x_1$ in the first constraint is an approximate value ranging from 5 to 9”, which perhaps is based on a personal understanding of the decision problem that this coefficient is “$7 \pm 2$”, but not certain. The value of $x_1$ is what is desired, but the uncertain coefficients in such a problem can be also expressed by fuzzy numbers, in addition to interval numbers. Rather, the application scenario of RMSGP can be that a DM says, “the coefficient of $x_1$ in the first constraint is, certainly, a value between 5 and 9”, based on the fixed available data to hand. Actually, both the value of $x_1$ and the value of “how much the coefficient is from 5 toward 9” is what is desired.

Oliveira and Antunes (2007) offered an illustrative overview of interval coefficient issues in MOLP models in a wider scope with a definition of “the intrinsic uncertainty associated with the model coefficients”. Causes that have induced “the model coefficients are not exactly known (the intrinsic uncertainty associated with the model coefficients)”, include situations where the relevant data is nonexistent or scarce, difficult to obtain or estimate, or where the system is subject to changes, etc. They found that interval programming allows a model with interval coefficients, on the mere assumption of “known information pertaining to the coefficient ranges”.

They categorized existing approaches mainly into satisficing approaches and optimizing approaches. Methods for satisficing approaches were introduced sequentially: for-LP methods, for-GP methods and interactive approaches. Then many methods and algorithms for optimizing approaches were introduced, according to their necessary/possible efficiency classification, as classified in the article.

In summary, situations pertaining to the programming model coefficients like, but not limited to situations where the relevant data is nonexistent or scarce, difficult to obtain or estimate, or where the system is subject to changes, are the main causes of not-exactly-known or unknown model coefficients and lead to intrinsic uncertainties associated with the model coefficients. A good example of this is, again, the DM’s consideration of coefficients before modeling: a guessed interval can be set and stated, according to any expectation of the coefficients, or expectations of the coefficients can be approximately set and stated.

However, although the models of RMSGP, when detailed, are quite similar to the interval numbers, in the application scenario of RMSGP (or MSGP), nothing is inherently uncertain. Neither does the model have intrinsic uncertain property, nor is its coefficient-relevant data non-deterministic (it is previously available). Neither is it difficult to estimate, nor is the model system subject to change. That is, the relevant data for setting the discrete constraint coefficients in the MSGP and for setting the in-between uncertain coefficients in RMSGP are uncertain, but not intrinsically uncertain (i.e., the in-between ranges are not intrinsically uncertain before modeling; they are defined ranges which are only uncertain during solution).

2.2.2. The %GP approach

In order to deal with the in-between uncertainty in the RMSGP model, a percentage-based distance measure (%dm) variable $s_i$ pertaining to each true decision variable $x_i$ is introduced. Then the definition of $s_i (a_{ij}, b_{ij})$ can be given as follows:
where $\delta_y$ is the range (distance) describing the extent to which $z_i^o(a_y, b_y)$ can vary from $a_y$ (i.e., $|b_y - a_y|$). $v_j$ is the by-percentage distance measure of the variation that describes how much, from 0% to 100%, a coefficient value departs from $a_y$ toward $b_y$.

Now consider the proper form of $z^o(a_y, \delta_y, v_j)$. Since its value depends on the two decision-contextual moderating constants (the starting value of the coefficient function $a_y$ and the possible variation range of the function $\delta_y$ defined previously) and a newly additional %dm decision variable $v_j$ (instead of $b_y$) to decide for $x_{ij}$, we can consider the functional form of $z^o(a_y, \delta_y, v_j)$ as follows:

$$z^o(a_y, \delta_y, v_j) = \begin{cases} a_y + \tau_j \times \delta_y, & \text{if } a_y \leq b_y \text{ (i.e., } \delta_y \neq 0) \\ \tau_j, & \text{if } a_y = b_y \text{ (i.e., } \delta_y = 0) \\ a_y - \tau_j \times \delta_y, & \text{if } a_y > b_y \text{ (i.e., } \delta_y = 0) \end{cases}$$

Note that we consider a case in which it is not necessarily for $a_y$ to be less than or equal to $b_y$. This consideration is practical because the direction of the coefficient of every constraint of each goal is relaxed arbitrarily (unlike the example case taken in MSGP). Taking the former example P1, if the coefficient of the first constraint is price, then the coefficient 3 or 6 may connote the price for VIPC or NC respectively in some currency for a certain product. However, if the coefficient of some constraint in the above case is not price, but cost on servicing, then the scenario can be inverted (e.g., serving a VIPC customer is often more expensive than serving an NC one).

Then, it is possible for the in-between relationship, in such a case, to be in-between 6 and 3. This is just to describe that both decision cases such as “in-between 3 and 6” and “in-between 6 and 3” can be taken into account by the proposed model. Fig. 1 shows the two general forms of the coefficient function discussed here.

Thus, by introducing %dm variables, the final model of Revised MSGP can be reconstructed as the percentage GP (%GP) model as follows:

(Percentage GP)

$$\min \ (+) (D^+ + D^-)$$

$$Z \times (B + A \times T) X = A + D^+ - D^-$$

where $Z$ is a $k \times n$ matrix of coefficients with each element $z^o(a_y, \delta_y, v_j)$ is a coefficient function in terms of a starting point $a_y$, a range parameter $\delta_y$, and a newly introduced %dm variable $v_j$; the matrix $B$, $A$, and $T$ are defined as follows:

$$B = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & \ldots & \ldots & \ldots \\ \vdots & \vdots & \ddots & \vdots \\ a_{k1} & \ldots & \ldots & a_{kn} \end{bmatrix}_{k \times N}$$

$$A = \begin{bmatrix} \delta_{11} \tau_1 & \delta_{12} \tau_2 & \ldots & \delta_{1n} \tau_n \\ \delta_{21} \tau_1 & \ldots & \ldots & \ldots \\ \vdots & \vdots & \ddots & \vdots \\ \delta_{k1} \tau_1 & \ldots & \ldots & \delta_{kn} \tau_n \end{bmatrix}_{k \times N}$$

$$T = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \vdots \\ \tau_n \end{bmatrix}_{n \times 1}$$

And if $A \times T$ is a square matrix (e.g., $k = n = 3$ in the example), then $A \times T$ can be further decomposed so that it is a multiplication of $A$ (containing ranges of coefficients only) and $T$ (containing %dm variables). That is,

$$A \times T = \begin{bmatrix} \delta_{11} \tau_1 & \delta_{12} \tau_2 & \ldots & \delta_{1n} \tau_n \\ \delta_{21} \tau_1 & \ldots & \ldots & \ldots \\ \vdots & \vdots & \ddots & \vdots \\ \delta_{k1} \tau_1 & \ldots & \ldots & \delta_{kn} \tau_n \end{bmatrix}_{k \times N}$$

As such, the thing of most importance now is that the $Z$ matrix that can be decomposed to a sum of a $k \times n$ matrix $B$ stating the “starting points” of coefficients and a product of the matrix $A$ stating the “physical ranges” of coefficients multiplied by the diagonal matrix $T$ (containing the %dm variables) that is to be determined (containing the auxiliary decision variables as its elements) in addition to $X$ (containing the true decision variables as its elements).

Taking the above example, the equation of P3 can be further rewritten as follows:

$$ZX = \begin{bmatrix} z^o_1(3,3, \tau_1) & z^o_2(2,0, \tau_2) & z^o_3(1,0, \tau_3) \\ z^o_1(4,0, \tau_1) & z^o_2(5,4, \tau_2) & z^o_3(2,0, \tau_3) \\ z^o_1(2,0, \tau_1) & z^o_2(5,0, \tau_2) & z^o_3(7,3, \tau_3) \end{bmatrix}_{3 \times 3} X \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{3 \times 1}$$

where

$$Z = \begin{bmatrix} 3 + 3 \times \tau_1 & 2 + 0 \times \tau_2 & 1 + 0 \times \tau_3 \\ 4 + 0 \times \tau_1 & 5 + 4 \times \tau_2 & 2 + 0 \times \tau_3 \\ 2 + 0 \times \tau_1 & 5 + 0 \times \tau_2 & 7 + 3 \times \tau_3 \end{bmatrix}_{3 \times 3}$$

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 5 & 2 \\ 2 & 5 & 7 \end{bmatrix}_{3 \times 3}$$

$$B = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 5 & 2 \\ 2 & 5 & 7 \end{bmatrix}_{3 \times 3} + D^- - D^+$$

$$D^- = \begin{bmatrix} 115 \\ 80 \\ 110 \end{bmatrix}_{3 \times 1}$$

2.2.3. The uncertainties and formulations of %GP

Based on the abovementioned, it can be concluded that most existing approaches tend to solve LP/GP/FP models whose coefficients have the intrinsic uncertainty property, while %GP does not, due to the inherent nature of the RMSGP’s in-between application scenario. The application scenario of the RMSGP involves coefficients which are uncertain, but not intrinsically uncertain. The intrinsic uncertainty property usually implies a non-deterministic decision model, whereas the in-between property leads to a deterministic model. Furthermore, in addition to the solution to the decision vector, per se, the use of RMSGP as a tool obtains more information, such as the values of coefficients and the additional information about how the finally determined coefficients are related to the two ends, and so on, all at best, expressed intuitively. Therefore, the %GP approach is proposed for the solution of RMSGP-modeled problems.

RMSGP can be viewed as a continuous version of the discontinuous MSGP model. Continuous optimization (Pinter, 2009) mainly deals with the objective function domain of definition which is a continuum, whereas in the RMSGP, the coefficient is a continuum. In contrast, discontinuous optimization (Mongeau, 2009) refers to a problem with an objective function, whose domain is a set of discrete points, and discontinuous optimization is a special case of continuous optimization. This is analogous to the relationship between the MSGP (with discontinuous coefficients) and RMSGP (with continuous coefficients). The use of %GP to solve RMSGP models relies on the continuous formulation of the discrete-continuous coefficients of MSGP. This is similar to the optimization problem.
reformulations, from discrete continuous to continuous (Stein, 2009), wherein decisions (i.e., decision variables are continuous instead of having discrete values) that are discrete are replaced by continuous decisions, while the %GP reformulates the coefficients.

In summary, for uncertainty in the LHS constraint coefficients, the scenarios of uncertain intervals (intrinsic uncertain), intrinsic uncertain fuzzy numbers (intrinsic uncertain), uncertain exclusive-or (non-intrinsic uncertain) and uncertain in-between (non-intrinsic uncertain) are illustrated and compared for any given multiplicative term pattern, cijxj in Table 1. There is also a pictorial view in Fig. 2, which shows the major difference between an intrinsic uncertain coefficient and a non-intrinsic uncertain coefficient.

2.3. An example of the %GP decision making scenario

Instead of showing a bunch of mathematical equations, for simplicity (Hempel, 1966) of demonstrating a new RMSGP model and the particular %GP approach to solve it, we examine a brief but fruitful example here.

Retailing company Y has a product-portfolio marketing policy and is now selling two kinds of products (products D and E). Y also adopts a PD policy that differentiates the selling price of product D from its NCs, but there is no such policy for product E. In addition, there is no gap in the total relevant cost when it is selling to VIPCs and NCs, but there is no such policy for Y also adopting a PD policy that differentiates the selling price of product D and E. When this simultaneous equations system is solved, it not only determines X  , which contains the proper quantities of the two product items (i.e., items D and E) to sell, but also determines T  , which contains individual percentages of the two products to be sold to the first group of customers (i.e., VIPC in this case). In addition, the multiplicative term of matrix (T  × X  ) connotes a very significant decision in practice: the product portfolio to exercise. By solving P5, we obtain the true decision variables (x1, x2), the %dm variables (τ1, τ2), and the surpluses/slacks as follows:

\[
X = (B + A \times T)X = B \times X + A \times (T \times X)
\]

As the solution has suggested, Y should sell a quantity of around 521 pieces of product D and a quantity of 1435 pieces of product E. By the way, Y should sell 27.10% of product D (about 141 pieces) and 18.179% of product E (about 261 pieces) to its VIPC and the rest amount of each product to its NC (i.e. sell 380 pieces of product D and 1174 pieces of product E to NC). Under this circumstance (when this marketing policy is adopted and executed), both goals completely achieved with no surpluses or slacks.

This decision can be represented as the matrix form for a clearer view as follows:

And thus,

\[
X' = \begin{bmatrix} 0.271051 \\ 0.1817944 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}
\]

Then another vector further suggests the by-product proper sales amounts to normal customers as follows:

\[
T'' \times X' = \begin{bmatrix} 0.271051 \\ 0.1817944 \end{bmatrix} \approx \begin{bmatrix} 141 \\ 261 \end{bmatrix}
\]

where I is an identical matrix with a proper rank order.
In this section, as an exemplar (Kuhn, 1974), we adopt the case from the MSGP study by Liao (2009). As the example data of the case has been formulated with %GP in the end of Section 2.2.2, we expand it back from the matrix format to the flat simultaneous case has been formulated with %GP in the end of Section 2.2.2, from the MSGP study by Liao (2009). As the example data of the company Y in %GP example.

Table 2

<table>
<thead>
<tr>
<th>Product</th>
<th>Sell to</th>
<th>Property</th>
<th>Value ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item D</td>
<td>VIP</td>
<td>Selling price</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Relevant cost</td>
<td>4</td>
</tr>
<tr>
<td>Normal</td>
<td></td>
<td>Selling price</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Relevant cost</td>
<td>4</td>
</tr>
<tr>
<td>Item E</td>
<td>VIP</td>
<td>Selling price</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Relevant cost</td>
<td>5</td>
</tr>
<tr>
<td>Normal</td>
<td></td>
<td>Selling price</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Relevant cost</td>
<td>4</td>
</tr>
<tr>
<td>Goals</td>
<td></td>
<td>Total selling revenue</td>
<td>As good as 11,500</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Total relevant cost</td>
<td>At best 9000</td>
</tr>
</tbody>
</table>

3. Modeling result comparison between %GP and MSGP

In this section, as an exemplar (Kuhn, 1974), we adopt the case from the MSGP study by Liao (2009). As the example data of the case has been formulated with %GP in the end of Section 2.2.2, we expand it back from the matrix format to the flat simultaneous form as P6 in Appendix B, and then solve it by LINGO (Schrage, 2002) to obtain the solutions as follows:

\[
(x_1^*, x_2^*, x_3^*, \tau_1^*, \tau_2^*, \tau_3^*, d_1^*, d_2^*, d_3^*, d_4^*)
\]

\[
= (16 \frac{2}{7}, 5.0, 4.0, 0.0, 100%, 0%, 0%, 0.3 \frac{2}{7}, 18 \frac{1}{7}, 0, 0, 0)
\]

As we can see, the %GP model suggests a totally different decision than does MSGP. The solution set \((x_1, x_2, x_3, b_1, b_2, b_3) = (11.54, 5.00, 4.46, 0.10, 0.0)\) of MSGP (Liao, 2009) suggests 6 as the possible coefficient for \(x_1\) in g1, 5 as that for \(x_2\) in g2, and 10 as that for \(x_3\) in g3. As stated by the authors of MSGP, g1 has a value of 83.70 (needs to reach 115), g2 has a value of 73.60 with a goal of 80, and g3 has a value of 109.85 with 110 expected.

In contrast, the solutions that are obtained by %GP suggest a 100% bias toward coefficient 6 for \(x_1\) in g1, and a 0% bias toward the coefficient base value 5 for \(x_2\) in g2, and a 0% bias toward the coefficient base value 7 for \(x_3\) in g3. Then, when we examine the accomplishments by %GP, g1 is approaching 115 with 111.714, g2 has a value of 98.143 over the goal 80, and g3 perfectly matches 110 as expected.

The results of the two models are summarized and shown in Table 3. As seen in this table, the decision suggested by the RMSGP model solved by %GP is much closer to the aspired goals in comparison with that suggested by the MSGP model. This implies that the RMSGP can really improve MSGP, on the base of the same assumption (equally-weighted-objectives) and the same data.

Table 3

<table>
<thead>
<tr>
<th>Category</th>
<th>Item</th>
<th>MSGP</th>
<th>RMSGP by %GP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decision variables solved</td>
<td>(x_1^*)</td>
<td>11.54</td>
<td>16.285714</td>
</tr>
<tr>
<td></td>
<td>(x_2^*)</td>
<td>5.00</td>
<td>5.00</td>
</tr>
<tr>
<td></td>
<td>(x_3^*)</td>
<td>4.46</td>
<td>4.00</td>
</tr>
<tr>
<td>Model-dependent variables and suggested coefficient</td>
<td>(b_1\tau_1(c_1))</td>
<td>0 (6)</td>
<td>100% (6)</td>
</tr>
<tr>
<td></td>
<td>(b_2\tau_2(c_2))</td>
<td>1 (5)</td>
<td>0% (5)</td>
</tr>
<tr>
<td></td>
<td>(b_3\tau_3(c_3))</td>
<td>0 (10)</td>
<td>0% (7)</td>
</tr>
<tr>
<td>Aspired levels of goal targets</td>
<td>(g_1)</td>
<td>115</td>
<td>83.70</td>
</tr>
<tr>
<td></td>
<td>(g_2)</td>
<td>80</td>
<td>73.60</td>
</tr>
<tr>
<td></td>
<td>(g_3)</td>
<td>110</td>
<td>73.85</td>
</tr>
<tr>
<td>Surplus/slacks when optimal solutions obtained</td>
<td>(d_1^<em>/d_1^</em>)</td>
<td>-31.30</td>
<td>-3285714</td>
</tr>
<tr>
<td></td>
<td>(d_2^<em>/d_2^</em>)</td>
<td>-6.40</td>
<td>-18.742857</td>
</tr>
<tr>
<td></td>
<td>(d_3^<em>/d_3^</em>)</td>
<td>-0.15</td>
<td>0</td>
</tr>
<tr>
<td>Total deviations induced (all goals are equally weighted)</td>
<td>(\sum d_i^*)</td>
<td>37.85</td>
<td>21.328571</td>
</tr>
</tbody>
</table>

4. Model boundaries and discussions

4.1. The improvement in the solution capability of RMSGP

Using the exemplar case, Section 3 shows the results for the RMSGP model (solved by %GP approach) against the traditional MSGP, as seen in Table 3. The solution method of the MSGP in Liao’s study actually solves a binary variable, in addition to each decision variable in a constraint that implies one of the two extreme ends of a coefficient, because the possible values are exclusive-ORed. The %GP approach of the RMSGP in this study extends the capability of RMSGP model (solved by %GP approach) against the traditional MSGP, as seen in Table 3. The solution method of the MSGP in Liao’s study actually solves a binary variable, in addition to each decision variable in a constraint that implies one of the two extreme ends of a coefficient, because the possible values are exclusive-ORed. The %GP approach of the RMSGP in this study extends the capability of RMSGP model against the traditional MSGP.

4.2. Discussions about the case solutions

If we simply compare the total deviations induced by the two models taking the example data from P1, it is not surprising that we arrive at the conclusion that %GP is far more effective in its capability to reach the aspired goal levels (i.e., we can argue that
in this example case, the achievement ability of %GP surpasses that of MSGP by 43.385% because $21,428571 \times 37.85 = 0.56615$. However, there are still some significant points worth noting as follows:

1. The example data taken from the study of MSGP is somewhat unusual. This leads us to believe that %GP is superior to MSGP. In fact, according to Chang (2007, 2008), the distribution of multiple aspiration levels pertaining to a goal should not be on a large-scale. For example, for multiple-choice aspiration levels 90, 100 and 110, 90–110 is an appropriate scale for a normal DM who is confused about his real target. The possible range of a coefficient should be in a similar scale. Consider a practical scenario in which both MSGP and %GP are applied, wherein a DM who has to sell a product to two groups of customers must decide which group he/she shall sell the product to or to determine what quantity he/she shall sell to the first group and the second. Will the prices marked for sale to the two groups of customers vary that much (e.g. 3 and 6, 5 and 9)? Thus, in practice, the multiple possible coefficients, either for the MSGP and the %GP model, do not vary that much.

2. In contrast to the more reasonable solution obtained in Section 2.2, in Section 3, the %GP model has suggested a biased decision based on the example data from the MSGP research. Suppose a case in which the coefficients are the prices marked for the sale of product types 1–3 to VIPC or NC correspondingly. The solution obtained by %GP suggests that the DM sell 100% of type 1 products to NC, 0% of both types 2 and 3 products to NC (and thus 100% of both types 2 and 3 products to VIPC). This is an extreme case for %GP in which the percentage determined does not move around in-between the minimal and the maximal possible values of a coefficient.) Superficially, the MSGP model suggests another product selling portfolio that is also biased (the binary sequence (0,1,0) results in the either-or-chosen possible coefficients (6, 5, 10)). This forces a decision to sell all the corresponding products of type 1–3 to NC, VIPC, and NC. Nevertheless, MSGP and %GP are by no means the same because MSGP offers no help for the DMs to determine a proper percentage of actual quantities of one type of product to sell to one group of customers.

3. Also, if the matrix $A \times T$ is a square matrix, under most circumstances, there can be ways to decompose it into a matrix $A$ of constants multiplied with a matrix $T$ of %dm variables. In these cases, $ZX = (B + A \times T)X$ is equivalent to $ZX = B \times X + A \times (T \times X)$. When we find the optimal solution for such a %GP model, not only are the true decision variables (in $X'$) and the percentage distance measurement variables (in $T'$) solved, so is the matrix $(T \times X)$ that represents the real decision to be executed. This point is especially meaningful for management as we have pointed out in Section 2.2, for the reason that we have not merely obtained the proper quantities to be planned to sell the product product-wisely, but also, we have also identified the details about the appropriate quantities of products to be sold to diverse market segments group-wisely. That is, in sum, the %GP model obtains solutions suggesting how many items of each type of product should be sold to each market segment, with related information about a proper product portfolio solved all together, at one stop.

4. As revealed in Section 3, the example case modeling of %GP introduces three additional real number %dm variables to the model, while the example case modeling of MSGP involves introducing three additional binary control variables to it. The performances of solutions are not benchmarked in this study. The set of three real number variables and the set of three binary variables do not differ greatly when finding the solution after all.

In sum, %GP is capable of solving RMSGP decision problems in which the coefficients are ranges instead of fixed values or discrete values, and a coefficient can vary continuously within the defined range. The practical means of %GP are also shown in Section 2.2 through solving a simple case that must determine the proper selling quantities, and their details, of two or multiple product types under a product portfolio marketing policy with PD. And the result of solving the exemplar problem in Table 3 has shown a great improvement from MSGP to RMSGP.

4.3. The role of the percentage vector $T$ in management

In the end of this section, we provide a short discussion about $T$, which is the matrix with %dm variables, in more detail. The proposed model solves both $X$ and $T$. Since we claimed in the previous sections that $T$ is “newly introduced”, we eventually found that it plays multiple roles with different functions in the %GP model. Let us examine them according to the following aspects:

1. $T$ as a distance measure of ranged coefficients: The element of $T$ is in fact that $t_i$ pertain to how the ranged coefficient is biased from its starting point when the optimal solution is obtained.

2. $T$ as a multiplicand of the multiplier $X$: As discussed in point 3 previously, $T$ is a multiplicand of $A$ and is a multiplicand of $X$. $A \times T$ is in fact “what the biased range of the coefficient is when the optimal solution is obtained”, and $T \times X$ is in fact “the decision values detailed by the percentage variables” in addition to “the decision values”, which is $X$.

3. $T$ as a percentage distribution measure of a decision in management: According to the former point and as demonstrated in the example in Section 2.2 and the case compared (Section 3), $T$ can measure the proper percentage distribution in relation to the values of true decision variables. This feature is especially useful for DMs who manage decision making problems with product portfolio marketing policy and price discrimination concerns. For example, telling a DM what percentages they should allocate to sell to VIPC and NC among the total sales quantity of product $D$ makes far more sense than directly telling him/her that they should sell certain amounts of items of product $D$ to VIPC and NC, let alone absurdly suggesting that they sell all of product $D$ to VIPC and sell all of product $E$ to NC, which could be the consequence of adopting MSGP. In short, this point implies that the %GP has the capability to offer more management, or even marketing imports and advisories.

5. Conclusion

MCGP has gained wide popularity since it was proposed by Chang (2007, 2008). It allows multiple aspiration levels for the RHS of goal constraints. On the other hand, MSGP, which allows multiple coefficient levels for decision variables in the LHS of goal constraints, has attracted more and more attention since it was proposed by Liao (2009). MCGP has already been enhanced with utility functions by Chang (2011), as well as with fuzzy set theory in a recent study (Bankian-Tabrizi et al., 2012). In contrast, we have seen few extension models of MSGP (Chang, Chen, & Zhuang, 2012).

As a remedy, this study aims to enhance the ability of the MSGP model when it is launched in practice, as well as to shed some light...
on research in this area. The original MSGP model has already been suitable for solving some specific real case problems. For example, companies can use it to determine which market segment (e.g., VIPC or NC7) should be the focus when adopting target marketing or niche marketing policies for various reasons. In contrast, the proposed RMSGP model is suitable for solving more problem cases, or niche marketing policies for various reasons. In contrast, the proposed model in this study will be more practical and can contribute to the study of GP extension models.

%GP is capable solving continuous decision variable coefficients with two ends. It not only conquers the DM’s critical problem between the theoretical modeling and the expectation of DMs in practice. The proposed RMSGP model is suitable for solving more problem cases, or niche marketing policies for various reasons. In contrast, the original MSGP model has already been solved by merely using MSGP, thus filling up the practical gap between the theoretical modeling and the expectation of DMs in practice.

Appendix A

The modeling example that demonstrates the power of %GP’s capability comprehensively in a brief way:

(P5) 
\[ \min z = d^+_1 + d^-_1 + d^+_2 + d^-_2 \]

s.t.
\[ \begin{align*}
(5 + (7 - 5)\tau_{11})x_1 + 6x_2 - d^+_1 + d^-_1 &= 11,500 \\
4x_1 + (5 - (5 - 4)\tau_{22})x_2 - d^+_2 + d^-_2 &= 9000 \\
d^+_i, d^-_i &\geq 0, \forall i \in \{1, 2\} \\
\tau_{11}, \tau_{22} &\geq 0, \tau_{11} = 1, \tau_{22} \leq 1 \\
d^+_i, d^-_i, \tau_{ij} &\in R
\end{align*} \]

Appendix B

The %GP modeling of the example case from the data of Liao’s MSGP study:

(P6) 
\[ \min z = d^+_1 + d^-_1 + d^+_2 + d^-_2 + d^+_3 + d^-_3 \]

s.t.
\[ \begin{align*}
(3 + (6 - 3)\tau_{11})x_1 + 2x_2 + x_3 - d^+_1 + d^-_1 &= 115 \\
4x_1 + (5 + (9 - 5)\tau_{22})x_2 + 2x_3 - d^+_2 + d^-_2 &= 80 \\
3.5x_1 + 5x_2 + (7 + (10 - 7)\tau_{33})x_3 - d^+_3 + d^-_3 &= 110 \\
x_2 + x_3 &\geq 9, \\
x_2 &\geq 5, x_1 + x_2 + x_3 &\geq 21 \\
d^+_i, d^-_i &\geq 0, \forall i \in \{1, 2, 3\} \\
\tau_{11}, \tau_{22}, \tau_{33} &\geq 0, \tau_{11} = 1, \tau_{22}, \tau_{33} \leq 1 \\
d^+_i, d^-_i, \tau_{ij} &\in R
\end{align*} \]

References