Diffusion Tensor Image Smoothing Using Efficient and Effective Anisotropic Filtering

Qing Xu, Adam W. Anderson, John C. Gore and Zhaohua Ding

Vanderbilt University Institute of Imaging Science, Vanderbilt University, 1161 21st Avenue South, Nashville, TN 37232-2310
Qing.xu.1@vanderbilt.edu

Abstract

To improve the accuracy of tissue structural and architectural characterization with diffusion tensor imaging, an anisotropic smoothing algorithm is presented for reducing noise in diffusion tensor images efficiently and effectively. The presented algorithm is based on previous anisotropic diffusion filtering, which is implemented with a straightforward but inefficient explicit numerical scheme. The main contribution of this paper is to improve the performance of the previous method considerably by using unconditionally stable and second order time accurate semi-implicit scheme. Our new method needs only few or even one iteration to achieve better smoothed images than what is generated by tens of iterations of the previous method, which makes it more attractive to practical use. Experiments with simulated and in vivo data have demonstrated the advantage of our new algorithm for denoising diffusion tensor images in terms of efficiency and effectiveness.

1. Introduction

Magnet resonance diffusion tensor imaging (DTI) has established to be a primary technique for non-invasive characterization of the structural and architectural features of living tissue [1]. As DTI is typically performed with echo-planar imaging sequences, the images acquired usually have very poor signal-to-noise ratio (SNR). High image noise is quite detrimental to accurate assessment of tissue property, most notably erroneous calculations of the principal diffusion direction [2] and an overestimate of fractional anisotropy (FA) due to sorting bias [3].

To improve SNR a plethora of image post processing techniques have been proposed for reducing noise in DTI data. These include non-linear diffusion filtering [4, 5], B-spline fitting [6] and more sophisticated regularization methods based on Markovian model [7], variational principles [8, 9], and Riemannian geometry [10], [11], [12]. This repository of smoothing techniques, however, has not established their practical utility due, in part, to the somewhat time-consuming iterative numerical implementation especially when the computation complexity increases with the number of weighting directions, or to a lack of rigorous validation with in vivo DTI data to prove their practical value.

In this work we proposed a highly efficient and effective method for anisotropic smoothing of diffusion tensor images by using an unconditionally stable and second order accurate semi-implicit (Craig-Sneyd) scheme [17]. The unconditional stability allows the use of very large step sizes so that our scheme requires much fewer iterations and thus is more efficient than commonly used schemes to achieve a certain degree of smoothing. Second order time accuracy enables our scheme to reduce noise more effectively than first order schemes [15] with the same total iteration time1. Both efficiency and effectiveness are evaluated quantitatively with simulated and in vivo DTI data. The proposed scheme works specifically for the anisotropic smoothing [13] with tensor diffusivity that allows both image detail enhancement and noise reduction. Although there are other efficient and accurate schemes for scalar diffusivity driven diffusion filtering [15, 16], this is the first effort to apply unconditionally stable and second order accurate schemes to tensor diffusivity driven diffusion filtering.

2. Anisotropic Noise Reduction in Diffusion Tensor Images

As implemented previously [13], noise in DTI data is reduced by anisotropically smoothing the diffusion weighted images (DWI) from which diffusion tensors are derived. The smoothing process is governed by the following diffusion equation:

$$\frac{\partial I_m}{\partial \tau} = \text{div}(T \cdot \nabla I_m)$$

(1)

where $I_m$ is the image intensity in weighting direction $m$.

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1Total iteration time represents accumulated time in the diffusion equation and is a parameter determining the total amount of smoothing. Computation time mentioned later refers to the time consumed on computers.
\( \nabla \) is a gradient operator, \( \text{div} \) is a divergence operator, \( t \) is the time variable. \( T \) is a structure tensor that provides the directionality of smoothing. It is constructed from a common gradient tensor \( G \), which is obtained by convolving the sum of outer product \( (\otimes) \) of \( \nabla I_m \) over all weighting directions with a Gaussian kernel \( K_\rho \):

\[
G = K_\rho * \sum (\nabla I_m \otimes \nabla I_m) \tag{2}
\]

To smooth images isotropically inside structures and anisotropically at structure boundaries, \( T \) is defined to be a normalized inverse of \( G \). Therefore, in homogeneous regions the magnitudes of the three eigenvalues of \( T \) are comparable, yielding a similar amount of smoothing along all directions (isotropic smoothing). At the structure boundaries, the eigenvalue of \( T \) is small across the boundaries, and large along them, thus permitting greater smoothing along the tangential direction of structure boundaries than perpendicular to it (anisotropic smoothing). To allow equal enhancement to homogeneous regions and structure boundaries, the trace of the structure tensor is normalized to be a constant \( C \); consequently, the total amount of smoothing, whether isotropic or anisotropic, is the same over the entire image.

### 3. Semi-implicit Scheme for Anisotropic Filtering

Before introducing the semi-implicit scheme, we first give a brief review of the explicit scheme. The standard explicit scheme for equation 1 is to approximate the time derivatives with forward-time differences and the spatial derivatives with central-space differences as follows,

\[
\frac{I_m^{n+1} - I_m^n}{\Delta t} = \sum_{i,j=1}^{3} \partial_{x_i} (T_{ij} \partial_{x_j} I_m^n) \tag{3}
\]

where \( \partial_{x_i} \) represents the central difference operator with respect to axis \( x_i \), one of the spatial coordinates; \( I_m^n \) is the DWI at time \( n\Delta t \) or \( n \)th iteration; \( T_{ij} \) is the \((i,j)\) component of structure tensor \( T \). The computational cost of each iteration is very low for the above explicit scheme, because \( I_m^{n+1} \) can be directly computed as follows,

\[
\tilde{T}_m^{n+1} = \tilde{T}_m^n + \Delta t \sum_{i,j=1}^{3} L_{ij} (\tilde{T}_m^n) \tilde{T}_m^n \tag{4}
\]

where \( \tilde{T}_m^n \) is a vector representation of \( I_m^n \), and \( L_{ij} \) is a linear operator matrix representing \( \partial_{x_i} (T_{ij} \partial_{x_j} ) \).

In spite of the simplicity, the explicit scheme however requires a very small time step size in order to ensure its stability [18]. This translates to more iterations it needs to take to reach a specific smoothing effect. Furthermore, the above scheme has only first order accuracy in time.

Unlike the explicit scheme, Craig-Sneyd approximates equation 1 with the following semi-implicit scheme:

\[
\bar{A}I_m^{n+1} = (A + B)\tilde{T}_m^n \tag{5}
\]

where

\[
A = \prod_{i,j=1}^{3} (1 - \theta \Delta t L_{ij})
\]

\[
B = \sum_{i,j=1}^{3} \Delta t L_{ij}
\]

and \( \theta \) is a real number that determines the implicitness of the scheme.

Equation 5 is still first order accurate in time because it has a mixed derivative in space and time [17]. Craig-Sneyd scheme employs a further iteration to time-center the mixed derivative, and thus gains second order time accuracy even in the presence of the mixed derivative. The two-iteration Craig-Sneyd scheme can be summarized by

\[
\bar{A}I_m^{n+1} = (A + B)\tilde{T}_m^n + \lambda M (\tilde{T}_m^{n+1} - \tilde{T}_m^n) \tag{6a}
\]

\[
\bar{A}I_m^{n+1} = (A + B)\tilde{T}_m^n + \lambda M (\tilde{T}_m^{n+1} - \tilde{T}_m^n) \tag{6b}
\]

where

\[
M = \Delta t \sum_{i,j=1}^{3} L_{ij}
\]

and \( \lambda \) is a real number.

Equation 6a serves as an estimator that gives an approximate solution for next time step, while equation 6b is actually a corrector that uses a part of the estimated solution to calculate its mixed derivative. The parameter \( \lambda \) controls how much of the mixed derivative is computed based on the estimated solution. Such a scheme is unconditionally stable and second order accurate in both time and space for three dimensional case when \( \lambda = \frac{1}{2} \), \( \theta = \frac{1}{2} \) [17].

In order to solve the linear system (of equations) in equation 6(a) efficiently, it is split into the following three systems:

\[
(1 - \theta \Delta t L_{1,1}) \tilde{T}_m^{n+1/3} = [1 + \theta \Delta t L_{1,1} + \Delta t \sum_{i=2}^{3} L_{i,i} + \Delta t \sum_{i,j=3(i\neq j)}^{3} L_{i,j} \tilde{T}_m^n]
\]

\[
(1 - \theta \Delta t L_{2,2}) \tilde{T}_m^{n+2/3} = \tilde{T}_m^{n+1/3} - (1 - \theta) \Delta t L_{2,2} \tilde{T}_m^n
\]
(1 - \theta \Delta t L_{3,3}) \tilde{T}_{n+1}^m = \tilde{T}_{n+2/3}^m - (1 - \theta) \Delta t L_{3,3} \tilde{T}_n^m \tag{7c}

where \tilde{T}_{n+1/3}^m and \tilde{T}_{n+2/3}^m denote intermediate variables. All the three linear systems in equation 7 are composed of tridiagonal systems that can be efficiently solved by a Thomas algorithm [14].

Similarly, equation 6(b) is split into the three systems below for efficient solutions by Thomas algorithm:

\begin{align*}
(1 - \theta \Delta t L_{2,2}) \tilde{T}_{n+1/3}^m &= [1 + (1 - \theta) \Delta t L_{2,2} + \Delta t \sum_{i,j=1}^3 L_{i,j} + \Delta t(1 - \lambda) \sum_{i,j=1}^3 L_{i,j} ij] \tilde{T}_n^m \\
+ \Delta t \lambda \sum_{i,j=1}^3 L_{i,j} \tilde{T}_{n+1}^m \\
(1 - \theta \Delta t L_{2,2}) \tilde{T}_{n+2/3}^m &= \tilde{T}_{n+2/3}^m - (1 - \theta) \Delta t L_{2,2} \tilde{T}_n^m \tag{8a} \\
(1 - \theta \Delta t L_{3,3}) \tilde{T}_{n+1}^m &= \tilde{T}_{n+2/3}^m - (1 - \theta) \Delta t L_{3,3} \tilde{T}_n^m \tag{8b}
\end{align*}

4. Experiments and Results

In the experiments both the efficiency and effectiveness are quantitatively evaluated on the simulated and in vivo diffusion tensor images. As our primary interest was to restore principal diffusion direction\(^2\) (PDD) for improving the accuracy of fiber tracking, effectiveness was assessed by percent angular difference improvement in PDD. Denoting \(\theta_0\), \(\theta_N\) to be the root mean square (RMS) angular difference of PDD (with respect to clean or “gold standard” data) before and after smoothing respectively, improvement in PDD was defined as:

\[
\frac{\theta_0 - \theta_N}{\theta_0} \times 100\% \tag{9}
\]

Efficiency was evaluated in terms of total computation time consumed by a COMPAQ Presario laptop (Mobile AMD Sempron 2800). Note that the number of iterations is inappropriate for efficiency evaluation, because one iteration of semi-implicit scheme is more computationally expensive than one iteration of explicit scheme.

4.1 Experiments with simulated DTI data

The simulated DTI volume consisted of six slices, each containing “fibers” along different orientations (see Figure 2a for the third slice). The “fibers” had mean diffusivity of \(0.7 \times 10^{-5}\) cm\(^2\)/s, and FA of 0.9, comparable to the measurements in the brain. Then the clean DWIs (64\(\times\)80\(\times\)6) with six weighting directions were corrupted with zero mean Gaussian noise at standard deviation (SD) of 5%, 10% and 15% times the maximum image intensity respectively. The third slice as shown in Figure 2a is chosen for quantitative evaluation, and the RMS angular difference is computed over the whole slice. As the maximum step size for stable implementation of 3D nonlinear anisotropic filtering is \(3/44\) s [18], in this work \(3/44\) s was used as a basic time unit (\(dt_0\)). The semi-implicit scheme with step size \(10dt_0\), \(20dt_0\), \(40dt_0\) and \(80dt_0\) and the explicit scheme with step size \(dt_0\) and \(20dt_0\) are performed on the simulated data.

Table 1. Comparisons of computation time and percent angular difference improvement of the PDD between the semi-implicit and explicit schemes

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Computation Time (seconds)</th>
<th>Percent PDD improvement</th>
<th>Number of iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semi-implicit scheme, total time = (80dt_0)</td>
<td>(153)</td>
<td>92%</td>
<td>(8)</td>
</tr>
<tr>
<td>Step size = (10dt_0)</td>
<td>(77)</td>
<td>93%</td>
<td>(4)</td>
</tr>
<tr>
<td>Step size = (20dt_0)</td>
<td>(38)</td>
<td>93%</td>
<td>(2)</td>
</tr>
<tr>
<td>Step size = (80dt_0)</td>
<td>(19)</td>
<td>85%</td>
<td>(1)</td>
</tr>
<tr>
<td>Explicit scheme, total time = (80dt_0)</td>
<td>(185)</td>
<td>61%</td>
<td>(80)</td>
</tr>
</tbody>
</table>

Table 1. demonstrates that one step of semi-implicit smoothing with step size \(80dt_0\) has much better PDD improvement than 80 steps of explicit smoothing with step size \(dt_0\). Although more computations are involved in each iteration of this semi-implicit scheme, it still gives roughly a ten fold speed-up over the explicit scheme in terms of total computation time. Moreover, the semi-implicit scheme with all step sizes gives greater PDD improvement (~90%) and thus is more effective than the explicit scheme (~60%). This is presumably attributable to the second order accuracy of the semi-implicit scheme. Figure 1 demonstrates that the semi-implicit scheme with step size \(80dt_0\) is stable and has smaller RMS angular difference in PDD than the explicit scheme with step size \(dt_0\) in any specified computation time, while the explicit scheme becomes unstable for step size \(20dt_0\). Figure 2 shows visually that the orientations of PDD are mostly restored with boundary preserved after one step of semi-implicit smoothing at \(40dt_0\).
4.2 Experiments with *in vivo* human DTI data

The *in vivo* data were acquired with a 3T Philips Intera Achieva MR scanner (Netherlands) and an eight-element SENSE coil. A volume of 256×256×120 mm$^3$ with an isotropic resolution of 2×2×2 mm$^3$ was scanned using 32 non-collinear weighting directions and a single shot, echoplanar, pulsed gradient spin echo imaging sequence with a $b$ value of 1000 s/mm$^2$. To generate high SNR data used as a “gold standard”, ten repeated scans were obtained, co-registered, and averaged, which yielded a volume with an SNR of ~75. A block of seven slices in the middle of this high SNR data was corrupted with zero mean Gaussian noise at SD = 5%, 10% and 15% times the maximum DWI intensity for smoothing tests. The middle slice of the test block was chosen for quantitative evaluation. Voxels with FA < 0.3 are excluded from RMS angular difference computation, because they typically reside in the cerebrospinal fluid or gray matter and have a poorly defined PDD regardless of the level of noise. We observe that RMS angular difference in PDD decreases after first several steps of diffusion filtering and then increases gradually due to excessive smoothing that smears out some of the structural details in the DTI data. This problem is in fact common to all diffusion filtering procedures [19]. Therefore, to achieve the best possible smoothing effect, an optimal smoothing time needs to be determined. With this in mind, taking the computational efficiency into consideration as well, one step of the semi-implicit scheme with step size of 5$dt_0$, 10$dt_0$, 15$dt_0$ as optimal choice were performed on images with noise SD = 5%, 10% and 15% respectively. As a comparison, anisotropic diffusion [13] and nonlinear diffusion [4] with the explicit scheme were also performed on the noisy datasets with step size of $dt_0$.

RMS angular difference in PDD is plotted with respect to computation time in Figure 3 for all the three noise levels and all the diffusion filtering schemes used. The performance of semi-implicit scheme is also shown to be superior to the explicit anisotropic filtering and Parker’s nonlinear diffusion in Figure 3 b-c in terms of both the smallest RMS angular difference in PDD and the corresponding computation time. Although the semi-implicit scheme consumes more computation time to reach the best PDD improvement than the explicit scheme for 5% noise level (Figure 3a), its best PDD improvement is almost twice that of explicit schemes. Quantitative analysis shows that the PDD improvement is 27% (1 iteration), 41% (1 iteration) and 50% (1 iteration) for noise at SD = 5%, 10% and 15% respectively with the semi-implicit scheme. In contrast, the best PDD improvement for noise at SD = 5%, 10% and 15% is 12% (5 iterations), 26% (11 iterations) and 37% (21 iterations) respectively for explicit anisotropic filtering, and 10% (8 iterations), 25% (19 iterations) and 34% (27 iterations) respectively for Parker’s nonlinear diffusion filtering. It should be noted that the RMS angular differences are only partly removed due in part to the systematic bias in the “gold standard” data, and to the fact that the angular difference in PDD itself is a biased estimator. Figure 4 evidently illustrates that the disarranged orientations of the PDDs due to SD = 10% noise corruption are largely restored with boundary preservation by one step of semi-implicit smoothing.

![Figure 1: Variations of RMS angular difference in PDD with computation time of smoothing. (a-c) show the results for the simulated data at the noise level SD = 5%, 10% 15% respectively.](image)
Figure 2: The effect of semi-implicit smoothing. The short line in each voxel represents the PDD. (a) PDDs in one middle slice of the clean simulated data. (b) one representative region containing boundary between two different fiber bundles in (a) was zoomed in to make the orientations clearly visualized. (c) PDDs of the same region in the nosied data that was corrupted with zero mean, SD = 10% Gaussian noise and (d) PDDs after one iteration of smoothing on noised data with step size 40Δt0.

Figure 3: Variations of RMS angular difference in PDD with computation time of smoothing. (a-c) show the results for in vivo human data at the noise level SD = 5%, 10% 15% respectively.
Figure 4: The effect of semi-implicit smoothing on in vivo data. (a) PDDs in one middle slice of the clean in vivo data. (b) one representative region in (a) was zoomed in and this region contained the boundary between fiber bundles along anterior-posterior and left-right directions (c) PDDs of the same region in the nosied data that was corrupted with zero mean, SD = 10% Gaussian noise and (d) PDDs after one iteration of smoothing on noised data with step size 10dt.<br>

5. Conclusions

In this paper, we applied a semi-implicit scheme to the previous anisotropic smoothing technique. Experiments with simulated DTI data demonstrate that the semi-implicit scheme can restore the PDD of the diffusion tensor much more efficiently and effectively than the original explicit scheme, with a ten fold speed-up and ~50% increase in the PDD improvement. Experiments with in vivo DTI data show that the best overall performance in terms of both efficiency and effectiveness can be achieved by one iteration of semi-implicit smoothing with a step size proportional to the noise level. This in effect turns the iterative smoothing algorithm into a “non-iterative” procedure, which offers a great potential of practical use.

References